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APPLICATION OF THE (G'/G)-EXPANSION METHOD FOR THE NONLINEAR DRINFELD-SOKOLOV AND GENERALIZED DRINFELD-SOKOLOV EQUATIONS

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Abstract. In this article, we establish the exact solutions for nonlinear Drinfeld-Sokolov (DS) and generalized Drinfeld-Sokolov (gDS) equations. Generalized $(\frac{G'}{G})$-expansion method is proposed to seek exact solutions of nonlinear evolution equations. This method is used to construct solitary and soliton solutions of nonlinear evolution equations. Also, for finding exact solutions are expressed three types of solutions that include hyperbolic function solution, trigonometric function solution and rational solution. The exact solutions with solitons and periodic structures are obtained. These solutions might play important role in engineering and physics fields. It is shown that this method, with the help of symbolic computation, provide a straightforward and powerful mathematical tool for solving problems in fluids science.

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1. Introduction

The investigation of the travelling wave solutions plays an important role in nonlinear sciences. In the recent decade, the study of nonlinear partial differential equations in modelling physical phenomena, has become an important tool. A variety of powerful methods has been presented, such as Hirota's bilinear method [1], the inverse scattering transform [2], sine-cosine method [3], homotopy perturbation method [20], homotopy analysis method [5, 6], variational iteration method [7, 8, 9], the $(\frac{G'}{G})$-expansion method [10, 11], tanh-function method [12], tanh-coth method [13, 14], Bäcklund transformation [15, 16], Exp-function method [17, 18, 19, 20, 21] and so on. Wang [22] introduced a new method called the $(\frac{G'}{G})$-expansion method to look for travelling wave solutions of NLEEs. Zhang et al. [23] examined the generalized $(\frac{G'}{G})$-expansion method and its applications. Authors of [24] used to mKdV equation with variable coefficients using the $(\frac{G'}{G})$-expansion method. Also, Bekir [25] used to application of the $(\frac{G'}{G})$-expansion method for nonlinear evolution equations. In this article we explain method which is called the $(\frac{G'}{G})$-expansion method to look for travelling wave solutions of nonlinear evolution equations. Here, we consider Drinfeld-Sokolov (DS) system and generalized Drinfeld-Sokolov (gDS) equations, as follow [27]

$$u_t + (v^2)_x = 0, \qquad v_t - av_{xxx} + 3bu_x v + 3kuv_x = 0, \tag{1}$$

where a, b and k are constants. Drinfeld-Sokolov system was introduced by Drinfeld and Sokolov as an example of a system of nonlinear equations possessing Lax pairs of a special form [26]. Next we consider family of generalizations of the Drinfeld-Sokolov (DS) equation following

$$u_t + (v^n)_x = 0, \qquad v_t - av_{xxx} + 3bu_x v + 3kuv_x = 0, \tag{2}$$

where a, b, n and k are constants. Here our aim is the determination of travelling wave solutions with compact and noncompact structures for the DS system, a generalized form of the DS system, and one type different of the DS system. Our aim of this paper is to obtain analytical solutions of nonlinear Drinfeld-Sokolov (DS) and generalized Drinfeld-Sokolov (gDS) equations and to determine the accuracy of the (G'/G)-expansion method in solving these kind of problems. The paper is organized as follows: In Section 2, we describe the (G'/G)-expansion method. In sections 3 and 4, we examine the application of aforementioned method for solving two nonlinear evolution equations. Also conclusion is given in Section 5. Finally some references are given at the end of this paper.

2. Basic idea of the (G'/G)-expansion method

We give a detailed description of method which was first presented by Wang [22].

Step 1. For a given NLPDE with independent variables $X = (x, t)$ and dependent variable u:

$$\mathcal{P}(u, u_t, u_x, u_{xx}, u_{tt}, u_{tx}, ...) = 0, \tag{3}$$

can be converted to an ODE

$$\mathcal{M}(u, -cu', u', u'', c^2u'', -cu'', ...) = 0, \tag{4}$$

the transformation $\xi = x - ct$ is wave variable. Also, c is constant to be determined later.
Step 2. We seek its solutions in the more general polynomial form as follows:

$$u(\xi) = a_0 + \sum_{k=1}^{m} a_k \left(\frac{G'(\xi)}{G(\xi)}\right)^k, \tag{5}$$

where $G(\xi)$ satisfies the second-order LODE in the form

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \tag{6}$$

where a_0, $a_k(k = 1, 2, ..., m)$, λ and μ are constants to be determined later, $a_m = 0$, but the degree of which is generally equal to or less than $m-1$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in Eq. (4).
Step 3. Substituting (5) and (6) into Eq. (4) with the value of m obtained in Step 1. Collecting the coefficients of $\left(\frac{G'(\xi)}{G(\xi)}\right)^k$ $(k = 0, 171, 172, ...)$, then setting each coefficient to zero, we can get a set of over-determined partial differential equations for $a_0, a_i(i = 1, 2, ..., n), \lambda, c$ and μ with the aid of symbolic computation *Maple*.
Step 4. Solving the algebraic equations in Step 3, then substituting $a_i, ..., a_m, c$ and general solutions of Eq. (6) into (5) we can obtain a series of fundamental solutions of Eq. (3) depending of the solution $G(\xi)$ of Eq. (6).

3. The Drinfeld-Sokolov Equation

We first consider the Drinfeld-Sokolov equation with the $(\frac{G'}{G})$-expansion method to the following time-dependent one dimensional DS equation

$$u_t + (v^2)_x = 0, \qquad v_t - av_{xxx} + 3bu_x v + 3kuv_x = 0, \tag{7}$$

where a, b and k are constants. And the wave variable $\eta = x - ct$ PDE transforms to an ODE

$$-cu' + (v^2)' = 0, \qquad cv' + av''' - 3bu'v - 3kuv' = 0, \tag{8}$$

where by integrating the first equation in the Eq. (8) and neglecting the constant of integration we get

$$cu = v^2. \tag{9}$$

Substituting (9) into the second equation of the system (8) and integrating we find

$$c^2 v + acv'' - (2b + k)v^3 = 0. \tag{10}$$

In order to determine value of m, we balance the linear term of the highest order v'' with the highest order nonlinear term v^3 in Eq. (10) we get

$$v^3(\xi) = a_m^3 \left(\frac{G'(\xi)}{G(\xi)}\right)^{3m} + ..., \tag{11}$$

$$v_\xi(\xi) = -ma_m \left(\frac{G'(\xi)}{G(\xi)}\right)^{m+1} + ...,$$

$$v_{\xi\xi}(\xi) = m(m+1)a_m \left(\frac{G'(\xi)}{G(\xi)}\right)^{m+2} + ...$$

In order to determine value of m, we balance v'' with v^3 in Eq. (10) we have

$$m + 2 = 3m, \tag{12}$$

we find $m = 1$. We can suppose that the solution of Eq. (10) is of the form

$$v(\xi) = a_0 + a_1 \left(\frac{G'(\xi)}{G(\xi)}\right), \qquad a_1 \neq 0, \tag{13}$$

and therefore

$$v^3(\xi) = a_1^3 \left(\frac{G'(\xi)}{G(\xi)}\right)^3 + 3a_0 a_1^2 \left(\frac{G'(\xi)}{G(\xi)}\right)^2 + 3a_0^2 a_1 \left(\frac{G'(\xi)}{G(\xi)}\right) + a_0^3, \tag{14}$$

and

$$v_{\xi\xi}(\xi) = 2a_1 \left(\frac{G'(\xi)}{G(\xi)}\right)^3 + 3a_1\lambda \left(\frac{G'(\xi)}{G(\xi)}\right)^2 + (a_1\lambda^2 + 2a_1\mu) \left(\frac{G'(\xi)}{G(\xi)}\right) + a_1\lambda\mu. \tag{15}$$

Substituting (13)–(15), and by using the well-known Maple software, we obtain the following results

$$a_0 = \frac{\lambda}{2}\sqrt{\frac{2ac}{2b+k}}, \qquad a_1 = \sqrt{\frac{2ac}{2b+k}}, \qquad c = \frac{a(\lambda^2 - 4\mu)}{2}, \qquad \mu = \mu, \qquad \lambda = \lambda. \tag{16}$$

Substituting (16) into expression (13), we get

$$v(\xi) = \frac{\lambda}{2}\sqrt{\frac{2ac}{2b+k}} + \sqrt{\frac{2ac}{2b+k}} \left(\frac{G'(\xi)}{G(\xi)}\right), \qquad \xi = x - \frac{a(\lambda^2 - 4\mu)}{2}t. \tag{17}$$

Substituting the general solutions of Eq. (6) into (17) we have three types of exact solutions of (7) as follows:
(1) When $\lambda^2 - 4\mu > 0$, we obtain hyperbolic function solution

$$v_1(x,t) = \frac{a(\lambda^2-4\mu)}{\sqrt{2b+k}}\left(\frac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right)}\right) \tag{18}$$

$$+\frac{\lambda}{2}\sqrt{\frac{2ac}{2b+k}}, \qquad \xi = x - \frac{a(\lambda^2-4\mu)}{2}t,$$

$$u_1(x,t) = \frac{a}{2(2b+k)}\left(\sqrt{\lambda^2-4\mu}\left(\frac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right)}\right)+\lambda\right)^2,$$

where $\xi = x - \frac{a(\lambda^2-4\mu)}{2}t$.
(2) When $\lambda^2 - 4\mu < 0$, we have trigonometric function solution

$$v_2(x,t) = \frac{a(4\mu-\lambda^2)}{\sqrt{2b+k}}\left(\frac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)}\right) \tag{19}$$

$$+\frac{\lambda}{2}\sqrt{\frac{2ac}{2b+k}}, \qquad \xi = x - \frac{a(\lambda^2-4\mu)}{2}t,$$

$$u_2(x,y) = \frac{a}{2(2b+k)}\left(\sqrt{4\mu-\lambda^2}\left(\frac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)}\right)+\lambda\right)^2,$$

where $\xi = x - \frac{a(\lambda^2-4\mu)}{2}t$.
(3) When $\lambda^2 - 4\mu = 0$, we get rational solution

$$v_3(x,t) = \sqrt{\frac{2ac}{2b+k}}\frac{C_2}{(C_1+C_2\xi)} + \frac{\lambda}{2}\sqrt{\frac{2ac}{2b+k}}, \qquad \xi = x - \frac{a(\lambda^2-4\mu)}{2}t, \tag{20}$$

$$u_3(x,t) = \frac{2a}{2b+k}\left(\frac{C_2}{C_1+C_2\xi}+\frac{\lambda}{2}\right)^2.$$

If $C_1 \neq 0, C_2 = 0, \lambda > 0, \mu = 0$, then (18) gives

$$v_4(x,t) = \frac{a\lambda^2}{\sqrt{2b+k}}\left(1+\tanh\frac{\lambda}{2}\xi\right), \qquad \xi = x - \frac{a\lambda^2}{2}t, \tag{21}$$

$$u_4(x,t) = \frac{a\lambda^2}{2(2b+k)}\left(1+\tanh\frac{\lambda}{2}\xi\right)^2, \qquad \xi = x - \frac{a\lambda^2}{2}t.$$

In particular, if $\lambda = 0$

Case 1: $\mu < 0$.

$$v_5(\xi) = \sqrt{\frac{-2ac\mu}{2b+k}}\left(\frac{C_1\sinh\sqrt{-\mu}\xi + C_2\cosh\sqrt{-\mu}\xi}{C_1\cosh\sqrt{-\mu}\xi + C_2\sinh\sqrt{-\mu}\xi}\right), \tag{22}$$

$$u_5(x,t) = -\frac{2a\mu}{2b+k}\left(\frac{C_1\sinh\sqrt{-\mu}\xi + C_2\cosh\sqrt{-\mu}\xi}{C_1\cosh\sqrt{-\mu}\xi + C_2\sinh\sqrt{-\mu}\xi}\right)^2.$$

Case 2: $\mu > 0$.

$$v_6(\xi) = \sqrt{\frac{2ac\mu}{2b+k}}\left(\frac{C_1\sin\sqrt{\mu}\xi + C_2\cos\sqrt{\mu}\xi}{C_1\cos\sqrt{\mu}\xi + C_2\sin\sqrt{\mu}\xi}\right), \tag{23}$$

$$u_6(x,t) = \frac{2a\mu}{2b+k}\left(\frac{C_1\sin\sqrt{\mu}\xi + C_2\cos\sqrt{\mu}\xi}{C_1\cos\sqrt{\mu}\xi + C_2\sin\sqrt{\mu}\xi}\right)^2.$$

But if $C_1 \neq 0$, $C_2 = 0$, then Eqs. (22) and (23) give respectively

$$v_5(\xi) = \sqrt{\frac{-2ac\mu}{2b+k}}\tanh(\sqrt{-\mu}\xi), \qquad u_5(x,t) = -\frac{2a\mu}{2b+k}\tanh^2(\sqrt{-\mu}\xi), \tag{24}$$

$$v_6(\xi) = \sqrt{\frac{2ac\mu}{2b+k}}\tan(\sqrt{\mu}\xi), \qquad u_6(x,t) = \frac{2a\mu}{2b+k}\tan^2(\sqrt{\mu}\xi), \tag{25}$$

which are the exact solutions of the Drinfeld-Sokolov equation. It can be seen that some results are similar to the results in [27].

4. A generalized Drinfeld-Sokolov system

In this section we study the generalized Drinfeld-Sokolov system with the (G'/G)-expansion method as follows

$$u_t + (v^n)_x = 0, \qquad v_t - av_{xxx} + 3bu_xv + 3kuv_x = 0, \tag{26}$$

where a, b, n and k are constants. The wave variable $\eta = x - ct$ PDE transforms to an ODE

$$-cu' + (v^n)' = 0, \qquad cv' + av''' - 3bu'v - 3kuv' = 0, \tag{27}$$

where by integrating the first equation in the system (8) and neglecting the constant of integration we get

$$cu = v^n. \tag{28}$$

Substituting (28) into the second equation of the system (27) and integrating we find

$$c^2 v + acv'' - \frac{3(2b+k)}{n+1} v^{n+1} = 0. \tag{29}$$

To get a closed form solution, we use the transformation

$$v(\eta) = w(\eta)^{\frac{1}{n}}, \tag{30}$$

that will carry Eq. (29) into the ODE

$$c^2 n^2 (n+1) w^2 - 3n^2 (k+bn) w^3 + acn(n+1) ww'' - ac(n^2-1)(w')^2 = 0, \tag{31}$$

we set

$$w(\xi) = a_0 + \sum_{k=1}^{m} a_k \left(\frac{G'(\xi)}{G(\xi)}\right)^k. \tag{32}$$

By the same manipulation as illustrated in the previous section, we can determine value of m by balancing w^3 and (ww'') or $(w')^2$ in Eq. (31), we find that $3m = 2m+2$, then conclude $m = 2$. With the aid (32) it is derived that

$$w(\xi) = a_0 + a_1 \left(\frac{G'(\xi)}{G(\xi)}\right) + a_2 \left(\frac{G'(\xi)}{G(\xi)}\right), \qquad a_2 \neq 0, \tag{33}$$

$$w^3(\xi) = \left(a_0 + a_1 \left(\frac{G'(\xi)}{G(\xi)}\right) + a_2 \left(\frac{G'(\xi)}{G(\xi)}\right)\right)^3, \tag{34}$$

and

$$\begin{aligned} w_{\xi\xi}(\xi) = {} & 6a_2 \left(\frac{G'(\xi)}{G(\xi)}\right)^4 + (2a_1 + 10a_2\lambda) \left(\frac{G'(\xi)}{G(\xi)}\right)^3 + \\ & (8a_2\mu + 3a_1\lambda + 4a_2\lambda^2) \left(\frac{G'(\xi)}{G(\xi)}\right)^2 + \\ & (6a_2\lambda\mu + 2a_1\mu + a_1\lambda^2) \left(\frac{G'(\xi)}{G(\xi)}\right) + 2a_2\mu^2 + a_1\lambda\mu. \end{aligned} \tag{35}$$

Substituting (33)–(35), we obtain the following results:
If $\lambda = 0$, then, by a similar derivation as illustrated in above, we obtain sets of non-trivial solutions:
(I) The first set:

$$a_0 = 0, \qquad a_1 = 0, \qquad c = -\frac{4a}{n}\mu, \qquad \mu = \mu, \qquad a_2 = \frac{4acn(n+1)}{3n^2(k+bn)}. \tag{36}$$

(II) The second set:

$$a_1 = 0, \qquad a_0 = a_2\mu, \qquad a_2 = \frac{2ac(n^2+3n+2)}{3n^2(k+bn)}, \qquad c = \frac{4a\mu}{n^2}, \qquad \mu = \mu. \tag{37}$$

By using (36) and (37), expression (33) can be written as

$$w(\xi) = \frac{4ac(n+1)}{3n(k+bn)}\left(\frac{G'(\xi)}{G(\xi)}\right)^2, \quad \xi = x + \frac{4a\mu}{n}t, \tag{38}$$

$$w(\xi) = \frac{2ac(n^2+3n+2)}{3n^2(k+bn)}\mu + \frac{2ac(n^2+3n+2)}{3n^2(k+bn)}\left(\frac{G'(\xi)}{G(\xi)}\right)^2, \quad \xi = x - \frac{4a\mu}{n^2}t. \tag{39}$$

When $\mu < 0$, we get

$$w_1(\xi) = -\frac{4ac(n+1)\mu}{3n(k+bn)}\left(\frac{C_1\sinh\sqrt{-\mu}\xi + C_2\cosh\sqrt{-\mu}\xi}{C_1\cosh\sqrt{-\mu}\xi + C_2\sinh\sqrt{-\mu}\xi}\right)^2, \tag{40}$$

where $\xi = x + \frac{4a\mu}{n}t$ and

$$w_2(\xi) = \frac{2ac(n^2+3n+2)}{3n^2(k+bn)}\mu\left[1 - \left(\frac{C_1\sinh\sqrt{-\mu}\xi + C_2\cosh\sqrt{-\mu}\xi}{C_1\cosh\sqrt{-\mu}\xi + C_2\sinh\sqrt{-\mu}\xi}\right)^2\right], \tag{41}$$

where $\xi = x - \frac{4a\mu}{n^2}t$. When $\mu > 0$, we have

$$w_3(\xi) = \frac{4ac(n+1)\mu}{3n(k+bn)}\left(\frac{-C_1\sin\sqrt{\mu}\xi + C_2\cos\sqrt{\mu}\xi}{C_1\cos\sqrt{\mu}\xi + C_2\sin\sqrt{\mu}\xi}\right)^2, \tag{42}$$

where $\xi = x + \frac{4a\mu}{n}t$ and

$$w_4(\xi) = \frac{2ac(n^2+3n+2)}{3n^2(k+bn)}\mu\left[1 + \left(\frac{-C_1\sin\sqrt{\mu}\xi + C_2\cos\sqrt{\mu}\xi}{C_1\cos\sqrt{\mu}\xi + C_2\sin\sqrt{\mu}\xi}\right)^2\right], \tag{43}$$

where $\xi = x - \frac{4a\mu}{n^2}t$. If $C_1 \neq 0, C_2 = 0, \mu < 0$, then (40) and (41) get

$$w_1(\xi) = -\frac{4ac(n+1)\mu}{3n(k+bn)}\tanh^2(\sqrt{-\mu}\xi), \qquad \xi = x + \frac{4a\mu}{n}t, \tag{44}$$

$$w_2(\xi) = \frac{2ac(n^2+3n+2)\mu}{3n^2(k+bn)}\operatorname{sech}^2(\sqrt{-\mu}\xi), \qquad \xi = x - \frac{4a\mu}{n^2}t. \tag{45}$$

But if $C_1 \neq 0, C_2 = 0, \mu > 0$, then (42) and (43) can be written as

$$w_3(\xi) = \frac{4ac(n+1)\mu}{3n(k+bn)}\tan^2(\sqrt{\mu}\xi), \qquad \xi = x + \frac{4a\mu}{n}t, \tag{46}$$

$$w_4(\xi) = \frac{2ac(n^2+3n+2)\mu}{3n^2(k+bn)}\sec^2(\sqrt{\mu}\xi), \qquad \xi = x - \frac{4a\mu}{n^2}t.$$

Case 1: $\mu < 0$.

Using (28) and (30), we get

$$v_1(\xi) = \sqrt[n]{-\frac{4ac(n+1)\mu}{3n(k+bn)}}\left(\frac{C_1\sinh\sqrt{-\mu}\xi + C_2\cosh\sqrt{-\mu}\xi}{C_1\cosh\sqrt{-\mu}\xi + C_2\sinh\sqrt{-\mu}\xi}\right)^{\frac{2}{n}}, \tag{47}$$

$$u_1(\xi) = -\frac{4a(n+1)\mu}{3n(k+bn)}\left(\frac{C_1\sinh\sqrt{-\mu}\xi + C_2\cosh\sqrt{-\mu}\xi}{C_1\cosh\sqrt{-\mu}\xi + C_2\sinh\sqrt{-\mu}\xi}\right)^{2},$$

where $\xi = x + \frac{4a\mu}{n}t$ and

$$v_2(\xi) = \sqrt[n]{\frac{2ac(n^2+3n+2)\mu}{3n^2(k+bn)}}\left[1 - \left(\frac{C_1\sinh\sqrt{-\mu}\xi + C_2\cosh\sqrt{-\mu}\xi}{C_1\cosh\sqrt{-\mu}\xi + C_2\sinh\sqrt{-\mu}\xi}\right)^{2}\right]^{\frac{1}{n}}, \tag{48}$$

$$u_2(\xi) = \frac{2a(n^2+3n+2)}{3n^2(k+bn)}\mu\left[1 - \left(\frac{C_1\sinh\sqrt{-\mu}\xi + C_2\cosh\sqrt{-\mu}\xi}{C_1\cosh\sqrt{-\mu}\xi + C_2\sinh\sqrt{-\mu}\xi}\right)^{2}\right],$$

where $\xi = x - \frac{4a\mu}{n^2}t$.

Case 2: $\mu > 0$.

Also, using (28) and (30) we obtain

$$v_3(\xi) = \sqrt[n]{\frac{4ac(n+1)\mu}{3n(k+bn)}}\left(\frac{-C_1\sin\sqrt{\mu}\xi + C_2\cos\sqrt{\mu}\xi}{C_1\cos\sqrt{\mu}\xi + C_2\sin\sqrt{\mu}\xi}\right)^{\frac{2}{n}}, \tag{49}$$

$$u_3(\xi) = \frac{4a(n+1)\mu}{3n(k+bn)}\left(\frac{-C_1\sin\sqrt{\mu}\xi + C_2\cos\sqrt{\mu}\xi}{C_1\cos\sqrt{\mu}\xi + C_2\sin\sqrt{\mu}\xi}\right)^{2},$$

where $\xi = x + \frac{4a\mu}{n}t$ and

$$v_4(\xi) = \sqrt[n]{\frac{2ac(n^2+3n+2)\mu}{3n^2(k+bn)}}\left[1 + \left(\frac{-C_1\sin\sqrt{\mu}\xi + C_2\cos\sqrt{\mu}\xi}{C_1\cos\sqrt{\mu}\xi + C_2\sin\sqrt{\mu}\xi}\right)^{2}\right]^{\frac{1}{n}}, \tag{50}$$

$$u_4(\xi) = \frac{2a(n^2+3n+2)}{3n^2(k+bn)}\mu\left[1+\left(\frac{-C_1\sin\sqrt{\mu}\xi + C_2\cos\sqrt{\mu}\xi}{C_1\cos\sqrt{\mu}\xi + C_2\sin\sqrt{\mu}\xi}\right)^2\right],$$

where $\xi = x - \frac{4a\mu}{n^2}t$. If particular, $C_1 \neq 0, C_2 = 0, \mu < 0, \mu > 0$ then (47)-(50) give respectively

$$v_1(\xi) = \sqrt[n]{-\frac{4ac(n+1)\mu}{3n(k+bn)}}\tanh^{\frac{2}{n}}(\sqrt{-\mu}\xi), \qquad \xi = x + \frac{4a\mu}{n}t, \tag{51}$$

$$u_1(\xi) = -\frac{4a(n+1)\mu}{3n(k+bn)}\tanh^2(\sqrt{-\mu}\xi), \qquad \xi = x + \frac{4a\mu}{n}t,$$

$$v_2(\xi) = \sqrt[n]{\frac{2ac(n^2+3n+2)\mu}{3n^2(k+bn)}}\operatorname{sech}^{\frac{2}{n}}(\sqrt{-\mu}\xi), \qquad \xi = x - \frac{4a\mu}{n^2}t, \tag{52}$$

$$u_2(\xi) = \frac{2ac(n^2+3n+2)\mu}{3n^2(k+bn)}\operatorname{sech}^2(\sqrt{-\mu}\xi), \qquad \xi = x - \frac{4a\mu}{n^2}t,$$

$$v_3(\xi) = \sqrt[n]{\frac{4ac(n+1)\mu}{3n(k+bn)}}\tanh^{\frac{2}{n}}(\sqrt{\mu}\xi), \qquad \xi = x + \frac{4a\mu}{n}t, \tag{53}$$

$$u_3(\xi) = \frac{4a(n+1)\mu}{3n(k+bn)}\tanh^2(\sqrt{\mu}\xi), \qquad \xi = x + \frac{4a\mu}{n}t,$$

$$v_4(\xi) = \sqrt[n]{\frac{2ac(n^2+3n+2)\mu}{3n^2(k+bn)}}\operatorname{sech}^{\frac{2}{n}}(\sqrt{\mu}\xi), \qquad \xi = x - \frac{4a\mu}{n^2}t, \tag{54}$$

$$u_4(\xi) = \frac{2ac(n^2+3n+2)\mu}{3n^2(k+bn)}\operatorname{sech}^2(\sqrt{\mu}\xi), \qquad \xi = x - \frac{4a\mu}{n^2}t,$$

which are the exact solutions of the generalized Drinfeld-Sokolov equation. It can be seen that the results are similar to the results in [27].

5. Conclusion

In this paper, we applied the generalized (G'/G)-expansion method for the Drinfeld-Sokolov and the generalized Drinfeld-Sokolov systems for constructing exact travelling wave solutions of nonlinear partial differential equations. These exact solutions include three types hyperbolic function solution, trigonometric function solution and rational solution. The validity of the method has been successfully applied to study two types of nonlinear equations such as Drinfeld-Sokolov system and the generalized Drinfeld-Sokolov system. We can successfully recover the previously known solitary wave solutions that had been found by other methods. In addition, this method allows us to perform complicated and tedious algebraic calculation on the computer. Some of the results are in agreement with the results reported by others in the literature and new results were formally developed in this work. It can be concluded that the generalized (G'/G)-expansion method is a very powerful and efficient technique in finding exact solutions for wide classes of problems. The solution procedure is very simple and the obtained solution is very concise.

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