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HYPERGRAPH RAMSEY NUMBERS INVOLVING PATHS

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Abstract. In this paper, we consider hypergraph Ramsey numbers for t-tight r-uniform paths and r-uniform stars. When the tightness of a path agrees with the cardinality of the center of a star, we give unconditional upper bounds and conditional lower bounds for the associated Ramsey number. Our techniques are sufficiently broad as to allow analogous results when the stars are replaced with certain tripartite hypergraphs. In particular, when $n \equiv 2 \ (mod\ m-1)$, we provide an exact evaluation of $R(P_m, K_{1,1,n-1}-e)$, where $K_{1,1,n-1}-e$ is the complete tripartite graph missing the single edge between the two partite sets having cardinality 1. We conclude by considering the Ramsey numbers for disjoint copies of paths and stars and the problem of obtaining similar results when the tightness of the path is not equal to the cardinality of the center of the star.

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1. Introduction

In 1974, Parsons [6] considered Ramsey numbers of the form $R(P_m, K_{1,n})$, defined to be the minimum $p \in \mathbb{N}$ such that every red/blue coloring of the edges in the complete graph K_p contains a red path on m vertices or a blue star on $n+1$ vertices. Explicit values were determined for all m and n, some using direct methods and others by recurrence. The motivation behind the present paper is to consider the analogous problem in the setting of r-uniform hypergraphs; our work builds upon recent work of Jackowska [4] who has looked at the Ramsey Numbers for paths in the context of 3-uniform hypergraphs. Before we proceed to coloring hyperedges, we turn our attention to the technical definitions of the objects under consideration.

For $r \geq 2$, an r-uniform hypergraph $H = (V(H), E(H))$ is defined to consist of a nonempty set $V(H)$ of vertices and a set $E(H)$ of different unordered r-tuples of distinct vertices. The elements in $E(H)$ are called hyperedges (or r-edges) and are of

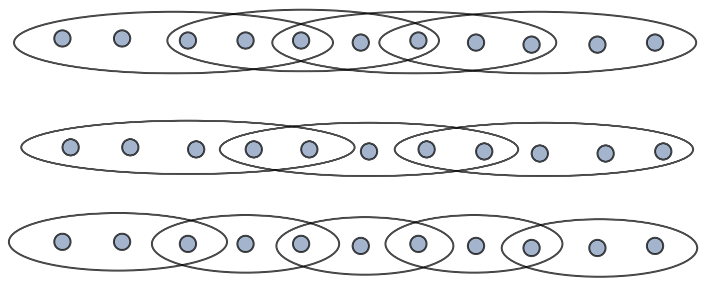

Figure 1: The paths $P^{(5)}_{3,11}$, $P^{(5)}_{2,11}$, and $P^{(3)}_{1,11}$.

the form $x_1x_2\cdots x_r$, where $x_1, x_2, \ldots, x_r$ are distinct vertices in $V(H)$. The degree of a vertex is the number of hyperedges it is contained in. We use the notation $K^{(r)}_p$ to denote the complete r-uniform hypergraph on p vertices in which every subset of r vertices forms a hyperedge.

A t-tight r-uniform path on $m \geq r$ vertices ($1 \leq t < r$), denoted $P^{(r)}_{t,m}$, is a sequence of distinct vertices $v_1 - v_2 - \cdots - v_m$ that form a connected r-uniform hypergraph with vertex set

$$V(P^{(r)}_{t,m}) = \{v_1, v_2, \ldots, v_m\}$$

and hyperedge set

$$E(P^{(r)}_{t,m}) = \{e_1, e_2, \ldots, e_k\}$$

such that for $1 \leq i \leq k$,

$$e_i = v_{(r-t)(i-1)+1}v_{(r-t)(i-1)+2}\cdots v_{(r-t)(i-1)+r}$$

and $m = r + (r-t)(k-1)$. Essentially, $P^{(r)}_{t,m}$ is a connected r-uniform hypergraph that can be formed hyperedge-by-hyperedge, with each new hyperedge including exactly t vertices from the previous hyperedge (with as many vertices as possible coming from the collection of vertices that have not been included in the previous hyperedges). For example, see Figure 1. When $t > \frac{r}{2}$, there exists some vertices in $P^{(r)}_{t,m}$ that have degree 3 or more. Otherwise, all vertices have degree at most 2.

The star $S^{(r)}_{t,n}$ is the r-uniform hypergraph with vertex set given by the disjoint union $V(S^{(r)}_{t.n}) = C \cup U$, where

$$C = \{v_1, v_2, \ldots v_t\} \quad \text{and} \quad U = \{u_1, u_2, \ldots, u_{n-t}\},$$

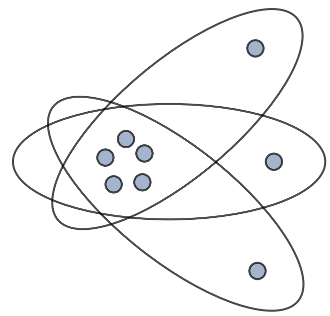

Figure 2: The star $S_{5,8}^{(6)}$.

and hyperedge set consisting of all hyperedges that include all t vertices in C along with $r-t$ vertices from U. For example, see Figure 2. The size of $S_{t,n}^{(r)}$ is given by $\binom{n-t}{r-t}$ and the set C is called the center of $S_{t,n}^{(r)}$.

If H_1 and H_2 are r-uniform hypergraphs, then the Ramsey number $R(H_1, H_2; r)$ is defined to be the least $p \in \mathbb{N}$ such that every red/blue coloring of the hyperedges of the complete r-uniform hypergraph $K_p^{(r)}$ of order p contains a red subhypergraph isomorphic to H_1 or a blue subhypergraph isomorphic to H_2. It will be useful to observe that

$$R(K_r^{(r)}, H_2; r) = |H_2| \tag{1}$$

since any red/blue coloring of a complete hypergraph contains a red $K_r^{(r)}$ precisely when there is some red hyperedge. If all hyperedges are blue, then $|H_2|$ vertices are required to have a blue subhypergraph isomorphic to H_2.

In the next section, we consider the Ramsey numbers $R(P_{t,m}^{(r)}, S_{t,n}^{(r)}; r)$, giving a general upper bound in Theorem 1 and a conditional lower bound in Theorem 2. Section 3 then shifts the focus to analogous results involving certain tripartite hypergraphs that contain the stars under consideration. In Section 4, we consider

Ramsey numbers for disjoint copies of the desired hypergraphs and we conclude in Section 5 with the problem of determining Ramsey numbers when the tightness of the given path is not equal to the central cardinality of the given star.

2. Ramsey Numbers for Paths vs. Stars

In a method of proof similar to that used by Burr in [2] for tree-star Ramsey numbers, and the authors' previous paper [1] (see Theorem 2.2) for hypergraph tree-star Ramsey numbers, we offer the following upper bound for $R(P_{t,m}^{(r)}, S_{t,n}^{(r)}; r)$.

Theorem 1. *For $r \geq 2$ and $1 \leq t < r$,*

$$R(P_{t,m}^{(r)}, S_{t,n}^{(r)}; r) \leq m + n - r.$$

Proof. We proceed by weak induction on the number k of hyperedges in $P_{t,m}^{(r)}$. When $k = 1$, $m = r$, and $P_{t,r}^{(r)}$ consists of just a single hyperedge (ie., it is isomorphic to $K_r^{(r)}$). From (1), it follows that

$$R(P_{t,r}^{(r)}, S_{t,n}^{(r)}; r) = n.$$

Now assume that the theorem is true for paths with k hyperedges:

$$R(P_{t,r+(r-t)(k-1)}^{(r)}, S_{t,n}^{(r)}; r) \leq r + (r-t)(k-1) + n - r = n + (r-t)(k-1).$$

Consider a red/blue coloring of the hyperedges in $K_{n+(r-t)k}^{(r)}$. By the inductive hypothesis, there exists a red $P_{t,r+(r-t)(k-1)}^{(r)}$ or a blue $S_{t,n}^{(r)}$. In the latter case, we are done. In the former case, let

$$v_1 - v_2 - \cdots - v_{r+(r-t)(k-1)}$$

be the sequence of distinct vertices that make up the red $P_{t,r+(r-t)(k-1)}^{(r)}$. There are

$$n + (r-t)k - (r + (r-t)(k-1)) = n - t$$

vertices in the $K_{(r-t)k+n}^{(r)}$ that are not in the red $P_{t,r+(r-t)(k-1)}^{(r)}$, the set of which we denote by $U = \{u_1, u_2, \ldots, u_{n-t}\}$. Now consider all hyperedges in the 2-coloring of $K_{(r-t)k+n}^{(r)}$ that contain all of the t vertices

$$v_{r+(r-t)(k-1)-(t-1)},\ v_{r+(r-t)(k-1)-(t-1)+1},\ \ldots,\ v_{r+(r-t)(k-1)}$$

and any $r-t$ vertices chosen from U. If any such hyperedge is red, it can be used to extend the red $P^{(r)}_{t,r+(r-t)(k-1)}$ to form a red $P^{(r)}_{t,r+(r-t)k}$. Otherwise, they are all blue and we have a blue $S^{(r)}_{t,n}$.

We will now use a constructive approach similar to that used by Parsons (see Theorem 1 of [6]), to obtain the following theorem for paths.

Theorem 2. *If* $n \equiv r \pmod{m-1}$, *then*

$$R(P^{(r)}_{1,m}, S^{(r)}_{1,n}; r) = m + n - r.$$

Proof. Theorem 1 provides the upper bound, so it remains to be shown that

$$R(P^{(r)}_{1,m}, S^{(r)}_{1,n}; r) \geq m + n - r.$$

Write $n - r = k(m-1)$ and consider $k+1$ copies of $K^{(r)}_{m-1}$. Color the hyperedges contained entirely within a copy of $K^{(r)}_{m-1}$ red and all interconnecting hyperedges blue. Clearly, no red $P^{(r)}_{1,m}$ exists. As for the largest blue star with $t = 1$, only $r-1$ vertices can come from the copy of $K^{(r)}_{m-1}$ that the center vertex is contained in (including the center vertex itself). All other vertices may be included. Thus, the largest blue star with $t = 1$ has order $r - 1 + k(m-1) = n - 1$. It follows that

$$R(P^{(r)}_{1,m}, S^{(r)}_{1,n}; r) \geq (k+1)(m-1) + 1 = m + n - r,$$

completing the proof of the theorem.

After proving the previous theorem, one might be tempted to conjecture that the upper bound given in Theorem 1 is always tight. However, this is not the case as can be quickly verified by noting that every 2-coloring of the hyperedges in $K^{(3)}_4$ results in either a red $P^{(3)}_{2,4}$ or a blue $S^{(3)}_{2,4}$. It follows that $R(P^{(3)}_{2,4}, S^{(3)}_{2,4}; 3) = 4$, not 5, which is the upper bound implied by Theorem 1.

While Parsons [6] was able to determine upper bounds for all path-star Ramsey numbers, the methods we have employed in this section do not work for arbitrary values of t. So, in the next section, we change directions to see how our methods may be modified to hypergraph Ramsey numbers involving paths and certain tripartite hypergraphs.

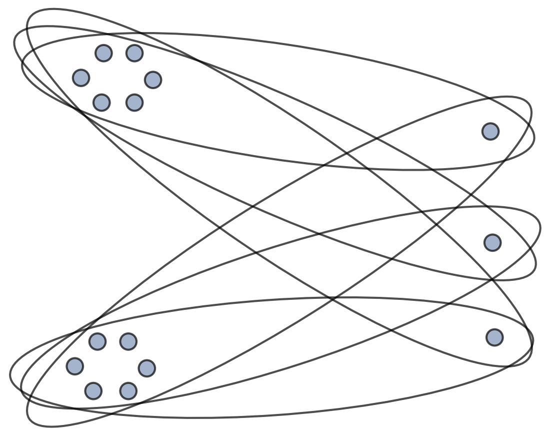

Figure 3: The tripartite hypergraph $H^{(7)}_{6,6,3}$.

3. Ramsey Numbers for Paths vs. Some Tripartite Hypergraphs

In the proof of Theorem 1, the inductive step considered the hyperedges containing the last t vertices in $P^{(r)}_{t,r+(r-t)(k-1)}$ (a t-tight path on k hyperedges) along with $r-t$ other vertices not contained in the path. If any such hyperedge is red, we are able to lengthen the red path by one hyperedge. Otherwise, all such hyperedges are blue and we obtain the desired star. Observing that a similar argument can be made for the other end of the path $P^{(r)}_{t,r+(r-t)(k-1)}$, we are able to strengthen this result from stars to certain tripartite hypergraphs with a few minor assumptions.

We say that an r-uniform hypergraph H is tripartite if $V(H)$ can be partitioned into partite sets V_1, V_2, and V_3 such that no hyperedge contains vertices from only one V_i. Such hypergraphs are weakly 3-colorable by assigning vertices colors based upon which partite set they reside in. The particular tripartite hypergraph that we will focus on is denoted $H^{(r)}_{t,t,c}$, where $1 \leq t < r$ and $r-t \leq c$. Its vertex set $V(H^{(r)}_{t,t,c})$ can be partitioned into partite sets V_1, V_2, and V_3 having cardinalities $|V_1| = t = |V_2|$ and $|V_3| = c$. The hyperedge set $E(H^{(r)}_{t,t,c})$ consists of all hyperedges that include all t vertices in V_1, along with all subsets of $r-t$ vertices from V_3, and hyperedges that include all t vertices from V_2, along with all subsets of $r-t$ vertices from V_3.

The hypergraph $H^{(r)}_{t,t,c}$ contains exactly $2\left(\begin{array}{c} c \\ r-t \end{array}\right)$ hyperedges and it can be observed that $H^{(r)}_{t,t,c}$ contains a subhypergraph isomorphic to $S^{(r)}_{t,c+t}$. For example, Figure 3 shows the hypergraph $H^{(7)}_{6,6,3}$ and it can be noted that $S^{(7)}_{6,9}$ is a subhypergraph. Thus, any coloring which avoids a $S^{(r)}_{t,c+t}$ in a given color also avoids an $H^{(r)}_{t,t,c}$ in the same color. From (1), it follows that

$$R(P^{(r)}_{t,r}, H^{(r)}_{t,t,n-t}) = n + t.$$

For larger paths, it becomes necessary to restrict to the cases in which $1 \leq t \leq \frac{r}{2}$. As the base case of our next theorem, we will need the fact that

$$R(P^{(r)}_{t,2r-t}, H^{(r)}_{t,t,n-t}) \leq n - t + r \quad \text{whenever} \quad 1 \leq t \leq \frac{r}{2}. \tag{2}$$

To see this, consider a red/blue coloring of $K^{(r)}_{n-t+r}$. If all hyperedges are blue, then there is certainly a blue $H^{(r)}_{t,t,n-t}$. Otherwise, let $e = v_1v_2\cdots v_r$ be a red hyperedge. There are $n-t$ other vertices, the set of which we denote V_3. Let

$$V_1 = \{v_1, v_2, \ldots, v_t\} \quad \text{and} \quad V_2 = \{v_{r-t+1}, v_{r-t+2}, \ldots, v_r\}$$

and observe that they are disjoint because of our assumption. Consider the hyperedges that include all vertices in V_1 and and $r-t$ vertices from V_3. If any such hyperedge is red, we obtain a red $P^{(r)}_{2r-t}$. Otherwise they are all blue. Also, consider the hyperedges that include all vertices from V_2 and any $r-t$ vertices from V_3. The same observation holds and we find that we either produce a red $P^{(r)}_{2r-t}$ or a blue $H^{(r)}_{t,t,n-t}$. While we may view stars as subhypergraphs of the tripartite hypergraphs we are considering, it is worth noting that Theorem 1 is not a consequence of the following theorem due to the necessary assumptions on t.

Theorem 3. *For $r \geq 2$, $m > r$, and $1 \leq t \leq \frac{r}{2}$,*

$$R(P^{(r)}_{t,m}, H^{(r)}_{t,t,n-t}; r) \leq m + n - r.$$

Proof. As in the proof of Theorem 1, we proceed by weak induction on $k \geq 2$, the number of hyperedges in $P^{(r)}_{t,m}$. Equation (2) provides us with the base case and the assumption $1 \leq t \leq \frac{r}{2}$ guarantees the existence of two disjoint sets of t vertices that can be used to form the partite sets V_1 and V_2 in $H^{(r)}_{t,t,n-t}$. Now assume that the theorem is true for paths having k hyperedges:

$$R(P^{(r)}_{t,r+(k-1)(r-t)}, H^{(r)}_{t,t,n-t}; r) \leq r + (k-1)(r-t) + n - r = (k-1)(r-t) + n.$$

Consider a red/blue coloring of the hyperedges in $K^{(r)}_{k(r-t)+n}$. From the inductive hypothesis, there exists a red $P^{(r)}_{t,r+(k-1)(r-t)}$ or a blue $H^{(r)}_{t,t,n-t}$. In the former case, suppose the red path is given by

$$v_1 - v_2 \cdots - v_{r+(k-1)(r-t)}$$

and let

$$V_1 = \{v_1, v_2, \ldots, v_t\}$$

and

$$V_2 = \{v_{r+(r-t)(k-1)-(t-1)},\ v_{r+(r-t)(k-1)-(t-1)+1},\ \ldots,\ v_{r+(r-t)(k-1)}\}.$$

Then V_1 and V_2 each have cardinality t and are disjoint since $1 \leq t \leq \frac{r}{2}$. There exists

$$k(r-t) + n - (r + (k-1)(r-t)) = n - t$$

vertices not contained in the red path and we refer to the set of such vertices as V_3. If any hyperedge containing all of the vertices in V_1 (or all of the vertices in V_2) along with any subset of $r-t$ vertices from V_3 is red, we obtain a red $P^{(r)}_{t,r+k(r-t)}$. Otherwise, they are all blue, and we obtain a blue $H^{(r)}_{t,t,n-t}$.

This general theorem leads us to the following result for graphs. We use the usual notation $K_{a,b,c}$ to denote the complete tripartite graph having partite sets with cardinalities a, b, and c.

Theorem 4. *If* $n \equiv 2 \pmod{m-1}$, $m > 2$, *and* $n \geq 2$ *then*

$$R(P_m, K_{1,1,n-1} - e) = m + n - 2,$$

where $K_{1,1,n-1} - e$ *is the complete tripartite graph missing a single edge between the two partite sets of cardinality* 1.

Proof. The upper bound is immediate from Theorem 3, thus it suffices to show that this bound is tight. By Theorem 2, it follows that

$$R(P_m, K_{1,n-1}) \geq m + n - 2,$$

and since $S_{1,n}$ is a subgraph of $K_{1,1,n-1} - e$, we obtain the necessary lower bound, proving equality.

4. Multiple Disjoint Copies of Paths and Stars/Tripartite Hypergraphs

The following lemma was originally proved by Burr, Erdős, and Spencer in [3] in the context of graphs. However, their proof is identical for any uniformity (see the proof in [5] for example), and so we omit it here.

Lemma 5. *Let F, U and H be r-uniform hypergraphs with $|V(U)| = u$ and $|V(H)| = h$. Then if $a, b \geq 1$ the following two inequalities hold:*

1. $R(F, U \cup H) \leq \max\{R(F,U) + h, R(F,H)\}$,
2. $R(aU, bH) \leq R(U,H) + (a-1)u + (b-1)h$.

Combining Theorem 3 with Lemma 5, we obtain the following immediate corollary.

Corollary 6. *For $r \geq 2$ and $a, b \geq 1$, we have*

$$R(aP_{t,m}^{(r)}, bS_{t,n}^{(r)}; r) \leq am + bn - r.$$

Furthermore, if we assume that $1 \leq t \leq \frac{r}{2}$, then

$$R(aP_{t,m}^{(r)}, bH_{t,t,n-t}^{(r)}; r) \leq am + bn - r + (b-1)t.$$

Finally, we will improve this upper bound slightly in the special case where $a = 1$ and $b = 2$ in the following theorem.

Theorem 7. *If $m > r \geq 2$ and $1 \leq t \leq \frac{r}{2}$, then*

$$R(P_{t,m}^{(r)}, 2S_{t,n}^{(r)}; r) \leq m + 2n - r - t.$$

Proof. Once again, we proceed by induction on k, the number of hyperedges in $P_{t,m}^{(r)}$. When $m = r$, equation (1) implies that

$$R(P_{t,r}^{(r)}, 2S_{t,n}^{(r)}; r) = 2n, \tag{3}$$

which does not agree with the bound given in the statement of the theorem. Hence, we must assume that $m > r$ (ie., $k > 1$). For $k = 2$, $m = 2r - t$, and we consider a red/blue coloring of the hyperedges in $K_{m+2n-r-t}^{(r)} = K_{r+2n-2t}^{(r)}$. With the assumption $1 \leq t \leq \frac{r}{2}$, we have at least $2n$ vertices and can apply (3), from which there exists a red $P^{(r)}_{t,r}$ or a blue $2S^{(r)}_{t,n}$. Assume the former case and denote the single red edge by $u_1u_2\cdots u_r$. Besides these vertices, there are $2(n-t)$ other vertices, which we partition into two disjoint sets U_1 and U_2 that each have cardinality $n-t$. Consider the hyperedges that include $u_1, u_2, \dots, u_t$ along with all of the vertices in U_1. If any such hyperedge is red, we obtain a red $P^{(r)}_{t,2r-t}$. Otherwise, all such hyperedges are blue and we obtain a blue $S^{(r)}_{t,n}$. Apply this same logic to the vertices $u_{r-t+1}, u_{r-t+2}, \dots, u_r$ along with all of the vertices in U_2. If any such hyperedge is red, we obtain a red $P^{(r)}_{t,2r-t}$. Otherwise, they are all blue and we obtain a blue $S^{(r)}_{t,n}$ that is disjoint from the other blue star. This handles our base case ($k=2$). Now suppose the theorem is true for a given $k \geq 2$:

$$R(P^{(r)}_{t,r+(k-1)(r-t)}, 2S^{(r)}_{t,n}; r) \leq (k-1)r + 2n - kt.$$

Now consider a red/blue coloring of the hyperedges in $K_{kr+2n-(k+1)t}$. By the inductive hypothesis, there exists a red $P^{(r)}_{t,r+(k-1)(r-t)}$ or a blue $2S^{(r)}_{t,n}$. Assume the former case and denote the vertices in the red path by

$$v_1 - v_2 - \cdots - v_{r+(k-1)(r-t)}.$$

There exists

$$kr + 2n - (k+1)t - (r + (k-1)(r-t)) = 2(n-t)$$

vertices not contained in this path, which we partition into two disjoint sets V_1 and V_2. Now we proceed as in the base case and consider the hyperedges formed using $v_1, v_2, \dots, v_t$ along with all of the vertices in V_1 and the hyperedges formed using

$$v_{r+(k-1)(r-t)-(t-1)}, v_{r+(k-1)(r-t)-(t-2)}, \dots, v_{r+(k-1)(r-t)}$$

along with all of the vertices in V_2. If any such hyperedge is red, we obtain a red path having $k+1$ hyperedges. Otherwise, all such hyperedges are blue and we obtain two disjoint blue stars on n vertices.

5. Paths and Stars Having Unequal Tightness and Center Cardinality

The techniques that we have used throughout this paper have all required that the tightness of the path under consideration be the same as the number of vertices contained in the center of the desired stars (or the cardinalities of two partite sets in certain tripartite hypergraphs). This leaves open the question of what can be done for Ramsey numbers involving “mismatched” values of t. That is, our techniques do not apply to Ramsey numbers involving paths and stars in which the tightness of the given path does not agree with the center cardinality of the star. We offer the following two theorems describing such cases.

Theorem 8. *Let* $r \geq 2$ *and* $n \geq r$*. Then*

$$R(P^{(r)}_{1,2r-1}, S^{(r)}_{r-1,n}; r) \leq 2n - 1.$$

Proof. Let $k = 2n - 2(r-1) - 1$ (note that k is odd). Consider an arbitrary red/blue coloring of the hyperedges in $K^{(r)}_{2r+k-2}$. Partition the vertex set into the disjoint union of three sets, A, B, and C, such that $|A| = r - 1 = |B|$ and $|C| = k$, where $k \geq 1$ is assumed to be odd. Let

$$A = \{x_1, x_2, \ldots, x_{r-1}\} \quad \text{and} \quad B = \{y_1, y_2, \ldots, y_{r-1}\}.$$

First, we consider the hyperedges that include all of the vertices in A and a single vertex from C. Denote the set of such hyperedges by E_A and define E_B analogously. If E_A and E_B each contain at least $\frac{k+1}{2}$ red hyperedges, then by the pigeonhole principle, there exist some $z \in C$ such that the hyperedges

$$x_1 x_2 \cdots x_{r-1} z \quad \text{and} \quad y_1 y_2 \cdots y_{r-1} z$$

are both red, forming a red $P^{(r)}_{1,2r-1}$. Otherwise, at least one of E_A and E_B contains at most $\frac{k-1}{2}$ red hyperedges (without loss of generality, assume it is E_A). Then E_A contains at least $\frac{k+1}{2}$ blue hyperedges, which form a blue $S^{(r)}_{r-1,r-1+\frac{k+1}{2}}$. Solving for n, $n = r - 1 + \frac{k+1}{2}$, gives the desired result.

Theorem 9. *Let* $r \geq 3$ *and* $n \geq r$*. Then*

$$R(P^{(r)}_{1,3r-2}, S^{(r)}_{2,n}; r) \leq n + 2(r-1).$$

Proof. Consider a red/blue coloring of the hyperedges in $K^{(r)}_{n+2(r-1)}$ that lacks a red $P^{(r)}_{1,3r-2}$. If this coloring lacks a blue $S^{(r)}_{2,n}$, then there must be some red hyperedge $e_1 = x_1 x_2 \cdots x_r$. Removing the vertices in e_1 results in a red/blue coloring of $K^{(r)}_{n+r-2}$, which must also contain a red hyperedge $e_2 = y_1 y_2 \cdots y_r$ if a blue $S^{(r)}_{2,n}$ is avoided. Thus, the original red/blue coloring of $K^{(r)}_{n+2(r-1)}$ contains nonadjacent red hyperedges e_1 and e_2. Now consider the hyperedges that include x_i and y_j along with $r-2$ vertices from the $n-2$ not included in e_1 or e_2. If any one of these hyperedges is red, then we can form a red $P^{(r)}_{1,3r-2}$ using e_1 and e_2. Otherwise, all such hyperedges are blue and we obtain a blue $S^{(r)}_{2,n}$.

Of course, the methods employed in the previous proofs do not readily extend to other tightnesses and center cardinalities. Thus, we conclude by encouraging the reader to consider the problem of determining upper and lower bounds for

$$R(P^{(r)}_{t_1,m}, S^{(r)}_{t_2,n}; r)$$

(or more generally, $R(P^{(r)}_{t_1,m}, H^{(r)}_{t_2,t_2,n-t_2}; r)$) when $t_1 \neq t_2$.

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