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A CLASSIFICATION OF CUBIC EDGE-TRANSITIVE GRAPHS OF ORDER $46P^2$

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Abstract. A graph is called *edge-transitive*, if its automorphisms group acts transitively on the set of its edges. In this paper, we classify all connected cubic edge-transitive graphs of order $46p^2$, where p is a prime.

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1. Introduction

Throughout this paper, graphs are assumed to be finite, simple, undirected and connected. For the group-theoretic concepts and notations not defined here we refer to [19].

For a graph X, we denote by $V(X)$, $E(X)$, $A(X)$ and $\mathrm{Aut}(X)$ the *vertex set*, the *edge set*, the *arc set* and *the full automorphisms group* of X, respectively. For $u, v \in V(X)$, denote by $\{u, v\}$ the edge incident to u and v in X.

Let G be a finite group and S a subset of G such that $1 \notin S$ and $S = S^{-1}$. The Cayley graph $X = Cay(G, S)$ on G with respect to S is defined to have vertex set $V(X) = G$ and edge set $E(X) = \{(g, sg) | g \in G, s \in S\}$. Clearly, $Cay(G, S)$ is connected if and only if S generates the group G. The automorphism group $\mathrm{Aut}(X)$ of X contains the right regular representation G_R of G, the acting group of G by right multiplication, as a subgroup, and G_R is regular on $V(X)$, that is, G_R is transitive on $V(X)$ with trivial vertex stabilizers. A graph X is isomorphic to a Cayley graph on a group G if and only if its automorphism group $\mathrm{Aut}(X)$ has a subgroup isomorphic to G, acting regularly on the vertex set.

An *s-arc* in a graph X is an ordered $(s+1)$-tuple $(v_0, v_1, \ldots, v_{s-1}, v_s)$ of vertices of X such that v_{i-1} is adjacent to v_i for $1 \le i \le s$ and $v_{i-1} \ne v_{i+1}$ for $1 \le i < s$. A graph X is said to be *s-arc-transitive* if $\mathrm{Aut}(X)$ acts transitively on the set of its s-arcs. In particular, 0-arc-transitive means vertex-transitive, and 1-arc-transitive means *arc-transitive* or *symmetric*. A graph X is said to be *s-regular*, if $\mathrm{Aut}(X)$ acts regularly on the set of its s-arcs. Tutte [21] showed that every finite connected cubic symmetric graph is s-regular for $1 \leq s \leq 5$. A subgroup of $\mathrm{Aut}(X)$ is said to be s-regular, if it acts regularly on the set of s-arcs of X. If a subgroup G of $\mathrm{Aut}(X)$ acts transitively on $V(X)$ and $E(X)$, we say that X is *G-vertex-transitive* and *G-edge-transitive*, respectively. In the special case, when $G =\mathrm{Aut}(X)$, we say that X is vertex-transitive and edge-transitive, respectively. It can be shown that a G-edge-transitive but not G-vertex-transitive graph X is necessarily bipartite, where the two parts of the bipartition are orbits of $G \leq\mathrm{Aut}(X)$. Moreover, if X is regular then these two parts have the same cardinality. A regular G-edge-transitive but not G-vertex-transitive graph will be referred to as a *G-semisymmetric graph*. In particular, if $G =\mathrm{Aut}(X)$ the graph is said to be semisymmetric.

The classification of cubic symmetric graphs of different orders is given in many papers. By [3, 4], the cubic s-regular graphs up to order 2048 are classified. Throughout this paper, p and q are prime numbers. The s-regular cubic graphs of some orders such as $2p^2$, $4p^2$, $6p^2$, $10p^2$ were classified in [9, 10, 11, 12]. Also recently, cubic s-regular graphs of order $2pq$ were classified in [25]. Also, the study of semisymmetric graphs was initiated by Folkman [14]. For example, cubic semisymmetric graphs of orders $6p^2$, $8p^2$, $4p^n$ and $2pq$ are classified in [17, 1, 2, 8]. In this paper we classify all cubic edge-transitive (symmetric and also semisymmetric) graphs of order $46p^2$ as follows.

Theorem 1. *Let p be a prime. Then the only connected cubic edge-transitive graph of order $46p^2$ is the 2-regular graph $C(N(23, 23, 23))$.*

2. Preliminaries

Let X be a graph and let N be a subgroup of $\mathrm{Aut}(X)$. For u, $v \in V(X)$, denote by $\{u, v\}$ the edge incident to u and v in X, and by $N_X(u)$ the set of vertices adjacent to u in X. The *quotient graph* X/N or X_N induced by N is defined as the graph such that the set Σ of N-orbits in $V(X)$ is the vertex set of X/N and $B, C \in \Sigma$ are adjacent if and only if there exist $u \in B$ and $v \in C$ such that $\{u, v\} \in E(X)$.

A graph $\widetilde{X}$ is called a *covering* of a graph X with projection $\wp : \widetilde{X} \to X$ if there is a surjection $\wp : V(\widetilde{X}) \to V(X)$ such that $\wp|_{N_{\widetilde{X}}(\tilde{v})} : N_{\widetilde{X}}(\tilde{v}) \to N_X(v)$ is a bijection for any vertex $v \in V(X)$ and $\tilde{v} \in \wp^{-1}(v)$. A covering graph $\widetilde{X}$ of X with a projection $\wp$ is said to be *regular* (or *K-covering*) if there is a semiregular subgroup K of the automorphism group $\mathrm{Aut}(\widetilde{X})$ such that graph X is isomorphic to the quotient graph $\widetilde{X}/K$, say by h, and the quotient map $\widetilde{X} \to \widetilde{X}/K$ is the composition $\wp h$ of $\wp$ and h.

Proposition 1. *[15, Theorem 9] Let X be a connected symmetric graph of prime valency and let G be an s-regular subgroup of $Aut(X)$ for some $s \geq 1$. If a normal subgroup N of G has more than two orbits, then it is semiregular and G/N is an s-regular subgroup of $Aut(X_N)$, where X_N is the quotient graph of X corresponding to the orbits of N. Furthermore, X is a N-regular covering of X_N.*

The next proposition is a special case of [23, Proposition 2.5].

Proposition 2. *Let X be a G-semisymmetric cubic graph with bipartition sets $U(X)$ and $W(X)$, where $G \leq A := Aut(X)$. Moreover, suppose that N is a normal subgroup of G. Then,*
(1) *If N is intransitive on bipartition sets, then N acts semiregularly on both $U(X)$ and $W(X)$, and X is an N-regular covering of a G/N-semisymmetric graph X_N.*
(2) *If 3 dose not divide $|Aut(X)/N|$, then N is semisymmetric on X.*

Proposition 3. *[7, Proposition 2.5] Let X be a connected cubic symmetric graph and G be an s-regular subgroup of $Aut(X)$. Then, the stabilizer G_v of $v \in V(X)$ is isomorphic to $\mathbb{Z}_3$, S_3, $S_3 \times \mathbb{Z}_2, S_4$, or $S_4 \times \mathbb{Z}_2$ for $s = 1, 2, 3, 4$ or 5, respectively.*

Proposition 4. *[18, Proposition 2.4] The vertex stabilizers of a connected G-semi symmetric cubic graph X have order $2^r \cdot 3$, where $0 \leq r \leq 7$. Moreover, if u and v are two adjacent vertices, then the edge stabilizer $G_u \cap G_v$ is a common Sylow 2-subgroup of G_u and G_v.*

Now, we have the following obvious fact in the group theory.

Proposition 5. *Let G be a finite group and let p be a prime. If G has an abelian Sylow p-subgroup, then p does not divide $|G' \cap Z(G)|$.*

Proposition 6. *[24, Proposition 4.4]. Every transitive abelian group G on a set Ω is regular and the centralizer of G in the symmetric group on Ω is G.*

The next two proposition are the result of [16 , Theorem 1.16].

Proposition 7. *Let G be a finite group and let p be a prime, where $p \mid |G|$ and $gcd(m, p) = 1$. Therefore, if $n_p(G) \ncong 1(modp^2)$, then there are $P, R \in Syl_p(G)$ such that $[P \cap R : P] = p$ and $[P \cap R : R] = p$.*

Proposition 8. *Let G be a finite group of order $p^k n$, where $k > 0$, p is a prime and $p \nmid |G|$. Moreover, suppose P and R are two distinct Sylow p-subgroups of G such that $[P \cap R : P] = p$. Then $[G : N_G(P \cap R)] = n/t$, where $t \nmid p, t > p$.*

3. Main results

Let p be an odd prime. Let $N(p,p,p) = \langle x^p = y^p = z^p = 1, [x,y] = z, [z,x] = [z,y] = 1\rangle$ be a finite group of order p^3 and $G = \langle a,b,c,d \mid a^2 = b^p = c^p = d^p = [a,d] = [b,d] = [c,d] = 1, d = [b,c], aba = b^{-1}, aca = c^{-1}\rangle$ be a group of order $2p^3$ and $S = \{a, ab, ac\}$. We write $C(N(p,p,p)) = Cay(G,S)$. By [13, Theorem 3.2], $C(N(p,p,p))$ is a 2-regular graph of order $2p^3$.

Let X be a cubic edge-transitive graph of order $46p^2$. By [22], every cubic edge and vertex-transitive graph is arc-transitive and consequently, X is either symmetric or semisymmetric. We now consider the symmetric case and then we have the following lemma.

Lemma 2. *Let p be a prime and let X be a cubic symmetric graph of order $46p^2$. Then X is isomorphic to the 2-regular graph $C(N(23,23,23))$.*

Proof. By [3, 4] there is no symmetric graph of order $46p^2$, where $p < 7$. If $p = 23$, then by [13, Theorem 3.2], X is isomorphic to the 2-regular graph $C(N(23,23,23))$.

To prove the lemma, we only need to show that no cubic symmetric graph of order $46p^2$ exist, for $p \geq 7$, $p \neq 23$. We suppose to the contrary that X is such a graph. Set $A := Aut(X)$. By Proposition 4, $|A_v| = 2^{s-1} \cdot 3$, where $1 \leq s \leq 5$ and hence $|A| = 2^s \cdot 3 \cdot 23 \cdot p^2$.

Let N be a minimal normal subgroup of A. Thus, $N \cong T \times T \times \cdots \times T = T^k$, where T is a simple group. Let N be unsolvable. By Proposition 1 N has at most two orbits on $V(X)$ and hence $23p^2 \mid |N|$. Since $p \geq 7, p \neq 23$ and $3^2 \nmid |A|$, one has $k = 1$ and hence $N \cong T$. So $|N| = 2^t.23.p^2$ or $2^t.3.23.p^2$, where $1 \leq t \leq s$. Let q be a prime .Then by [6], a non-abelian simple $\{2,p,q\}$-group is one of the following groups

$$A_5, A_6, PSL(2,7), PSL(2,8), PSL(2,17), PSL(3,3), PSU(3,3), PSU(4,2) \quad (1)$$

With orders $2^2.3.5, 2^3.3^2.5, 2^3.3.7, 2^3.3^2.7, 2^4.3^2.17, 2^4.3^3.13, 2^5.3^3.7, 2^6.3^4.5$, respectively. This implies that for $p \geq 7$, there is no simple group of order $2^t.23.p^2$. Hence $|N| = 2^t.3.23.p^2$.

Assume that L is a proper subgroup of N. If L is unsolvable, then L has a non-abelian simple composite factor L_1/L_2. Since $p \geq 11$ and $|L_1/L_2||2^t.3.23.p^2$, by simple group listed in 1, L_1/L_2 cannot be a $\{2,3,23\}-, \{2,3,p\} - or\{2,23,p\}$-group. Thus, L_1/L_2 is a $\{2,3,23,p\}$-group. One may assume that $|L| = 2^r.3.23.p^2$ or $2^r.3.23.p$, where $r \geq 2$. Let $|L| = 2^r.3.23.p^2$. Then $|N : L| \leq 8$ because $|N| = 2^t.3.23.p^2$. Consider the action of N on the right cosets of by right multiplication, and the simplicity of N implies that this action is faithful. It follows $N \leq S_8$ and hence $p \leq 7$. Since $p \geq 7$, one has $p = 7$ and hence $N = 2^t.3.23.7^2$. But by [6], there is no non-abelian simple group of order $2^t.3.23.7^2$, a contradiction. Thus, L is solvable and hence N is a minimal non-abelian simple group, that is, N is a non-abelian simple group and every proper subgroup of N is solvable. By [20, Corollary 1], N is one of the groups in Table I. It can be easily verified that the order of groups in Table I are not of the form $2^r.3.23.p^2$. Thus $|L| = 2^r.3.23.p$. By the same argument as in the preceding paragraph (replacing N by L) L is one of the groups in Table I. Since $|L| = 2^r.3.23.p$, the possible candidates for L is $PSL(2, m)$. Clearly, $m = p$. We show that $|L| < 10^{25}$. If $23 \nmid (p-1)/2$, then $(p-1)/2|96$, which implies that $p \leq 193$. If $p = 193$, then $2^6||L|$, a contradiction. Thus $p < 193$ and hence $p \leq 97$ because $(p-1)/2|96$. It follows that $|L| \leq 96.23.97 = 214176$. If $23|(p-1)/2$, Then $p+1|96$. Consequently $p \leq 47$, implying $|L| \leq 96.23.47 < 214176$. Thus, $|L| \leq 214176$. Then by [6], is isomorphic to $PSL(2, 23)$ or $PSL(2, 47)$. It follows that $p = 11$ or 47 and hence $|N| = 2^t.3.23.11^2$ or $2^t.3.23.47^2$, which is impossible by [6].

Table I. The possible for non-abelian simple group N

N	|N|
$PSL(2,m), m > 3$ a prime and $m^2 \neq 3 \pmod{p^2}$	$\frac{1}{2}m(m-1)(m+1)$
$PSL(2,2^n), n$ a prime	$2^n(2^{2n}-1)$
$PSL(2,3^n), n$ an odd prime	$\frac{1}{2}3^n(3^{2n}-1)$
$PSL(3,3), n$ a prime	$\frac{1}{3}.3^3.2^4$
Suzuki group $Sz(2^n), n$ an odd prime	$2^{2n}(2^{2n}+1)(2n-1)$

Hence, N is solvable and so elementary abelian. Again by Proposition 1, N is semiregular, implying $|N| \mid 46p^2$. Consequently, $N \cong \mathbb{Z}_2$, $\mathbb{Z}_p \times \mathbb{Z}_p$, $\mathbb{Z}_p$ or $\mathbb{Z}_{23}$. If $N \cong \mathbb{Z}_2$, then by Proposition 1, X_N is a cubic graph of odd order $23p^2$, a contradiction. Also, if $N \cong \mathbb{Z}_p \times \mathbb{Z}_p$, then by Proposition 1, X_N is a cubic symmetric graph of order 46. But, by [3, 4] there is no symmetric cubic graph of order 46, a contradiction. Suppose now that $N \cong \mathbb{Z}_p$. Set $C := C_A(N)$ the centralizer of N in A. Let K be a Sylow p-subgroup of A. Since K is an abelian group and $N < K$, $p^2 \mid |C|$. Suppose that C' is the derived subgroup of C. This forces $p^2 \nmid |C'|$ and hence C' has more than two orbits on $V(X)$. By Proposition 1, C' is semiregular and consequently $|C'| \mid 46p^2$. Since C/C' is an abelian group and $p^2 \nmid |C'|$, then C/C' has a normal Sylow p-subgroup, say H/C', which is normal in A/C'. Thus $H \lhd A$ and $p^2 \mid |H|$. Also $|H| \mid 46p^2$ because $|C'| \mid 46p^2$ and $|H/C'| \mid p^2$. Hence H has a characteristic Sylow p-subgroup of order p^2, say K, which is normal in A. Then by Proposition 1, X_K is a cubic symmetric graph of order 46, a contradiction.

Now, suppose that $N \cong \mathbb{Z}_{23}$. Since N has more than two orbits, then by Proposition 1, N is semiregular and the quotient X_N is a cubic A/N-symmetric graph of order $2p^2$ and A/N is an arc-transitive subgroup of $\mathrm{Aut}(X_N)$. Suppose first that

$p = 7$ and T/N be a minimal normal subgroup of A/N. Thus by [11, Lemma 3.1], T/N is 7-subgroup abelian elementary. So $|T/N| = 7$ or 7^2. Consequently $|T| = 23.7$ or 23.7^2. It is easy to see that the Sylow 7-subgroup of T is normal in A, and by the same argument as the previous paragraph, we get a similar contradiction.

We suppose now $p = 11$ and let M/N be the Sylow p-subgroup of A/N. Then, M/N by [10, Lemma 3.1], is normal in A/N. It follows that M is normal in A and $|M/N| = 11^2$. It implies that $|M| = 23.11^2$. Let n_{11} be the number of the Sylow 11-subgroups of M. Thus $n_{11} \mid 23$. So $n_{11} = 1$ or 23. If $n_{11} = 1$, then the Sylow 11-subgroup of M is normal in A, so we get a contradiction. Also, if $n_{11} = 23$, then by Proposition 7, M has two distinct Sylow 11-subgroups, say P and R, such that $[P \cap R : P] = 11$ and $[P \cap R : R] = 11$. Let $N_M(P \cap R)$ be normalizer $P \cap R$ in M. According to Proposition 8 , $[M : N_M(P \cap R] = 1$ and hence $P \cap R$ is normal in M. Since M is characteristic in A, so $P \cap R$ is normal in A. Again A has a normal subgroup of order $p(= 11)$, a contradiction.

We now suppose that $p \geq 13$, $p \neq 23$. Then [11, Theorem 3.2], the Sylow p-subgroup of $\mathrm{Aut}(X_N)$ is normal. Consequently, the Sylow p-subgroup of A/N, say M/N, is normal. Thus, M is normal in A and $|M| = 23p^2$. It follows that the Sylow p-subgroup of A, say K, is normal. Then by Proposition 1, X_K is a cubic symmetric graph of order 46, a contradiction. Hence, the result now follows.

Now, we study the semisymmetric case, and we have the following lemma.

Lemma 3. *Let p be a prime. Then, there is no cubic semisymmetric graph of order $46p^2$.*

Proof. Let X be a cubic semisymmetric graph of order $46p^2$. Denote by $U(X)$ and $W(X)$ the bipartition sets of X, where $|U(X)| = |W(X)| = 23p^2$. For $p = 2, 3$, by [5] there is no cubic semisymmetric graph of order $46p^2$. Thus we can assume that $p \geq 5$. Set $A := Aut(X)$ and let $Q := O_p(A)$ be the maximal normal p-subgroup of A. By Proposition 4, we have $|A_v| = 2^r \cdot 3$, where $0 \leq r \leq 7$ and hence $|A| = 2^r \cdot 3 \cdot 23 \cdot p^2$. Let N be a minimal normal subgroup of A. If N is unsolvable, then $N \times T \times = T^k$, where T is a non-abelian $\{2, 3, 23\}$ or $\{2, 3, 23, p\}$-simple group. By [6], $T \cong A_5$, $PSL(2, 7)$, $PSL(2, 23)$ or $PSL(2, 47)$ with orders $2^2 \cdot 3 \cdot 7, 2^3 \cdot 3 \cdot 7, 2^3 \cdot 3 \cdot 11 \cdot 23$ and $2^4 \cdot 3 \cdot 23 \cdot 47$, respectively. But $3^2 \nmid |N|$ and hence $k = 1$. So $N \cong T$. Since $3 \nmid |A/N|$, by Proposition 3, N must be semisymmetric on X and then $23p^2 \mid |N|$, a contradiction. So N is solvable and so elementary abelian. Thus N acts intransitively on $U(X)$ and $W(X)$ and by Proposition 2, it is semiregular on each partition. Hence $|N| \mid 23p^2$. So $|N| = 23, p$ or p^2. We show that $|Q| = p^2$ as follows.

First Suppose that $Q = 1$. It implies that $N \cong \mathbb{Z}_{23}$. Let X_N be the quotient graph of X relative to N, where X_N is a cubic A/N-semisymmetric graph of order

$2p^2$. By [11], X_N is a vertex-transitive graph. So X_N is a cubic symmetric graph of order $2p^2$. Suppose that T/N be a minimal normal subgroup in A/N. First suppose that $p = 5$, by [11, Lemma 3.1], T/N is 5-subgroup abelian elementary. So $|T/N| = 5$ or 5^2 and hence $|T| = 23 \cdot 5$ or $23 \cdot 5^2$. It follows the Sylow 5-subgroup T is normal in A. This is a contrary with $|Q| = 1$.

Now, suppose $p = 7, 11$. Then, by similar argument as above, we get a contradiction.

Therefore, we can suppose that $p \geq 13$. By [11, Lemma 3.1], Sylow p-subgroup of A/N is normal, say M/N. So $|M/N| = p^2$ and hence $|M| = 23p^2$. Clearly, the Sylow p-subgroup M is normal in A, a contradiction.

We now suppose that $|Q| = p$. Since $|N| \mid 23p^2$, then we have two cases: $N \cong \mathbb{Z}_{23}$ and $N \cong \mathbb{Z}_p$.

Case I. $N \cong \mathbb{Z}_{23}$. By Proposition 2, X_N is a cubic A/N-semisymmetric graph of order $2p^2$. Let T/N be a minimal normal subgroup of A/N. If T/N is an unsolvable group, then by [6], $T/N \cong PSL(2, 7)$. Thus $|T| = 2^3 \cdot 3 \cdot 23 \cdot 7$. Since $3 \nmid |A/T|$, then by Proposition 2, T is semisymmetric on X. Consequently $7^2 \mid |T|$, a contradiction. Hence T/N is solvable and so elementary abelian. If $|T/N| = p^2$, then $|T| = 23p^2$. By a similar way as above, we get, the Sylow p-subgroup of T is characteristic and consequently normal in A. It contradicts our assumption that $|Q| = p$. Therefore T/N intransitively on bipartition sets of X_N and by Proposition 2, it is semiregular on each partition, which force $|T/N| \mid p^2$. Hence $|T/N| = p$ and so $|T| = 23p$. Since T acts intransitively on bipartition sets of X, by Proposition 2, X_T is a cubic A/T-semisymmetric graph of order $2p$. Let K/T be a minimal normal subgroup of A/T. Clearly $N \lhd K$. If K/N is unsolvable then by [6], $K/N \cong PSL(2, 7)$ and so $|K| = 2^3 \cdot 3 \cdot 23 \cdot 7$. Since $K \lhd A$ and 3 dose not divide $|A/K|$, then by Proposition 2, K is semisymmetric on X. Therefore $23 \cdot 7^2 \mid |K|$, a contradiction. It follows that K/N is solvable and since N is solvable, K is solvable. Consequently K/T is solvable and so elementary abelian. If K/T acts transitively on any partition of X_T, then by Proposition 6, K/T is regular and hence $|K/T| = p$. Therefore, $|K| = 23p^2$. Similarly as the case $|Q| = 1$, in this case, we get that $p \neq 5, 7, 11$ and the Sylow p-subgroup K is characteristic and so normal in A, a contrary to this fact that $|Q| = p$. Thus K/T acts intransitively on each partition of X_T and by Proposition 2, K/T is semiregular on two partitions. It implies that $|K/T| = p$ and so $|K| = 23p^2$, a similar contradiction is obtained.

Case II. $N \cong \mathbb{Z}_p$. By Proposition 2, X_N is a cubic A/N-semisymmetric graph of order $46p$. Let T/N be a minimal normal subgroup of A/N. By a similar way as above, T/N is solvable and so elementary abelian. By Proposition 2, T/N is semiregular. It implies that $|T/N| \mid 23p$. If $|T/N| = p$, then $|T| = p^2$, a contrary to this fact that $|Q| = p$. Hence $|T/N| = 23$ and so $|T| = 23p$. By Proposition 2, X_T

is a cubic A/T-semisymmetric graph of order $2p$. Thus by a similar way as case I, we get a contradiction. Therefore $|Q| = p^2$ and so by Proposition 2, X is a regular Q-covering of an A/Q-semisymmetric graph of order 46. But it is impossible because by [4, 5] there is no edge-transitive graph of order 46. The result now follows.

Proof of Theorem Now we complete the proof of the main theorem. Let X is a connected cubic edge-transitive graph of order $46p^2$, where p is a prime. We know that every cubic edge-transitive graph is either symmetric or semisymmetric. Therefore, by Lemmas 2 and 3 the proof is completed.

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