Acta Universitatis Apulensis No. 50/2017
ISSN: 1582-5329 pp. 1-10
http://www.uab.ro/auajournal/ doi: 10.17114/j.aua.2017.50.01

EXACT CONTROLLABILITY FOR THE WAVE PROBLEM WITH ROBIN CONDITIONS ON AN ε- PERIODIC DOMAIN

M. Dumitrache and C. Gheldiu

Abstract. The paper presents the study of the exact controllability on an ε-periodic domain lying along two directions. The exact control is applied on a part of the boundary domain, in the case of the wave problem with Robin conditions. The result is a plane wave problem, with convection term end exactly controlled by a control which represents a combination between the limit of the initial control and the convection of the limit.

2010 *Mathematics Subject Classification*: 35B27, 76M50.

Keywords: two-scale convergence, homogenization, HUM, Robin conditions.

1. Introduction

The article studies in the homogenization of a wave problem with Robin condition controlled by an exact intern control exerted on a part of the border of the structure. The problem was studied on a fixed domain in [3]. The structure is three-dimensional, rectangular type, denoted by Ω, with the inferior base fixed in the plane XOY(or X_1OX_2) and it is consisted of deformable solid. In the interior of the structure we have a parallelepiped which has the same median plan with the initial parallelepiped and in which are distributed empty spheres (holes) with period ε, but only following the directions OX_1 and OX_2. The thickness of the initial parallelepiped is $k\varepsilon \quad (k > 0)$ and the thickness of the included parallelepiped is $hk\varepsilon \quad (0 < h < 1)$, we will denote by Γ_h - the median plan, Γ_ε^+ - the upper face and Γ_ε^- - the base – the lower face. The domain which is occupied by the material is denoted by Ω_ε and it is an ε periodically perforated domain following the directions OX_1 and OX_2 only in the band size $hk\varepsilon$. The domain is similarly to the domain from [5]. On the cover Γ_ε^+ is applied a force v_ε which determines oscillations in whole structure Ω_ε, v_ε satisfying the exact control condition for Ω_ε. We made the construction of the control $v_\varepsilon$using the HUM method introduced by Lions in [4]. For the homogenization of the wave problem we applied the dilatation method and the two-scale convergence method.

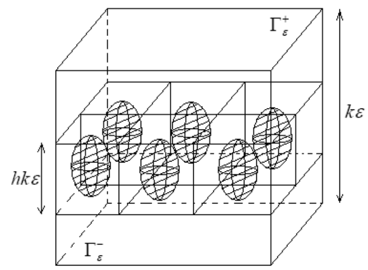

Figure 1: The domain Ω_ε

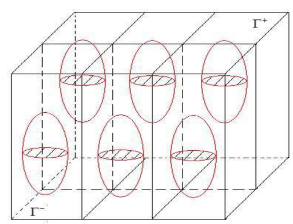

Figure 2: The domain Ω_ε^*

2. The Statement of Problem of the Free Waves with Robin Conditions

First, we made dilatation $z = \frac{x_3}{k\varepsilon}$ that transforms the partial perforated domain Ω_ε into Ω_ε^*, the domain where the base is in the plan X_1OX_2, the superior cover Γ_ε^+ is transformed into Γ^+, the thickness of the structure is 1, and the middle parallelepiped has the thickness h.

We consider the domain Ω_ε^* covered with the grid εY^*, where Y^* is the periodicity cell, definite by $Y^* = Y \backslash T$, where $Y = (0,1)^3$ is the representative cell and T is the hole from the interior of Y, transformed from the initial sphere with the dilatation $z = \frac{x_3}{k\varepsilon}$. We denote by $S_h^{+,-}$ the covers of Y^*. Initial, the cell Y^* is distributed in the parallelepiped Ω with the period ε.

Now, we consider the wave problem on Ω_ε

$$\begin{cases} u_\varepsilon^{''} - \Delta u_\varepsilon + q u_\varepsilon = 0_\varepsilon \text{ in } \Omega_\varepsilon \times (0,T) \\ \frac{\partial u_\varepsilon}{\partial \nu} + a u_\varepsilon = 0 \text{ on } \Gamma_\varepsilon^+ \times (0,T) \\ \frac{\partial u_\varepsilon}{\partial \nu} = 0 \text{ on } \left(\partial T_\varepsilon \cup \partial \Omega_\varepsilon^\infty\right) \times (0,T) \\ u_\varepsilon = 0 \text{ on } \Gamma_\varepsilon^- \\ u_\varepsilon(0) = u_\varepsilon^0, \; u_\varepsilon'(0) = u_\varepsilon^1 \text{ in } \Omega_\varepsilon \end{cases} \tag{1}$$

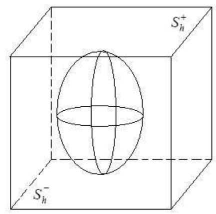

Figure 3: The cell Y^*

where $T_\varepsilon = \varepsilon T$ such that $T_\varepsilon \cap \partial\Omega = \emptyset$, $\partial\Omega_\varepsilon^\infty$ is the lateral border of Ω_ε.

We consider the following conditions satisfied:

i) $q = q\left(\frac{x_1}{\varepsilon}, \frac{x_2}{\varepsilon}\right) = q(y_1, y_2)$ and $0 < m \leq q(y_1, y_2) = q(y_\alpha) \leq M$ a.e. Y^*; $a = a\left(\frac{x_1}{\varepsilon}, \frac{x_2}{\varepsilon}\right) = a(y_1, y_2) = a(y_\alpha)$ with the property $0 < \alpha \leq a(y_\alpha) \leq \beta$ a.e. S_h^+; $q \in L^\infty_{(1,2)per}(Y^*)$, $\quad a \in L^\infty_{(1,2)per}\left(S_h^+\right)$.

ii) $\left(u_\varepsilon^0, u_\varepsilon^1\right) \in V_\varepsilon \times L^2(\Omega_\varepsilon)$ where V_ε is the Hilbert space $V_\varepsilon = \left\{u \in H^1(\Omega_\varepsilon) : u = 0 \text{ on } \Gamma_\varepsilon^-\right\}$, the norm induced by the space $H^1(\Omega_\varepsilon)$, and the condition $u_\varepsilon^0 \in L^2(\Gamma_\varepsilon^+)$.

After the dilatation operation, we multiply the first equation of the system (1) by u'_ε, we integrate by parts on $\Omega_\varepsilon \times (0,T)$ and we obtain

$$\begin{gathered}\frac{1}{2}\int_0^T \frac{d}{dt}\int_{\Omega_\varepsilon^*} \left(u'_\varepsilon\right)^2 dx_\alpha dz dt+ \\ +\frac{1}{2}\int_0^T \frac{d}{dt}\int_{\Omega_\varepsilon^*} \left[\frac{\partial u_\varepsilon}{\partial x_\alpha}\cdot\frac{\partial u_\varepsilon}{\partial x_\alpha} + \frac{1}{(k\varepsilon)^2}\left(\frac{\partial u_\varepsilon}{\partial z}\right)^2\right]^2 dx_\alpha dz dt+ \\ +\frac{1}{2}\int_0^T \frac{d}{dt}\int_{\Omega_\varepsilon^*} \mu\left(\frac{x_\alpha}{\varepsilon}\right)(u_\varepsilon)^2 dx_\alpha dz dt+ \\ +\frac{1}{2}\int_0^T \frac{d}{dt}\int_{\Gamma^+} a\left(\frac{x_\alpha}{\varepsilon}\right)(u_\varepsilon)^2 dx_{\sigma^\varepsilon} dt = 0\end{gathered} \tag{2}$$

and we denote the energy of the system by:

$$E_u(t) = \frac{1}{2}\int_{\Omega_\varepsilon^*} \left(u'_\varepsilon\right)^2 dx_\alpha dz + \frac{1}{2}\int_{\Omega_\varepsilon^*} \left[\frac{\partial u_\varepsilon}{\partial x_\alpha}\cdot\frac{\partial u_\varepsilon}{\partial x_\alpha} + \frac{1}{(k\varepsilon)^2}\left(\frac{\partial u_\varepsilon}{\partial z}\right)^2\right] dx_\alpha dz+$$

$$+\frac{1}{2}\int_{\Omega_\varepsilon^*} q\left(\frac{x_\alpha}{\varepsilon}\right)(u_\varepsilon)^2 dx_\alpha dz + \frac{1}{2}\int_{\Gamma^+} a\left(\frac{x_\alpha}{\varepsilon}\right)(u_\varepsilon)^2 d\sigma^\varepsilon(x_\alpha)$$

so, the relation (2) implies

$$E_u(T) = E_u(0) =$$

$$= \frac{1}{2}\left\|u_\varepsilon^1\right\|^2_{L^2(\Omega^*_\varepsilon)} + \frac{1}{2}\left\|u_\varepsilon^0\right\|^2_{V_\varepsilon} + \frac{1}{2}\int\limits_{\Omega^*_\varepsilon} q\left(\frac{x_\alpha}{\varepsilon}\right)\left(u_\varepsilon^0\right)^2 dx_\alpha dz +$$

$$+\frac{1}{2}\int\limits_{\Gamma^+} a\left(\frac{x_\alpha}{\varepsilon}\right)\left(u_\varepsilon^0\right)^2 d\sigma^\varepsilon(x_\alpha) \leq C_1\left\|u_\varepsilon^0\right\|^2_{V_\varepsilon} + \frac{1}{2}\left\|u_\varepsilon^1\right\|^2_{L^2(\Omega^*_\varepsilon)}.$$

We use the conservation of the energy and the conditions i), ii), and we obtain

$$E_u(t) \leq C$$

so, we get

$$\left\|u_\varepsilon^1\right\|_{L^2(\Omega^*_\varepsilon)} \leq C,\ \ \|u_\varepsilon\|_{V_\varepsilon} \leq C,\ \ \|u_\varepsilon\|_{L^2(\Gamma^+)} \leq C$$

$$\left(\|u_\varepsilon\|_{V_\varepsilon} = \int\limits_{\Omega^*_\varepsilon}\left[\frac{\partial u_\varepsilon}{\partial x_\alpha}\cdot\frac{\partial u_\varepsilon}{\partial x_\alpha} + \frac{1}{(k\varepsilon)^2}\left(\frac{\partial u_\varepsilon}{\partial z}\right)^2\right] dx_\alpha dz\right)$$

which implies the next two-scale convergences:

$$\begin{cases} u_\varepsilon \xrightarrow{2s} u(x_\alpha) \in H^1\left(\Gamma_h^+\right),\ u'_\varepsilon \xrightarrow{2s} u'(x_\alpha) \in H^{-1}\left(\Gamma_h^+\right) \\ \nabla u_\varepsilon \xrightarrow{2s} \nabla_{x_\alpha} u(x_\alpha) + \nabla_{y_\alpha} U(y_\alpha, z) + k^{-1}\nabla_z U(y_\alpha, z) \end{cases}$$

where

$$U \in L^2\left(0, T; H^1_{(1,2)per}(Y)/\mathbb{R}\right)$$

and from ii) we obtain

$$u_\varepsilon^0 \xrightarrow{2s} \frac{u^0(x_\varepsilon)}{(measY^*)},\ u_\varepsilon^1 \xrightarrow{2s} \frac{u^1(x_\alpha)}{(measY^*)}.$$

3. The Homogenization of the Free Waves Problem with Robin Conditions

For problem (1) we apply the two-scale convergence method [1] and we find the plan hyperbolic limit problem:

$$
\begin{gathered}
(measY^*)\, u''(x_\alpha) - \\
-\frac{\partial}{\partial x_\alpha}\left(A_{\alpha\beta}\frac{\partial u}{\partial x_\beta}\right) + b_\alpha \frac{\partial u}{\partial x_\alpha} + \lambda u(x_\alpha) = 0 \text{ in } \Gamma_h^+ \times (0,T), \\
u(x_\alpha) = 0 \text{ on } \partial\Gamma_h^+ \times (0,T), \\
u(0) = \frac{u^0}{measY^*}, u'(0) = \frac{u^1}{measY^*} \text{ in } \Gamma^+,
\end{gathered}
$$

where

$$
y = (y_\alpha, z)
$$

$$
A_{\alpha\beta} = \int_{Y^*} \frac{\partial\left(y_\alpha + \chi^\alpha(y)\right)}{\partial y_\gamma} \cdot \frac{\partial\left(y_\beta + \chi^\beta(y)\right)}{\partial y_\gamma} dy + \frac{1}{k^2}\int_{Y^*} \frac{\partial \chi^\alpha}{\partial z}(y) \cdot \frac{\partial \chi^\beta}{\partial z}(y)\, dy
$$

$$
b_\alpha = -\int_{Y^*}\left[\frac{\partial \gamma}{\partial y_\alpha}(y) + \frac{1}{k}\frac{\partial \gamma}{\partial z}(y)\right] dy + \int_{S_h^+} a(y_\alpha) \cdot \chi^\alpha(y_\alpha, 1)\, d\sigma(y_\alpha)
$$

$$
\lambda = \int_{Y^*} q(y_\alpha)\, dy + \int_{S_h^+} a(y_\alpha) \cdot \gamma(y_\alpha, 1)\, d\sigma(y_\alpha),
$$

where the correctors $\chi^\beta(y)$, $\gamma(y) \in H^1_{(1,2)per}(Y)$, $(\beta = 1,2)$ verifies the weak microscopic problems

$$
\int_{Y^*} \frac{\partial\left(y_\beta + \chi^\beta(y)\right)}{\partial y_\alpha} \cdot \frac{\partial q}{\partial y_\alpha} dy + \frac{1}{k^2}\int_{Y^*} \frac{\partial \chi^\beta}{\partial z}(y) \cdot \frac{\partial q}{\partial z}(y)\, dy = 0,
$$

$$
\begin{gathered}
\int_{Y^*}\left[\frac{\partial \gamma}{\partial y_\alpha}(y) \cdot \frac{\partial q}{\partial y_\alpha}(y) + \frac{1}{k^2}\frac{\partial \gamma}{\partial z}(y) \cdot \frac{\partial q}{\partial z}(y)\right] dy + \\
+\frac{1}{k}\int_{S_h^+} \alpha(y_\alpha) \cdot q(y_\alpha, 1)\, d\sigma(y_\alpha) = 0,
\end{gathered}
$$

$\forall q \in H^1_{(1,2)per}(Y^*/\mathbb{R})$.

4. The HUM Method for the Construction of the Exact Control of the Problem (1)

We consider the system

$$\left\{\begin{array}{l} \phi_\varepsilon'' - \Delta\phi_\varepsilon + q\phi_\varepsilon = 0 \text{ in } \Omega_\varepsilon \times (0,T)\,, \\ \frac{\partial\phi_\varepsilon}{\partial\nu} + a\phi_\varepsilon = 0 \text{ on } \Gamma_\varepsilon^+ \times (0,T)\,, \\ \phi_\varepsilon = 0 \text{ on } \Gamma_\varepsilon^- \times (0,T)\,, \\ \frac{\partial\phi_\varepsilon}{\partial\nu} = 0 \text{ on } \left(\partial T_\varepsilon \cup \partial\Omega_\varepsilon^\infty\right) \times (0,T)\,, \\ \phi_\varepsilon(0) = \phi_\varepsilon^0,\ \phi_\varepsilon'(0) = \phi_\varepsilon^1, \end{array}\right. \tag{3}$$

where $\left(\phi_\varepsilon^0,\, \phi_\varepsilon^1\right) \in L^2(\Omega_\varepsilon) \times V_\varepsilon'$, $\left\|\phi_\varepsilon^0\right\|_{L^2(\Omega_\varepsilon)} \leq C$, $\left\|\phi_\varepsilon^1\right\|_{V_\varepsilon'} \leq C$ and the retrograde system:

$$\left\{\begin{array}{l} y_\varepsilon'' - \Delta y_\varepsilon + qu_\varepsilon = 0 \text{ in } \Omega_\varepsilon \times (0,T)\,, \\ \frac{\partial y_\varepsilon}{\partial\nu} + ay_\varepsilon = -\phi_\varepsilon \text{ on } \Gamma_\varepsilon^+ \times (0,T)\,, \\ y_\varepsilon = 0 \text{ on } \Gamma_\varepsilon^- \times (0,T)\,, \\ \frac{\partial y_\varepsilon}{\partial\nu} = 0 \text{ on } \left(\partial T_\varepsilon \cup \partial\Omega_\varepsilon^\infty\right) \times (0,T)\,, \\ y_\varepsilon(T) = y_\varepsilon'(T) = 0 \text{ in } \Omega_\varepsilon \end{array}\right. \tag{4}$$

and we consider the application

$$\begin{gathered} \Lambda_\varepsilon : F_\varepsilon \to F_\varepsilon' \quad \Lambda_\varepsilon\left(\phi_\varepsilon^0,\, \phi_\varepsilon^1\right) = \left(y_\varepsilon'(0)\,, -y_\varepsilon(0)\right) \Rightarrow \\ \left\langle \Lambda_\varepsilon\left(\phi_\varepsilon^0,\, \phi_\varepsilon^1\right), \left(\phi_\varepsilon^0,\, \phi_\varepsilon^1\right)\right\rangle_{F_\varepsilon^1, F_\varepsilon} = \int\limits_{\Omega_\varepsilon} \left[y_\varepsilon'(0) \cdot \phi_\varepsilon^0 - y_\varepsilon(0) \cdot \phi_\varepsilon^1\right] dx, \end{gathered} \tag{5}$$

where $F_\varepsilon = L^2(\Omega_\varepsilon) \times V_\varepsilon'$ and $F_\varepsilon' = L^2(\Omega_\varepsilon) \times V_\varepsilon$.

We multiply the first equation from the system (4) by ϕ_ε, we integrate by parts two times on $\Omega_\varepsilon \times (0,T)$ and we obtain

$$0 = \int\limits_0^T \int\limits_{\Omega_\varepsilon} \left(y_\varepsilon'' - \Delta y_\varepsilon + qu_\varepsilon\right) \phi_\varepsilon dxdt =$$

$$= \left[\int\limits_{\Omega_\varepsilon} \left(y_\varepsilon'\phi_\varepsilon - y_\varepsilon\phi_\varepsilon'\right) dx\right] \Bigg|_0^T - \int\limits_0^T \int\limits_{\Gamma_\varepsilon^+} \left(\frac{\partial y_\varepsilon}{\partial\nu} \cdot \phi_\varepsilon - y_\varepsilon \cdot \frac{\partial\phi_\varepsilon}{\partial\nu}\right) d\sigma^\varepsilon(x)\, dt +$$

$$+ \int\limits_0^T \int\limits_{\Omega_\varepsilon} y_\varepsilon \left(\phi_\varepsilon'' - \Delta\phi_\varepsilon + q\phi_\varepsilon\right) dxdt =$$

$$= \int\limits_{\Omega_\varepsilon} [y'_\varepsilon(T)\,\phi_\varepsilon(T) - y_\varepsilon(T)\,\phi'_\varepsilon(T) - y'_\varepsilon(0)\,\phi_\varepsilon(0) + y_\varepsilon(0)\,\phi'_\varepsilon(0)]dx + \int\limits_0^T \int\limits_{\Gamma_\varepsilon^+} \phi_\varepsilon^2 d\sigma^\varepsilon(x)dt.$$

Using the relation (5) we have:

$$\left\langle \Lambda_\varepsilon\left(\phi_\varepsilon^0,\, \phi_\varepsilon^1\right),\left(\phi_\varepsilon^0,\, \phi_\varepsilon^1\right)\right\rangle_{F'_\varepsilon, F_\varepsilon} = \int\limits_0^T \int\limits_{\Gamma_\varepsilon^+} \phi_\varepsilon^2 d\sigma^\varepsilon(x)dt =$$

$$= \|\phi_\varepsilon\|^2_{L^2\left(0,T;L^2\left(\Gamma_\varepsilon^+\right)\right)} = \left\|\left(\phi_\varepsilon^0,\, \phi_\varepsilon^1\right)\right\|^2_{F_\varepsilon} \tag{6}$$

we deduce that

$$\left\|\Lambda_\varepsilon\left(\phi_\varepsilon^0,\, \phi_\varepsilon^1\right)\right\|_{F'_\varepsilon} = \left\|\left(\phi_\varepsilon^0,\, \phi_\varepsilon^1\right)\right\|_{F_\varepsilon} = \left(\left\|\phi_\varepsilon^0\right\|^2_{L^2(\Omega_\varepsilon)} + \left\|\phi_\varepsilon^1\right\|^2_{V'_\varepsilon}\right)^{1/2} \leq C \tag{7}$$

it means that Λ_ε is bounded and from relation (6) results that we can apply Lax-Milgram, so that Λ_ε is an isomorphism from F_ε to F'_ε.

Now, we consider the system (1) to which we attach an application $v_\varepsilon = -\phi_\varepsilon$ on Γ_ε^+ and we have

$$\begin{cases} u''_\varepsilon - \Delta u_\varepsilon + q u_\varepsilon = 0 \text{ in } \Omega_\varepsilon \times (0,T)\,, \\ \frac{\partial u_\varepsilon}{\partial \nu} + a u_\varepsilon = \nu_\varepsilon \text{ on } \Gamma_\varepsilon^+ \times (0,T)\,, \\ u_\varepsilon = 0 \text{ on } \Gamma_\varepsilon^- \times (0,T)\,, \\ \frac{\partial u_\varepsilon}{\partial \nu} = 0 \text{ on } \left(\partial T_\varepsilon \cup \partial \Omega_\varepsilon^\infty\right) \times (0,T)\,, \\ u_\varepsilon(0) = u_\varepsilon^0, u'_\varepsilon(0) = u_\varepsilon^1 \text{ in } \Omega_\varepsilon. \end{cases} \tag{8}$$

But Λ_ε is an isomorphism, so result

$$y'_\varepsilon(0) = u_\varepsilon^1,\ y_\varepsilon(0) = u_\varepsilon^0$$

and because $v_\varepsilon = -\phi_\varepsilon$ we observe that y_ε is the solution of the problem (8) which has unique solution, so

$$y_\varepsilon = u_\varepsilon \Rightarrow u_\varepsilon(T) = u'_\varepsilon(T) = 0$$

and the system (1) accepts an exact control $v_\varepsilon \in L^2\left(0,T;L^2\left(\Gamma_\varepsilon^+\right)\right)$.

5. The Limit of the Exact Control

Because $v_\varepsilon = -\phi_\varepsilon$, it is enough to study the convergence of ϕ_ε. The first equation of the system (3) is multiplied with ϕ_ε, then we integrate it by parts on $\Omega_\varepsilon \times (0,T)$, we take into account the conditions satisfied by ϕ_ε^0 and ϕ_ε^1, we find like in section 1:

$$\|\phi_\varepsilon\|_{L^2(0,T;L^2(\Omega_\varepsilon))} \leq C$$

and from equations (6), (7), we find

$$\|\phi_\varepsilon\|_{L^2(0,T;L^2(\Gamma^+))} \leq C$$

and from relation (3) we get the estimation

$$\left\|\phi_\varepsilon^0\right\|_{L^2(\Omega_\varepsilon)} \leq C.$$

From all these relations we obtain the two-scale convergences

$$\phi_\varepsilon \overset{2s}{\to} \phi\left(x_\alpha\right), \ \phi_\varepsilon^0 \overset{2s}{\longrightarrow} \frac{\phi^0\left(x_\alpha\right)}{\left(measY^*\right)}.$$

Because ϕ_ε^1 isn't a regular function, we apply the regularization method of a problem (3), resulting o problem with regular conditions, we compute the limit for it (like for problem (1)) and finally we obtain the next limit problem for the control limit:

$$\begin{cases} \left(measY^*\right)\phi''\left(x_\alpha\right) - \frac{\partial}{\partial x_\alpha}\left(A_{\alpha\beta}\frac{\partial\phi}{\partial x_\beta}\left(x_\alpha\right)\right) + \\ = b_\alpha\frac{\partial\phi}{\partial x_\alpha}\left(x_\alpha\right) + \lambda\phi\left(x_\alpha\right) = 0 \text{ in } \Gamma^+ \times (0,T), \\ \phi = 0 \text{ on } \Gamma^+ \times (0,T), \\ \phi(0) = \frac{\phi^0}{measY^*}, \phi'(0) = \frac{\phi^{1,*}}{measY^*} \text{ in } \Gamma^+ \end{cases}$$

where $\phi^{1,*}\left(x_\alpha\right)$ is equal with

$$\phi^{1,*}\left(x_\alpha\right) = \frac{\partial g_1^*}{\partial x_1}\left(x_\alpha\right) + \frac{\partial g_2^*}{\partial x_2}\left(x_\alpha\right)$$

where we have the convergence

$$\mathrm{g}_\beta^\varepsilon \overset{2s}{\longrightarrow} g^*\left(x_\alpha\right), \ \beta = 1,2$$

and

$$\mathrm{g}_\beta^\varepsilon\left(x_\alpha, z\right) = \frac{\partial\rho_\varepsilon}{\partial x_\alpha} \cdot \frac{\partial\chi^\beta}{\partial x_\alpha}\left(\frac{x_\alpha}{\varepsilon}, z\right) + \frac{1}{k^2}\frac{\partial\rho_\varepsilon}{\partial z}\left(x_\alpha, z\right) \cdot \frac{\partial\chi^\beta}{\partial z}\left(x_\alpha, z\right), \ \beta = 1,2$$

with ρ_ε is the solution of the elliptical problem a little regular:

$$\begin{cases} -\left[\frac{\partial^2\rho_\varepsilon}{\partial x_1^2}+\frac{\partial^2\rho_\varepsilon}{\partial x_2^2}+\frac{1}{k^2}\frac{\partial^2\rho_\varepsilon}{\partial z^2}\right]+ \\ +q\left(\frac{x_\alpha}{\varepsilon}\right)\rho_\varepsilon\left(x_\alpha,z\right)=-\phi_\varepsilon^1 \text{ in } \Omega_\varepsilon^*, \\ \frac{\partial\rho_\varepsilon}{\partial\nu}=0 \text{ on } \left(\partial T_\varepsilon\cup\partial\Omega_\varepsilon^\infty\right), \\ \rho_\varepsilon=0 \text{ on } \Gamma^-, \\ \frac{\partial\rho_\varepsilon}{\partial\nu}+a\left(\frac{x_\alpha}{\varepsilon}\right)\rho_\varepsilon\left(x_\alpha,z\right)=0 \text{ on } \Gamma^+. \end{cases}$$

This regularization method is in [2], and $\phi^{1,*}$ is obtained in [6].

Finally, we obtain a macroscopic problem of controlled waves:

$$\begin{cases} \left(measY^*\right)u''-\frac{\partial}{\partial x_\alpha}\left(A_{\alpha\beta}\frac{\partial u}{\partial x_\beta}\right)+ \\ +b_\alpha\frac{\partial u}{\partial x_\alpha}+\lambda u=F\left(x_\alpha\right) \text{ in } \Gamma^+\times(0,T), \\ u=0 \text{ on } \partial\Gamma^+\times(0,T), \\ u(0)=\frac{u^0}{measY^*}, u'(0)=\frac{u^1}{measY^*} \text{ in } \Gamma^+, \end{cases}$$

where the control of the limit problem is

$$F\left(x_\alpha\right)=\left[\int_{S_h^+}\chi^\beta\left(y_\alpha,1\right)d\sigma\left(y_\alpha\right)\right]\cdot\frac{\partial v}{\partial x_\beta}\left(x_\alpha\right)+\left[\int_{S_h^+}\gamma\left(y_\alpha,1\right)d\sigma\left(y_\alpha\right)\right]\cdot v\left(x_\alpha\right)$$

where $v=-\phi\in L^2\left(0,T;L^2\left(\Gamma^+\right)\right)$.

References

[1] A. Ainouz, *Two-scale homogenization of a Robin problem in perforated media*, Applied Mathematical Sciences, 1, 36 (2007), 1789-1802.

[2] D. Cioranescu, P. Donato, *Exact internal controllability in perforated domains*, J. Math. Pures et Appl., nr 68(1989), 185-213.

[3] V. Komornik, *Exact controllability and stabilization. The multiplier method*, Masson (Paris), Series Editors: P. G. Ciarlet and J.L. Lions (1994).

[4] J.L. Lions, *Controlabilite exacte. Perturbations et stabilization des systemes distribues*, Tomes 1 et 2. Masson(Paris), 1988.

[5] V. Lukes, E. Rohan, *Modeling of acoustic transmission through perforated layer*, Applied and Computational Mechanics 1(2007), 137-142.

[6] L. Tartar, *Compensated compactness and applications to partial differential equations in Nonlinear Analysis and Mechanics*, Heriott Watt Symposium, 4, Pitman n. 39, 136-212.

Mihaela Dumitrache
Department of Mathematics and Computer Science, Faculty of Science, Physical Education and Computer Science,
University of Pitesti,
Pitesti, Romania
email: *mihaela_dumitrache_1@yahoo.com*

Camelia Gheldiu
Department of Mathematics and Computer Science, Faculty of Science, Physical Education and Computer Science,
University of Pitesti,
Pitesti, Romania
email: *camelia.gheldiu@upit.ro*