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ON ϕ-PSEUDO SYMMETRIC PARA-SASAKIAN MANIFOLDS

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Abstract. The object of the present paper is to study ϕ-pseudo symmetric and ϕ-pseudo Ricci symmetric Para-Sasakian manifolds with respect to Levi-Civita connection and quarter-symmetric metric connection and obtain a necessary and sufficient condition of a ϕ-pseudo symmetric Para-Sasakian manifold with respect to quarter symmetric metric connection to be ϕ-pseudo symmetric Para-Sasakian manifold with respect to Levi-Civita connection.

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1. Introduction

The study of Riemann symmetric manifolds began with the work of Cartan [6]. A Riemannian manifold (M^n, g) is said to be locally symmetric due to Cartan [6] if its curvature tensor R satisfies the relation $\nabla R = 0$, where ∇ denotes the operator of covariant differentiation with respect to the metric tensor g.

During the last five decades the notion of locally symmetric manifolds has been weakened by many authors in several ways to a different extent such as recurrent manifold by Walker [53], semisymmetric manifold by Szabó [46], pseudosymmetric manifold in the sense of Deszcz [20], pseudosymmetric manifold in the sense of Chaki [7].

A non-flat Riemannian manifold $(M^n, g)(n > 2)$ is said to be pseudosymmetric in the sense of Chaki [7] if it satisfies the relation

$$\begin{aligned}(\nabla_W R)(X,Y,Z,U) &= 2A(W)R(X,Y,Z,U) + A(X)R(W,Y,Z,U) \\ &+ A(Y)R(X,W,Z,U) + A(Z)R(X,Y,W,U) \\ &+ A(U)R(X,Y,Z,W), \end{aligned} \tag{1}$$

i.e.,

$$\begin{aligned}(\nabla_W R)(X,Y)Z &= 2A(W)R(X,Y)Z + A(X)R(W,Y)Z \\ &+ A(Y)R(X,W)Z + A(Z)R(X,Y)W \\ &+ g(R(X,Y)Z,W)\rho\end{aligned} \tag{2}$$

for any vector field X, Y, Z, U and W, where R is the Riemannian curvature tensor of the manifold, A is a non-zero 1-form such that $g(X,\rho) = A(X)$ for every vector field X. Such an n-dimensional manifold is denoted by $(PS)_n$.

Every recurrent manifold is pseudosymmetric in the sense of Chaki [7] but not conversely. Also the pseudosymmetry in the sense of Chaki is not equivalent to that in the sense of Deszcz [20]. However, pseudosymmetry by Chaki will be the pseudosymmetry by Deszcz if and only if the non-zero 1-form associated with $(PS)_n$, is closed. Pseudosymmetric manifolds in the sense of Chaki have been studied by Chaki and Chaki [9], Chaki and De [10], De [12], De and Biswas [14], De, Murathan and Özgür [17], Özen and Altay ([34], [35]), Tarafder ([49], [50]), Tarafder and De [51] and others.

A Riemannian manifold is said to be Ricci symmetric if its Ricci tensor S of type (0,2) satisfies $\nabla S = 0$, where ∇ denotes the Riemannian connection. During the last five decades, the notion of Ricci symmetry has been weakened by many authors in several ways to a different extent such as Ricci recurrent manifold [36], Ricci semisymmetric manifold [46], pseudo Ricci symmetric manifold by Deszcz [21], pseudo Ricci symmetric manifold by Chaki [8].

A non-flat Riemannian manifold (M^n, g) is said to be pseudo Ricci symmetric [8] if its Ricci tensor S of type (0,2) is not identically zero and satisfies the condition

$$(\nabla_X S)(Y,Z) = 2A(X)S(Y,Z) + A(Y)S(X,Z) + A(Z)S(Y,X), \tag{3}$$

for any vector field X, Y, Z, where A is a nowhere vanishing 1-form and ∇ denotes the operator of covariant differentiation with respect to the metric tensor g. Such an n-dimensional manifold is denoted by $(PRS)_n$. The pseudo Ricci symmetric manifolds have been also studied by Arslan et. al [4], Chaki and Saha [11], De and Mazumder [16], De, Murathan and Özgür [17], Özen [33] and many others.
The relation (3) can be written as

$$(\nabla_X Q)(Y) = 2A(X)Q(Y) + A(Y)Q(X) + S(Y,X)\rho, \tag{4}$$

where ρ is the vector field associated to the 1-form A such that $A(X) = g(X,\rho)$ and Q is the Ricci operator, i.e., $g(QX,Y) = S(X,Y)$ for all X, Y.

As a weaker version of local symmetry, the notion of locally ϕ-symmetric Sasakian manifolds was introduced by Takahashi [47]. Generalizing the notion of locally ϕ-symmetric Sasakian manifolds, De, Shaikh and Biswas [18] introduced the notion of ϕ-recurrent Sasakian manifolds. In this connection De [13] introduced and studied ϕ-symmetric Kenmotsu manifolds and in [19] De, Yildiz and Yaliniz introduced and studied ϕ-recurrent Kenmotsu manifolds. In this connection it may be mentioned that Shaikh and Hui studied locally ϕ-symmetric β-kenmotsu manifolds [41] and extended generalized ϕ-recurrent β-Kenmotsu Manifolds [42], respectively. Also in [37] Prakash studied concircularly ϕ-recurrent Kenmotsu Manifolds. In [44] Shukla and Shukla studied ϕ-Ricci symmetric Kenmotsu manifolds. Also Shukla and Shukla [45] studied ϕ-symmetric and ϕ-Ricci symmetric Para-Sasakian manifolds. Recently the present author [26] studied ϕ-pseudo symmetric and ϕ-pseudo Ricci symmetric Kenmotsu manifolds.

Motivated by the above studies the present paper deals with the study of ϕ-pseudo symmetric and ϕ-pseudo Ricci symmetric Para-Sasakian manifolds. The paper is organized as follows. Section 2 is concerned with Para-Sasakian manifolds. Section 3 consists with the study of quarter symmetric metric connection. In section 4, we study ϕ-pseudo symmetric Para-Sasakian manifolds. Section 5 is devoted with the study of ϕ-pseudo Ricci symmetric Para-Sasakian manifolds.

In [22] Friedmann and Schouten introduced the notion of semisymmetric linear connection on a differentiable manifold. Then in 1932 Hayden [24] introduced the idea of metric connection with torsion on a Riemannian manifold. A systematic study of the semisymmetric metric connection on a Riemannian manifold has been given by Yano in 1970 [54]. In 1975, Golab introduced the idea of a quarter symmetric linear connection in differentiable manifolds.

A linear connection $\overline{\nabla}$ in an n-dimensional differentiable manifold M is said to be a quarter symmetric connection [23] if its torsion tensor τ of the connection $\overline{\nabla}$ is of the form

$$\begin{aligned} \tau(X,Y) &= \overline{\nabla}_X Y - \overline{\nabla}_Y X - [X,Y] \\ &= \eta(Y)\phi X - \eta(X)\phi Y, \end{aligned} \tag{5}$$

where η is a 1-form and ϕ is a tensor of type (1,1). In particular, if $\phi X = X$ then the quarter symmetric connection reduces to the semisymmetric connection. Thus the notion of quarter symmetric connection generalizes the notion of the semisymmetric connection. Again if the quarter symmetric connection $\overline{\nabla}$ satisfies the condition

$$(\overline{\nabla}_X g)(Y,Z) = 0 \tag{6}$$

for all X, Y, $Z \in \chi(M)$, where $\chi(M)$ is the Lie algebra of vector fields on the manifold M, then $\overline{\nabla}$ is said to be a quarter symmetric metric connection. Quarter symmetric metric connection have been studied by many authors in several ways to a different extent such as [2], [3], [5], [25], [27], [28], [29], [31], [32], [38], [39], [40], [43], [48], [52]. Recently Kumar, Venkatesha and Bagewadi [30] studied ϕ-recurrent Para-Sasakian manifolds admitting quarter symmetric metric connection.

Motivated by the above studies in this paper we also study of ϕ-pseudo symmetric and ϕ-pseudo Ricci symmetric Para-Sasakian manifolds with respect to quarter symmetric metric connection. Section 6 is devoted to the study of ϕ-pseudo symmetric Para-Sasakian manifolds with respect to quarter symmetric metric connection and obtain a necessary and sufficient condition of a ϕ-pseudo symmetric Para-Sasakian manifold with respect to quarter symmetric metric connection to be ϕ-pseudo symmetric Para-Sasakian manifold with respect to Levi-Civita connection.

In [45] Shukla and Shukla proved that a ϕ-symmetric Para-Sasakian manifold is an Einstein manifold. In this paper we obtain the Ricci tensor of a ϕ-pseudo symmetric Para-Sasakian manifold with respect to quarter symmetric metric connection and it is proved that a ϕ-symmetric Para-Sasakian manifold with respect to quarter symmetric metric connection is an η-Einstein manifold. In section 7, we have studied ϕ-pseudo Ricci symmetric Para-Sasakian manifolds with respect to quarter symmetric metric connection.

2. Para-Sasakian manifolds

An n-dimensional differentiable manifold M is called an almost paracontact manifold if it admits an almost paracontact structure (ϕ, ξ, η) consisting of a (1,1) tensor field ϕ, a vector field ξ, and an 1-form η satisfying

$$\phi^2 X = X - \eta(X)\xi, \tag{7}$$

$$\eta(\xi) = 1, \quad \phi\xi = 0, \qquad \eta(\phi X) = 0. \tag{8}$$

If g is a compatible Riemannian metric with (ϕ, ξ, η), that is,

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \qquad \eta(X) = g(X, \xi), \tag{9}$$

$$g(\phi X, Y) = g(X, \phi Y) \tag{10}$$

for all vector fields X, Y on M, then M becomes a almost paracontact Riemannian manifold equipped with an almost paracontact Riemannian structure (ϕ, ξ, η, g).

An almost paracontact Riemannian manifold is called a Para-Sasakian manifold if it satisfies

$$(\nabla_X \phi)(Y) = -g(X, Y)\xi - \eta(Y)\phi X + 2\eta(X)\eta(Y)\xi, \tag{11}$$

where ∇ denotes the Riemannian connection of g. From the above equation it follows that

$$\nabla_X \xi = \phi(X), \quad (\nabla_X \eta)(Y) = g(X, \phi Y) = (\nabla_Y \eta)(X). \tag{12}$$

In an n-dimensional para-Sasakian manifold M, the following relations hold ([1], [30]):

$$\eta(R(X,Y)Z) = \eta(Y)g(X,Z) - \eta(X)g(Y,Z), \tag{13}$$
$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{14}$$
$$R(\xi,X)Y = \eta(Y)X - g(X,Y)\xi, \tag{15}$$
$$S(X,\xi) = -(n-1)\eta(X), \quad Q\xi = -(n-1)\xi, \tag{16}$$
$$S(\phi X,\phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y), \tag{17}$$
$$(\nabla_W R)(X,Y)\xi = g(\phi X,W)Y - g(\phi Y,W)X - R(X,Y)\phi W \tag{18}$$

for any vector field X, Y, Z and W on M and R is the Riemannian curvature tensor and S is the Ricci tensor of type (0,2) such that $g(QX,Y) = S(X,Y)$.

Definition 1. *A Para-Sasakian manifold M is said to be η-Einstein if its Ricci tensor S of type (0,2) is of the form*

$$S = ag + b\eta \otimes \eta, \tag{19}$$

where a, b are smooth functions on M.

3. Quarter symmetric metric connection

Let M be an n-dimensional Para-Sasakian manifold and ∇ be the Levi-Civita connection on M. A quarter symmetric metric connection $\overline{\nabla}$ in a Para-Sasakian manifold is defined by ([23], [30])

$$\overline{\nabla}_X Y = \nabla_X Y + H(X,Y), \tag{20}$$

where H is a tensor of type (1,1) such that

$$H(X,Y) = \frac{1}{2}\big[\tau(X,Y) + \tau'(X,Y) + \tau'(Y,X)\big] \tag{21}$$

and

$$g(\tau'(X,Y),Z) = g(\tau(Z,X),Y). \tag{22}$$

From (5) and (22), we get

$$\tau'(X,Y) = \eta(X)\phi Y - g(\phi X,Y)\xi. \tag{23}$$

Using (5) and (23) in (21), we obtain

$$H(X,Y) = \eta(Y)\phi X - g(\phi X,Y)\xi. \tag{24}$$

Hence a quarter symmetric metric connection $\overline{\nabla}$ in a Para-Sasakian manifold is given by

$$\overline{\nabla}_X Y = \nabla_X Y + \eta(Y)\phi X - g(\phi X, Y)\xi. \tag{25}$$

If R and $\overline{R}$ are respectively the curvature tensor of Levi-Civita connection ∇ and the quarter symmetric metric connection $\overline{\nabla}$ in a Para-Sasakian manifold then we have [30]

$$\begin{aligned}\overline{R}(X,Y)Z &= R(X,Y)Z + 3g(\phi X, Z)\phi Y - 3g(\phi Y, Z)\phi X \\ &+ \big[\eta(X)Y - \eta(Y)X\big]\eta(Z) - \big[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)\big]\xi.\end{aligned} \tag{26}$$

From (26) we have

$$\overline{S}(Y,Z) = S(Y,Z) + 2g(Y,Z) - (n+1)\eta(Y)\eta(Z), \tag{27}$$

where $\overline{S}$ and S are respectively the Ricci tensor of a Para-Sasakian manifold with respect to the quarter symmetric metric connection and Levi-Civita connection.
Also from (27), we have

$$\overline{r} = r + (n-1), \tag{28}$$

where $\overline{r}$ and r are the scalar curvatures with respect to quarter symmetric metric connection and Levi-Civita connection respectively.
From (14), (18), (25) and (26), we get

$$\begin{aligned}(\overline{\nabla}_W \overline{R}(X,Y)\xi &= 2g(\phi X, W)[Y + \eta(Y)\xi] \\ &- 2g(\phi Y, W)[X + \eta(X)\xi] - R(X,Y)\phi W.\end{aligned} \tag{29}$$

Again from (25) and (26), we have

$$g((\overline{\nabla}_W \overline{R})(X,Y)Z, U) = -g((\overline{\nabla}_W \overline{R})(X,Y)U, Z). \tag{30}$$

Again from (14), (15) and (26), we obtain

$$\overline{R}(X,Y)\xi = 2[\eta(X)Y - \eta(Y)X], \tag{31}$$

$$\overline{R}(X,\xi)Y = 2[g(X,Y)\xi - \eta(Y)X]. \tag{32}$$

Also in view of (16) we get from (27) that

$$\overline{S}(Y,\xi) = -2(n-1)\eta(Y). \tag{33}$$

4. ϕ-pseudo symmetric Para-Sasakian manifolds

Definition 2. *A Para-Sasakian manifold $M^n(\phi,\xi,\eta,g)$ $(n>2)$ is said to be ϕ-pseudo symmetric* [26] *if the curvature tensor R satisfies*

$$\begin{aligned}\phi^2((\nabla_W R)(X,Y)Z) &= 2A(W)R(X,Y)Z + A(X)R(W,Y)Z \\ &+ A(Y)R(X,W)Z + A(Z)R(X,Y)W \\ &+ g(R(X,Y)Z,W)\rho\end{aligned} \tag{34}$$

for any vector field X, Y, Z and W, where A is a non-zero 1-form.
In particular, if $A=0$ then the manifold is said to be ϕ-symmetric [45].

We now consider a Para-Sasakian manifold (M^n,g), which is ϕ-pseudo symmetric. Then by virtue of (7), it follows from (34) that

$$\begin{aligned}& (\nabla_W R)(X,Y)Z - \eta((\nabla_W R)(X,Y)Z)\xi \\ =\ & 2A(W)R(X,Y)Z + A(X)R(W,Y)Z + A(Y)R(X,W)Z \\ +\ & A(Z)R(X,Y)W + g(R(X,Y)Z,W)\rho\end{aligned} \tag{35}$$

from which it follows that

$$\begin{aligned}& g((\nabla_W R)(X,Y)Z,U) - \eta((\nabla_W R)(X,Y)Z)\eta(U) \\ =\ & 2A(W)g(R(X,Y)Z,U) + A(X)g(R(W,Y)Z,U) + A(Y)g(R(X,W)Z,U) \\ +\ & A(Z)g(R(X,Y)W,U) + g(R(X,Y)Z,W)A(U).\end{aligned} \tag{36}$$

Taking an orthonormal frame field and then contracting (36) over X and U and then using (7), we get

$$\begin{aligned}& (\nabla_W S)(Y,Z) - g((\nabla_W R)(\xi,Y)Z,\xi) \\ =\ & 2A(W)S(Y,Z) + A(Y)S(W,Z) + A(Z)S(Y,W) \\ +\ & A(R(W,Y)Z) + A(R(W,Z)Y).\end{aligned} \tag{37}$$

Using the relation $g((\nabla_W R)(X,Y)Z,U) =$ - $g((\nabla_W R)(X,Y)U,Z)$ and (10), (15), (18), we have

$$g((\nabla_W R)(\xi,Y)Z,\xi) = 0. \tag{38}$$

By virtue of (38) it follows from (37) that

$$\begin{aligned}(\nabla_W S)(Y,Z) &= 2A(W)S(Y,Z) + A(Y)S(W,Z) + A(Z)S(Y,W) \\ &+ A(R(W,Y)Z) + A(R(W,Z)Y).\end{aligned} \tag{39}$$

This leads to the following:

Theorem 1. *A ϕ-pseudo symmetric Para-Sasakian manifold is pseudo Ricci symmetric if and only if*

$$A(R(W,Y)Z) + A(R(W,Z)Y) = 0,$$

for any vector fields W, Y and Z.

Setting $Z = \xi$ in (35) and using (13) and (14), we get

$$\begin{aligned} & A(\xi)R(X,Y)W + R(X,Y)\phi W \\ = \;& g(W,\phi X)Y - g(W,\phi Y)X - 2A(W)\{\eta(X)Y - \eta(Y)X\} \\ - \;& A(X)\{\eta(W)Y - \eta(Y)W\} - A(Y)\{\eta(X)W - \eta(W)X\} \\ - \;& \{\eta(X)g(Y,W) - \eta(Y)g(X,W)\}\rho. \end{aligned} \tag{40}$$

Replacing W by ϕW in (40) and using (7) and (14), we get

$$\begin{aligned} & R(X,Y)W + A(\xi)R(X,Y)\phi W \\ = \;& g(X,W)Y - g(Y,W)X - 2A(\phi W)\{\eta(X)Y - \eta(Y)X\} \\ + \;& \{A(X)\eta(Y) - A(Y)\eta(X)\}\phi W \\ - \;& \{\eta(X)g(Y,\phi W) - \eta(Y)g(X,\phi W)\}\rho. \end{aligned} \tag{41}$$

From (40) and (41), we obtain

$$\begin{aligned} & [1 - \{A(\xi)\}^2]R(X,Y)W \\ = \;& g(X,W)Y - g(Y,W)X - A(\xi)[g(W,\phi X)Y - g(W,\phi Y)X] \\ - \;& 2[A(\phi W) - A(\xi)A(W)][\eta(X)Y - \eta(Y)X] \\ + \;& [A(X)\eta(Y) - A(Y)\eta(X)]\phi W + A(\xi)A(X)[\eta(W)Y - \eta(Y)W] \\ + \;& A(\xi)A(Y)[\eta(X)W - \eta(W)X] - [\eta(X)g(Y,\phi W) - \eta(Y)g(X,\phi W)]\rho \\ + \;& A(\xi)[\eta(X)g(Y,W) - \eta(Y)g(X,W)]\rho, \end{aligned} \tag{42}$$

provided that $1 - \{A(\xi)\}^2 \neq 0$. From (42), we get

$$\begin{aligned} \big[1 - \{A(\xi)\}^2\big]S(Y,W) \;=\; & \big[\{A(\xi)\}^2 - (n-1)\big]g(Y,W) \\ + \;& [n - 2A(\xi)]g(Y,\phi W) + 2(n-1)A(\phi W)\eta(Y) \\ - \;& A(\xi)\big[2nA(W)\eta(Y) + (n-2)A(Y)\eta(W)\big]. \end{aligned} \tag{43}$$

This leads to the following:

Theorem 2. *In a ϕ-pseudo symmetric Para-Sasakian manifold, the curvature tensor R and the Ricci tensor S are respectively given by* (42) *and* (43)*.*

5. ϕ-pseudo Ricci symmetric Para-Sasakian manifolds

Definition 3. *A Para-Sasakian manifold $M^n(\phi, \xi, \eta, g)$ $(n > 2)$ is said to be ϕ-pseudo Ricci symmetric [26] if the Ricci operator Q satisfies*

$$\phi^2((\nabla_X Q)(Y)) = 2A(X)QY + A(Y)QX + S(Y, X)\rho \tag{44}$$

for any vector field X, Y, where A is a non-zero 1-form.

In particular, if $A = 0$, then (44) turns into the notion of ϕ-Ricci symmetric Para-Sasakian manifold introduced by Shukla and Shukla [45].

Let us take a Para-Sasakian manifold (M^n, g), which is ϕ-pseudo Ricci symmetric. Then by virtue of (7) it follows from (44) that

$$(\nabla_X Q)(Y) - \eta((\nabla_X Q)(Y))\xi = 2A(X)QY + A(Y)QX + S(Y, X)\rho$$

from which it follows that

$$\begin{aligned} & g(\nabla_X Q(Y), Z) - S(\nabla_X Y, Z) - \eta((\nabla_X Q)(Y))\eta(Z) \\ = \;& 2A(X)S(Y, Z) + A(Y)S(X, Z) + S(Y, X)A(Z). \end{aligned} \tag{45}$$

Putting $Y = \xi$ in (45) and using (12) and (16), we get

$$\begin{aligned} & A(\xi)S(X, Z) + S(\phi X, Z) \\ = \;& -(n-1)g(\phi X, Z) + (n-1)\big[2A(X)\eta(Z) + A(Z)\eta(X)\big]. \end{aligned} \tag{46}$$

Replacing X by ϕX in (46) and using (7), we obtain

$$S(X, Z) + A(\xi)S(\phi X, Z) = -(n-1)g(X, Z) + 2(n-1)A(\phi X)\eta(Z). \tag{47}$$

From (46) and (47), we have

$$\begin{aligned} \big[1 - \{A(\xi)\}^2\big]S(X, Z) &= -(n-1)g(X, Z) \\ &+ 2(n-1)\big[A(\phi X) - A(\xi)A(X)\big]\eta(Z) \\ &+ (n-1)A(\xi)\big[g(\phi X, Z) - A(Z)\eta(X)\big]. \end{aligned} \tag{48}$$

This leads to the following:

Theorem 3. *In a ϕ-pseudo Ricci symmetric Para-Sasakian manifold, the Ricci tensor is of the form* (48).

In particular, if $A = 0$ then from (48), we get $S(X, Z) = -(n-1)g(X, Z)$, which implies that the manifold under consideration is Einstein.
This leads to the following:

Corollary 4. [45] *A ϕ-Ricci symmetric Para-Sasakian manifold is an Einstein manifold.*

6. ϕ-pseudo symmetric Para-Sasakian manifolds with respect to quarter symmetric metric connection

Definition 4. *An n-dimensional Para-Sasakian manifold M^n $(n > 2)$ is said to be ϕ-pseudo symmetric with respect to quarter symmetric metric connection if the curvature tensor $\overline{R}$ with respect to quarter symmetric metric connection satisfies*

$$\begin{aligned}\phi^2((\overline{\nabla}_W\overline{R})(X,Y)Z) &= 2A(W)\overline{R}(X,Y)Z + A(X)\overline{R}(W,Y)Z \\ &+ A(Y)\overline{R}(X,W)Z + A(Z)\overline{R}(X,Y)W \\ &+ g(\overline{R}(X,Y)Z,W)\rho\end{aligned} \tag{49}$$

for any vector field X, Y, Z and W, where A is a non-zero 1-form.
In particular, if $A = 0$ then the manifold is said to be ϕ-symmetric Para-Sasakian manifold with respect to quarter symmetric metric connection.

We now consider a Para-Sasakian manifold M^n $(n > 2)$, which is ϕ-pseudo symmetric with respect to quarter symmetric metric connection. Then by virtue of (7), it follows from (49) that

$$\begin{aligned}&(\overline{\nabla}_W\overline{R})(X,Y)Z - \eta((\overline{\nabla}_W\overline{R})(X,Y)Z)\xi \\ =\ & 2A(W)\overline{R}(X,Y)Z + A(X)\overline{R}(W,Y)Z + A(Y)\overline{R}(X,W)Z \\ +\ & A(Z)\overline{R}(X,Y)W + g(\overline{R}(X,Y)Z,W)\rho\end{aligned} \tag{50}$$

from which it follows that

$$\begin{aligned}&g((\overline{\nabla}_W\overline{R})(X,Y)Z,U) - \eta((\overline{\nabla}_W\overline{R})(X,Y)Z)\eta(U) \\ =\ & 2A(W)g(\overline{R}(X,Y)Z,U) + A(X)g(\overline{R}(W,Y)Z,U) + A(Y)g(\overline{R}(X,W)Z,U) \\ +\ & A(Z)g(\overline{R}(X,Y)W,U) + g(\overline{R}(X,Y)Z,W)A(U).\end{aligned} \tag{51}$$

Taking an orthonormal frame field and then contracting (51) over X and U and then using (7) and (9), we get

$$\begin{aligned}&(\overline{\nabla}_W\overline{S})(Y,Z) - g((\overline{\nabla}_W\overline{R})(\xi,Y)Z,\xi) \\ =\ & 2A(W)\overline{S}(Y,Z) + A(Y)\overline{S}(W,Z) + A(Z)\overline{S}(Y,W) \\ +\ & A(\overline{R}(W,Y)Z) + A(\overline{R}(W,Z)Y).\end{aligned} \tag{52}$$

Using (10), (15), (29) and (30), we have

$$g((\overline{\nabla}_W\overline{R})(\xi,Y)Z,\xi) = -g((\overline{\nabla}_W\overline{R})(\xi,Y)\xi,Z) = 3g(W,\phi Y)\eta(Z). \tag{53}$$

By virtue of (53) it follows from (52) that

$$\begin{aligned}(\overline{\nabla}_W\overline{S})(Y,Z) &= 2A(W)\overline{S}(Y,Z) + A(Y)\overline{S}(W,Z) + A(Z)\overline{S}(Y,W) \\ &+ A(\overline{R}(W,Y)Z) + A(\overline{R}(W,Z)Y) - 3g(W,\phi Y)\eta(Z).\end{aligned} \tag{54}$$

This leads to the following:

Theorem 5. *A ϕ-pseudo symmetric Para-Sasakian manifold with respect to quarter symmetric metric connection is pseudo Ricci symmetric with respect to quarter symmetric metric connection if and only if*

$$A(\overline{R}(W,Y)Z) + A(\overline{R}(W,Z)Y) - 3g(W,\phi Y)\eta(Z) = 0.$$

Setting $Z = \xi$ in (52) and using (53), we get

$$\begin{aligned} & (\overline{\nabla}_W \overline{S})(Y,\xi) - 3g(W,\phi Y) \\ = \; & 2A(W)\overline{S}(Y,\xi) + A(Y)\overline{S}(W,\xi) + A(\xi)\overline{S}(Y,W) \\ + \; & A(\overline{R}(W,Y)\xi) + A(\overline{R}(W,\xi)Y). \end{aligned} \tag{55}$$

We know that

$$(\overline{\nabla}_W \overline{S})(Y,\xi) = \overline{\nabla}_W \overline{S}(Y,\xi) - \overline{S}(\overline{\nabla}_W Y,\xi) - \overline{S}(Y,\overline{\nabla}_W \xi). \tag{56}$$

Using (12), (16), (25), (27) and (33) in (56) we obtain

$$(\overline{\nabla}_W \overline{S})(Y,\xi) = -2[S(Y,\phi W) + g(Y,\phi W)]. \tag{57}$$

In view of (27), (31)-(33) and (57), we have from (55) that

$$\begin{aligned} & A(\xi)S(Y,W) + 2S(Y,\phi W) \\ = \; & -4A(\xi)g(Y,W) - 7g(Y,\phi W) \\ + \; & (n+1)A(\xi)\eta(Y)\eta(W) - 4nA(W)\eta(Y) - 2(n-2)A(Y)\eta(W). \end{aligned} \tag{58}$$

Replacing W by ϕW in (58) and using (7) and (8), we get

$$\begin{aligned} & A(\xi)S(Y,\phi W) + 2S(Y,W) \\ = \; & -4A(\xi)g(Y,\phi W) - 7g(Y,W) \\ - \; & (2n-9)\eta(Y)\eta(W) - 4nA(\phi W)\eta(Y). \end{aligned} \tag{59}$$

From (58) and (59), we obtain

$$\begin{aligned} & [4 - \{A(\xi)\}^2]S(Y,W) \\ = \; & 2[2\{A(\xi)\}^2 - 7]g(Y,W) - A(\xi)g(Y,\phi W) \\ - \; & [(n+1)\{A(\xi)\}^2 + 2(2n-9)]\eta(Y)\eta(W) \\ + \; & A(\xi)[4nA(W)\eta(Y) + 2(n-2)A(Y)\eta(W)]. \end{aligned} \tag{60}$$

This leads to the following:

Theorem 6. *In a ϕ-pseudo symmetric Para-Sasakian manifold with respect to quarter symmetric metric connection the Ricci tensor is given by* (60)*.*

In particular, if $A = 0$ then (60) reduces to

$$S(Y,W) = -\frac{7}{2}g(Y,W) - \frac{1}{2}(2n-9)\eta(Y)\eta(W), \tag{61}$$

which implies that the manifold under consideration is η-Einstein.
This leads to the following:

Corollary 7. *A ϕ-symmetric Para-Sasakian manifold with respect to quarter symmetric metric connection is an η-Einstein manifold.*

Using (30) in (50), we get

$$\begin{aligned}(\overline{\nabla}_W\overline{R})(X,Y)Z &= g((\overline{\nabla}_W\overline{R})(X,Y)\xi,Z)\xi + 2A(W)\overline{R}(X,Y)Z \\ &+ A(X)\overline{R}(W,Y)Z + A(Y)\overline{R}(X,W)Z \\ &+ A(Z)\overline{R}(X,Y)W + g(\overline{R}(X,Y)Z,W)\rho.\end{aligned} \tag{62}$$

In view of (26) and (29) it follows from (62) that

$$\begin{aligned}(\overline{\nabla}_W\overline{R})(X,Y)Z &= \big[2g(W,\phi X)\{g(Y,Z)+\eta(Y)\eta(Z)\} \\ &- 2g(W,\phi Y)\{g(X,Z)+\eta(X)\eta(Z)\} \\ &- R(X,Y,\phi W,Z)\big]\xi \\ &+ 2A(W)\big[R(X,Y)Z + 3g(\phi X,Z)\phi Y \\ &- 3g(\phi Y,Z)\phi X + \{\eta(X)Y-\eta(Y)X\}\eta(Z) \\ &- \{\eta(X)g(Y,Z)-\eta(Y)g(X,Z)\}\xi\big] \\ &+ A(X)\big[R(W,Y)Z + 3g(\phi W,Z)\phi Y \\ &- 3g(\phi Y,Z)\phi W + \{\eta(W)Y-\eta(Y)W\}\eta(Z) \\ &- \{\eta(W)g(Y,Z)-\eta(Y)g(W,Z)\}\xi\big] \\ &+ A(Y)\big[R(X,W)Z + 3g(\phi X,Z)\phi W \\ &- 3g(\phi W,Z)\phi X + \{\eta(X)W-\eta(W)X\}\eta(Z) \\ &- \{\eta(X)g(W,Z)-\eta(W)g(X,Z)\}\xi\big] \\ &+ A(Z)\big[R(X,Y)W + 3g(\phi X,W)\phi Y \\ &- 3g(\phi Y,W)\phi X + \{\eta(X)Y-\eta(Y)X\}\eta(W) \\ &- \{\eta(X)g(Y,W)-\eta(Y)g(X,W)\}\xi\big] \\ &+ \big[R(X,Y,Z,W) + 3g(\phi X,Z)g(\phi Y,W) \\ &- 3g(\phi Y,Z)g(\phi X,W) \\ &+ \{\eta(X)g(Y,W)-\eta(Y)g(X,W)\}\eta(Z) \\ &- \{\eta(X)g(Y,Z)-\eta(Y)g(X,Z)\}\eta(W)\big]\rho\end{aligned} \tag{63}$$

for arbitrary vector fields X, Y, Z and W. This leads to the following:

Theorem 8. *A Para-Sasakian manifold is ϕ-pseudo symmetric with respect to quarter symmetric metric connection if and only if the relation* (63) *holds.*

Now by virtue of the relation $g((\nabla_W R)(X,Y)Z,U) = -g((\nabla_W R)(X,Y)U,Z)$ and (18) it follows from (35) that

$$\begin{aligned}(\nabla_W R)(X,Y)Z &= \big[g(W,\phi X)g(Y,Z) - g(W,\phi Y)g(X,Z) \\ &- R(X,Y,\phi W,Z)\big]\xi + 2A(W)R(X,Y)Z \\ &+ A(X)R(W,Y)Z + A(Y)R(X,W)Z \\ &+ A(Z)R(X,Y)W + R(X,Y,Z,W)\rho.\end{aligned} \tag{64}$$

From (63) and (64), we can state the following:

Theorem 9. *A ϕ-pseudo symmetric Para-Sasakian manifold is invariant under quarter symmetric metric connection if and only if the relation*

$$\begin{aligned}&\big[g(W,\phi X)\{g(Y,Z) + 2\eta(Y)\eta(Z)\} - g(W,\phi Y)\{g(X,Z) + 2\eta(X)\eta(Z)\}\big]\xi \\ +\ & 2A(W)\big[3g(\phi X,Z)\phi Y - 3g(\phi Y,Z)\phi X + \{\eta(X)Y - \eta(Y)X\}\eta(Z) \\ -\ & \{\eta(X)g(Y,Z) - \eta(Y)g(X,Z)\}\xi\big] + A(X)\big[3g(\phi W,Z)\phi Y - 3g(\phi Y,Z)\phi W \\ +\ & \{\eta(W)Y - \eta(Y)W\}\eta(Z) - \{\eta(W)g(Y,Z) - \eta(Y)g(W,Z)\}\xi\big] \\ +\ & A(Y)\big[3g(\phi X,Z)\phi W - 3g(\phi W,Z)\phi X + \{\eta(X)W - \eta(W)X\}\eta(Z) \\ -\ & \{\eta(X)g(W,Z) - \eta(W)g(X,Z)\}\xi\big] + A(Z)\big[3g(\phi X,W)\phi Y - 3g(\phi Y,W)\phi X \\ +\ & \{\eta(X)Y - \eta(Y)X\}\eta(W) - \{\eta(X)g(Y,W) - \eta(Y)g(X,W)\}\xi\big] \\ +\ & \big[3g(\phi X,Z)g(\phi Y,W) - 3g(\phi Y,Z)g(\phi X,W) + \{\eta(X)g(Y,W) \\ -\ & \eta(Y)g(X,W)\}\eta(Z) - \{\eta(X)g(Y,Z) - \eta(Y)g(X,Z)\}\eta(W)\big]\rho = 0\end{aligned}$$

holds for arbitrary vector fields X, Y, Z and W.

7. ϕ-pseudo Ricci symmetric Para-Sasakian manifolds with respect to quarter symmetric metric connection

Definition 5. *A Para-Sasakian manifold M^n $(n > 2)$ is said to be ϕ-pseudo Ricci symmetric with respect to quarter symmetric metric connection if the Ricci operator Q satisfies*

$$\phi^2((\overline{\nabla}_X \overline{Q})(Y)) = 2A(X)\overline{Q}Y + A(Y)\overline{Q}X + \overline{S}(Y,X)\rho \tag{65}$$

for any vector field X, Y, where A is a non-zero 1-form.

In particular, if $A = 0$, then (65) turns into the notion of ϕ-Ricci symmetric Para-Sasakian manifold with respect to quarter symmetric metric connection.

Let us take a Para-Sasakian manifold M^n $(n > 2)$, which is ϕ-pseudo Ricci symmetric with respect to quarter symmetric metric connection. Then by virtue of (7) it follows from (65) that

$$(\overline{\nabla}_X\overline{Q})(Y) - \eta((\overline{\nabla}_X\overline{Q})(Y))\xi = 2A(X)\overline{Q}Y + A(Y)\overline{Q}X + \overline{S}(Y,X)\rho$$

from which it follows that

$$\begin{aligned} & g(\overline{\nabla}_X\overline{Q}(Y),Z) - \overline{S}(\overline{\nabla}_XY,Z) - \eta((\overline{\nabla}_X\overline{Q})(Y))\eta(Z) \\ = \; & 2A(X)\overline{S}(Y,Z) + A(Y)\overline{S}(X,Z) + \overline{S}(Y,X)A(Z). \end{aligned} \tag{66}$$

Putting $Y = \xi$ in (66) and using (12), (16), (25), (27) and (33), we get

$$\begin{aligned} A(\xi)S(X,Z) + 2S(\phi X,Z) &= -2A(\xi)g(X,Z) - 4ng(\phi X,Z) \\ &+ (n+1)A(\xi)\eta(X)\eta(Z) \\ &+ 2(n-1)\big[2A(X)\eta(Z) + A(Z)\eta(X)\big]. \end{aligned} \tag{67}$$

Replacing X by ϕX in (67) and using (7) and (8), we obtain

$$\begin{aligned} & A(\xi)S(\phi X,Z) + 2S(X,Z) \\ = \; & -4ng(X,Z) - 2A(\xi)g(\phi X,Z) \\ + \; & 2(n+1)\eta(X)\eta(Z) + 4(n-1)A(\phi X)\eta(Z). \end{aligned} \tag{68}$$

From (67) and (68), we have

$$\begin{aligned} & [4 - \{A(\xi)\}^2]S(X,Z) \\ = \; & 2[\{A(\xi)\}^2 - 4n]g(X,Z) + 4(n-1)A(\xi)g(\phi X,Z) \\ + \; & (n+1)[4 - \{A(\xi)\}^2]\eta(X)\eta(Z) + 8(n-1)A(\phi X)\eta(Z) \\ - \; & 2(n-1)A(\xi)\big[2A(X)\eta(Z) + A(Z)\eta(X)\big]. \end{aligned} \tag{69}$$

This leads to the following:

Theorem 10. *In a ϕ-pseudo Ricci symmetric Para-Sasakian manifold with quarter symmetric metric connection the Ricci tensor is of the form* (69).

In particular, if $A = 0$ then from (69), we get

$$S(X,Z) = -2ng(X,Z) + (n+1)\eta(X)\eta(Z), \tag{70}$$

which implies that the manifold under consideration is η-Einstein. This leads the following:

Corollary 11. *A ϕ-Ricci symmetric Para-Sasakian manifold with quarter symmetric metric connection is an η-Einstein manifold.*

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