

COMMENT ON "ON THE INFLUENCE OF VARIABLE VISCOSITY  
ON LAMINAR MAGNETOHYDRODYNAMIC THERMAL  
OSCILLATORY FLOW PAST A LIMITING SURFACE WITH  
VARIABLE SUCTION" [ACTA UNIVERSITATIS APULENSIS, NO.  
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ABSTRACT. In the present comment, we point out some weak points in the above referenced paper.

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1. DISCUSSION

In the above paper (Gbadeyan et al. [1]), the influence of variable viscosity on laminar magneto-hydrodynamic thermal oscillatory flow past a limiting surface with variable suction has been studied. Approximate solutions are obtained for the expression for velocity, induced magnetic and temperature when the magnetic Prandtl number  $P_m = 1$ . All the results have been presented for water at  $20^{\circ}C$  with Prandtl numbers 7.0. However, there are two weak points in this paper and therefore the presented results do not have any practical value. This argument is explained below:

1. In the transformed energy equation (11) the Prandtl number (P) has been assumed constant across the boundary layer. All the presented results concern for  $P=7.0$ . Since, the Prandtl number is a function of viscosity and viscosity is a functions of temperature. Taking into account that temperature varies across the boundary layer, the Prandtl number varies, too. Taking the constant Prandtl number inside the boundary layer is a wrong assumption and leads to unrealistic results as mentioned by Pantokratoras [2, 3]. Such types of problem can be treated properly either with the direct solution of the initial boundary layer equations and treating the fluid properties as functions of temperature [2-3] or considering the Prandtl number as a variable in the transformed equations [4-5].

2. The important new thing in this work is the assumption that, except for the applied external uniform magnetic field, the electrically conducting fluid induces a new magnetic field. However, the importance of the induced magnetic field depends on the magnetic Reynolds number which is defined as follows [6]:

$$R_m = \mu\sigma ul, \quad (1)$$

where,  $\mu$  is the magnetic permeability,  $\sigma$  is the fluid electrical conductivity,  $u$  is the characteristic velocity of the flow, and  $l$  is the characteristic length scale. If the magnetic Reynolds number is much smaller than unity ( $R_m \ll 1$ ) then the induced magnetic field is negligible and the imposed external magnetic field is unaffected by the moving conducting fluid [6]. In the above work (Gbadeyan et al. [1]), the author took into account the induced magnetic field without any reference to the magnetic Reynolds number which is the suitable criterion.

Let us calculate here  $R_m$  for water ( $Pr=7.0$  at  $20^{\circ}C$ ). Water electrical conductivity at  $20^{\circ}C$  is  $10^{-4}\Omega^{-1}m^{-1}$ , [7, 8], whereas water magnetic permeability is  $1.257 * 10^{-6}$  Vs /Am, [9]. For a typical velocity  $u=1$  m/s and a typical length scale  $l=0.1$  m, the magnetic Reynolds number (dimensionless) is

$$R_m \cong 1.257 * 10^{-11}. \quad (\text{Sharma [10]}) \quad (2)$$

Instead of using the above magnetic Reynolds number, the author used the parameter  $P_m$  named as Magnetic Prandtl number (dimensionless),

$$P_m = \sigma\mu_0\nu_0. \quad (3)$$

where,  $\sigma$  is the fluid electrical conductivity,  $\mu_0$  is the magnetic permeability  $\nu_0$  is the fluid kinematic viscosity. In this paper (Gbadeyan et al. [1]), all the presented results are for water ( $P=7.0$ ) and  $P_m = 1.0$ .

Let us calculate the  $P_m$  for water at  $20^{\circ}C$ . The water kinematic viscosity at  $20^{\circ}C$  is  $9.8 * 10^{-7}$   $m^2/s$  [11] and we have

$$P_m \cong 1.23 * 10^{-16}. \quad (4)$$

In conclusion, for the used fluid (water), the magnetic Reynolds number as well as the magnetic Prandtl number is very small and completely different from the values used in the results. Water cannot induce a significant magnetic field, hence, the results presented in the above paper do not have any practical value.

Taking into the above arguments, it is clear that the results included in the paper (Gbadeyan et al. [1]) are wrong both from a theoretical and practical point of view.

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