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RESULTS ON MEROMORPHIC FUNCTIONS SHARING TWO SETS WITH REDUCED CARDINALITY AND WEIGHT

Arindam Sarkar, Paulomi Chattopadhyay

Abstract. We prove two uniqueness theorems of two nonconstant meromorphic functions sharing two sets which improve results of H.X.Yi and W.R.Lu, I.Lahiri, Fang-Lahiri and Banerjee.

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1. Introduction and necessary background materials.

Let f and g be two non constant meromorphic functions defined in the open complex plane $\mathbb{C}$. If for some $a \in \mathbb{C}\cup\{\infty\}$, f and g have the same set of a-points with the same multiplicities, we say that f and g share the value a CM (Counting Multiplicities)and if we do not consider the multiplicities, then f and g are said to share the value a IM (Ignoring Multiplicities). We do not explain the standard notations and definitions of the value distribution theory as these are available in [9]. Let S be a set of distinct elements of $\mathbb{C}\cup\{\infty\}$ and $E_f(S) = \bigcup_{a\in S}\{z : f(z) - a = 0\}$, where each zero is counted according to its multiplicity. If we do not count the multiplicity then we replace the above set by $\overline{E}_f(S)$. If $E_f(S) = E_g(S)$ we say that f and g share the set S CM. On the other hand if $\overline{E}_f(S) = \overline{E}_g(S)$, we say that f and g share the set S IM. When we let r, a real number, tend towards ∞ we will always assume that while approaching to ∞, r may avoid some subset E, say, of the real line of finite measure, not necessarily the same at every occurrence.

In 1976 F.Gross proposed the following question in [8].

Question A. *Can one find finite sets S_j,$j = 1, 2$ such that any two nonconstant entire functions f and g satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1, 2$ must be identical ?*

Gross also raised question about the cardinalities of such sets if it exist.

Yi[17] and independently Fang and Xu[5] gave the one and same positive answer to this question. Now it is natural to ask the following question.

Question B. *Can one find finite sets S_j, $j = 1, 2$ such that any two nonconstant meromorphic functions f and g satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1, 2$ must be identical ?*

In 1994 Yi[15] gave an affirmative answer to Question B and proved that there exist two finite sets S_1(with two elements)and S_2(with nine elements) such that any two nonconstant meromorphic functions f and g satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1, 2$ must be identical.

In 1996 Li and Yang [13] proved that there exist two finite sets S_1(with one element)and S_2(with fifteen elements) such that any two nonconstant meromorphic functions f and g satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1, 2$ must be identical.

In 1997 Fang and Guo[4] obtained a better result than that of Li and Yang. They succeeded in establishing the above result with two sets with less cardinalities namely S_1 with one element and S_2 with nine elements.

Suppose that the polynomial $P(w)$is defined by

$$P(w) = aw^n - n(n-1)w^2 + 2n(n-2)bw - (n-1)(n-2)b^2 \tag{1}$$

where $n \geq 3$ is an integer and a and b are two nonzero complex numbers satisfying $ab^{n-2} \neq 2$. We also define

$$R(w) = \frac{aw^n}{n(n-1)(w-\alpha_1)(w-\alpha_2)}, \tag{2}$$

where $\alpha_1, \alpha_2$are two distinct roots of $n(n-1)w^2 - 2n(n-2)bw + (n-1)(n-2)b^2 = 0$. It can be shown that $P(w)$ has only simple roots.{See [1,2].}

In 2002 Yi[19] proved the following result in which he not only reduced the cardinalities of the set S but also relaxed the sharing of the poles from CM to IM.

Theorem A.[19] *Let $S = \{w \mid P(w) = 0\}$, where $P(w)$is given by (1)and $n(\geq 8)$. Suppose that f and g are two nonconstant meromorphic functions such that $E_f(S) = E_g(S)$ and $\overline{E}_f(\{\infty\}) = \overline{E}_g(\{\infty\})$ then $f \equiv g$.*

As a consequence of Question B, Yi and Lü[20] raised the following question in 2004.

Question C. *Can one find finite sets S_j,$j = 1, 2$ such that any two nonconstant meromorphic functions f and g satisfying for $\overline{E}_f(S_j) = \overline{E}_g(S_j)$ $j = 1, 2$ must be identical ?*

In this direction they established the following results which also improved results already obtained by Yi[16].

Theorem B.[20] *Let $S = \{w \mid P(w) = 0\}$, where $P(w)$is given by (1)and $n(\geq 12)$. Suppose that f and g are two nonconstant meromorphic functions such that $\overline{E}_f(S) = \overline{E}_g(S)$ and $E_f(\{\infty\}) = E_g(\{\infty\})$ then $f \equiv g$.*

Theorem C.[20] *Let $S = \{w \mid P(w) = 0\}$, where $P(w)$is given by (1)and $n(\geq 13)$. Suppose that f and g are two nonconstant meromorphic functions such that $\overline{E}_f(S) = \overline{E}_g(S)$ and $\overline{E}_f(\{\infty\}) = \overline{E}_g(\{\infty\})$ then $f \equiv g$.*

In 2001 Lahiri introduced the notion of weighted sharing as follows.

Definition 1.[10,11] *Let k be a nonnegative integer or infinity. For $a \in \mathbb{C} \cup \{\infty\}$ we denote by $E_k(a; f)$ the set of all a-points of f where an a-point of multiplicity m is counted m times if $m \leq k$ and $k + 1$times if $m > k$. If $E_k(a; f) = E_k(a; g)$, we say that f and g share the value a with weight k.*

The definition implies that if f, g share a value a with weight k, then z_0 is a zero of $f - a$ with multiplicity $m(\leq k)$ if and only if it is a zero of $g - a$ with multiplicity $m(\leq k)$ and z_0 is a zero of $f - a$ of multiplicity $m(> k)$ if and only if it is a zero of $g - a$ with multiplicity $n(> k)$ where m is not necessarily equal to n.

We write f, g share (a, k) to mean f, g share the value a with weight k. Clearly if f, g share (a, k) then f, g share (a, p) for all integers p , $0 \leq p < k$. Also we note that f, g share a value a IM or CM if and only f, g share $(a, 0)$ or (a, ∞) respectively.

Definition 2.[11] *Let S be a set of distinct elements of $\mathbb{C} \cup \{\infty\}$ and k be a positive integer or ∞. We denote by $E_f(S, k)$the set $\bigcup_{a \in S} E_k(a; f)$. Clearly $E_f(S) = E_f(S, \infty)$ and $\overline{E}_f(S) = E_f(S, 0)$.*

Recently Banerjee[1] improved and supplemented Theorem A and Theorem B as follows.

Theorem D.[1] *Let $S = \{w \mid P(w) = 0\}$, where $P(w)$is given by (1)and $n(\geq 8)$. Suppose that f and g are two nonconstant meromorphic functions such that $E_f(S, 2) = E_g(S, 2)$ and $E_f(\{\infty\}, 0) = E_g(\{\infty\}, 0)$ then $f \equiv g$.*

Theorem E.[1] *Let $S = \{w \mid P(w) = 0\}$, where $P(w)$is given by (1)and $n(\geq 9)$. Suppose that f and g are two nonconstant meromorphic functions such that $E_f(S, 1) = E_g(S, 1)$ and $E_f(\{\infty\}, 0) = E_g(\{\infty\}, 0)$ then $f \equiv g$.*

Theorem F.[1] *Let $S = \{w \mid P(w) = 0\}$, where $P(w)$is given by (1)and $n(\geq 12)$. Suppose that f and g are two nonconstant meromorphic functions such that $E_f(S, 0) = E_g(S, 0)$ and $E_f(\{\infty\}, 3) = E_g(\{\infty\}, 3)$ then $f \equiv g$.*

Note that none of the above mentioned theorems of Banerjee improves Theorem C, which has been claimed to be the best result till date in[20]. In a most recent paper Banerjee, however established the following result as a special case of which one can obtain Theorem C as well as Theorem F.

Theorem G.[2] *Let $S = \{w \mid P(w) = 0\}$, where $P(w)$is given by (1)and $n(\geq 9)$. If f and g be two nonconstant meromorphic functions such that $E_f(S, 0) = E_g(S, 0)$*

and $E_f(\{\infty\},k)=E_g(\{\infty\},k)$ *and*

$$\frac{11}{4}min\{\Theta_f,\Theta_g\}>\frac{9}{2}+\frac{2(n-3)}{(n-5)\{(n-2)k+(n-3)\}}+\frac{10}{n-5}-\frac{n}{2}$$

then $f\equiv g$, *where* $\Theta_f=\Theta(0;f)+\Theta(b;f)$ *and* Θ_g *is defined similarly.*

Remark 1. In Theorem G when $n\geq 12$ and $k=3$ we get Theorem F. Again when $n\geq 13$ and $k=0$ we get Theorem C. Thus Theorem G improves both Theorems C and F.

Strictly speaking Theorem G is a generalization of Theorems C and F rather than direct improvements since it can neither reduce the cardinality of the shared set S in Theorem C nor it reduces the weight of the shared set $\{\infty\}$ in Theorem F. In this paper we propose our first theorem below as a corollary of which we may get the desired improvements of Theorem C and Theorem F.

Theorem 1. *Let* $S=\{w\mid P(w)=0\}$, *where* $P(w)$*is given by (1)and* $n(\geq 9)$. *If* f *and* g *be two nonconstant meromorphic functions such that* $E_f(S,0)=E_g(S,0)$ *and* $E_f(\{\infty\},k)=E_g(\{\infty\},k)$ *and*

$$\begin{aligned}min\{3\Theta(0;f)+2\Theta(b;f),3\Theta(0;g)+2\Theta(b;g)\} &> 4+\frac{8}{n-5}\\ &+\frac{2n-6}{(n-5)\{(n-2)k+(n-3)\}}-\frac{n}{2}\end{aligned}$$

then $f\equiv g$.

Following corollary is a natural consequence of above theorem.

Corollary 1. *Let* $S=\{w\mid P(w)=0\}$, *where* $P(w)$*is given by (1)and* $n(\geq 12)$. *If* f *and* g *be two nonconstant meromorphic functions such that* $E_f(S,0)=E_g(S,0)$ *and* $E_f(\{\infty\},0)=E_g(\{\infty\},0)$ *then* $f\equiv g$.

Recently Banerjee also obtained the following results in two different papers where he has considered the shared set S with less number of elements to obtain the uniqueness of functions under different conditions improving some previous results.

Theorem H.[2] *Let* $S=\{w\mid P(w)=0\}$, *where* $P(w)$*is given by (1)and* $n(\geq 6)$. *If* f *and* g *be two nonconstant meromorphic functions such that* $E_f(S,2)=E_g(S,2)$ *and* $E_f(\{\infty\},0)=E_g(\{\infty\},0)$ and

$$2min\{\Theta_f,\Theta_g\}>3+\frac{3}{2(n-3)}+\frac{6}{3n-11}-\frac{n}{2}$$

then $f\equiv g$, *where* $\Theta_f=\Theta(0;f)+\Theta(b;f)$ *and* Θ_g *is defined similarly.*

Theorem I.[3] *Let $S = \{w \mid P(w) = 0\}$, where $P(w)$is given by (1)and $n(\geq 7)$. If f and g be two nonconstant meromorphic functions such that $E_f(S, 2) = E_g(S, 2)$ and $E_f(\{\infty\}, \infty) = E_g(\{\infty\}, \infty)$ and*

$$min\{\Theta_f^1, \Theta_g^1\} > 7 + \frac{2}{n-3} - n$$

then $f \equiv g$, where $\Theta_f^1 = 4\Theta(0; f) + 4\Theta(b; f) + \Theta(\infty; f)$ and Θ_g^1 is defined similarly.

In our next Theorem we improve Theorem I by reducing the cardinality of the set S from 7 to 5 and extending the Theorem for any weight k, for the shared set $\{\infty\}$. Also we claim that our next result will also improve Theorem H.Thus our next result will combine both Theorems H and I in an improved result. Note that in the definition of the polynomial $P(w)$, we require $ab^{n-2} \neq 2$. For our purpose, in addition to it we assume $ab^{n-2} \neq 1$, by which the polynomial $P(w)$ will not lose any of its properties mentioned above. Thus from now on our set S is given by $S = \{w \mid P(w) = 0\}$ where $P(w)$ is given by (1) with $ab^{n-2} \neq 2, 1$.

We state below our next Theorem:

Theorem 2. *Let $S = \{w \mid P(w) = 0\}$, where $P(w)$is given by (1)and $n(\geq 5)$ and $ab^{n-2} \neq 2, 1$. Suppose that f and g are two nonconstant meromorphic functions such that $E_f(S, 2) = E_g(S, 2)$ and $E_f(\{\infty\}, k) = E_g(\{\infty\}, k)$where k is a nonnegative integer or ∞.*

If

$$min\{\Theta_f^1, \Theta_g^1\} > 7 + \frac{2}{n-3} + \frac{8n-24}{(3n-11)\{(n-2)k+n-3\}} - n,$$

then $f \equiv g$ where Θ_f^1 and Θ_g^1 are same as Theorem I.

Remark 2. When $k = \infty$ in Theorem 2 we get the conclusion of Theorem I with the shared set S containing less number of elements(five elements). Thus Theorem 2 improves Theorem I.

When $n \geq 8$ in Theorem 2 we obtain Theorem D. Thus Theorem 2 improves Theorem D. Also it is easy to verify that the condition on ramification index in this theorem is weaker than the condition in the Theorem H for $n = 6$ and $n = 7$. Since when $n \geq 8$ the condition on ramification indices cease to exist both in Theorems H and 2, Theorem 2 improves Theorem H.

We close this section with a few more definitions.

Definition 3. *For $a \in \mathbb{C} \cup \{\infty\}$ For a positive integer m we denote by $N(r, a; f \mid\geq m)$ the counting function of those a-points of f whose multiplicities are not less than m where each a-point is counted according to its multiplicity. We agree to write $\overline{N}(r, a; f \mid\geq m)$ to denote the corresponding reduced counting function.*

Definition 4.[10,18,20] *Let f and g be two nonconstant meromorphic functions such that f and g share (a,k) where $a \in \mathbb{C}\cup\{\infty\}$. Let z_0 be an a-point of f with multiplicity p, an a-point of g of multiplicity q. We denote by $\overline{N}_L(r,a;f)(\overline{N}_L(r,a;g))$ the counting function of those a-points of f and g where $p>q(q>p)$, by $\overline{N}_E^{(k+1}(r,a;f)$ the counting functions of those a-points of f and g where $p=q\geq k+1$ each point in these counting functions is counted only once. In the same way we can define $\overline{N}_E^{(k+1}(r,a;g)$. Clearly $\overline{N}_E^{(k+1}(r,a;f)=\overline{N}_E^{(k+1}(r,a;g)$. We denote by $\overline{N}_*(r,a;f,g)$ the reduced counting function of those a-points of f whose multiplicities differ from the corresponding a-points of g. Clearly $\overline{N}_*(r,a;f,g)=\overline{N}_*(r,a;g,f)$ and $\overline{N}_*(r,a;f,g)$ $=\overline{N}_L(r,a;f)+\overline{N}_L(r,a;g)$. We also denote by $N_E^{1)}(r,a;f)$ the counting function of those a-points of f, g for which $p=q=1$.*

Definition 5.[1] *Let f and g share the value 1 IM.Let z_0 be a 1-point of f and g with multiplicities p and q respectively. Let s be a positive integer. We denote by $\overline{N}_{f>s}(r,1;g)$ the reduced counting function of those 1-points of f and g such that $p>q=s$.*

2. Lemmas

In this section we present some lemmas which will be required to establish our results. In the lemmas several times we use the function H defined by $H=\frac{F''}{F'}-\frac{2F'}{F-1}-\frac{G''}{G'}+\frac{2G'}{G-1}$.

Let f and g be two nonconstant meromorphic functions and

$$F=R(f),G=R(g), \tag{3}$$

where $R(w)$ is given by (2). From (2) and (3) it is clear that

$$T(r,f)=\frac{1}{n}T(r,F)+S(r,f),T(r,g)=\frac{1}{n}T(r,G)+S(r,g). \tag{4}$$

Lemma 1.[2] *Let F and G be given by (3)where $n\geq 3$ is an integer and $H\not\equiv 0$. If F, G share $(1,m)$and f,g share (∞,k), where $0\leq m<\infty$. Then*

$$\begin{aligned}\{\frac{n}{2}+1\}\{T(r,f)+T(r,g)\} \leq\ & 2[\overline{N}(r,0;f)+\overline{N}(r,0;g)+\overline{N}(r,b;f)+\overline{N}(r,b;g)] \\ & +\overline{N}(r,\infty;f)+\overline{N}(r,\infty;g)+\overline{N}_*(r,\infty;f,g) \\ & -(m-\frac{3}{2})\overline{N}_*(r,1;F,G)+S(r,f)+S(r,g).\end{aligned}$$

Lemma 2.[1] *Let F and G be given by (3) and $H \not\equiv 0$. If F, G share $(1,m)$ and f,g share (∞,k), where $0 \le m < \infty, 0 \le k < \infty$, then*

$$\begin{aligned}[(n-2)k+n-3]\overline{N}(r,\infty;f \mid\ge k+1) &= [(n-2)k+n-3]\overline{N}(r,\infty;g \mid\ge k+1)\\ &\le \overline{N}(r,0;f)+\overline{N}(r,0;g)\\ &\quad +\overline{N}_*(r,1;F,G)+S(r,f)+S(r,g).\end{aligned}$$

Lemma 3.[1] *Let F and G be given by (3) and $H \not\equiv 0$. If F, G share $(1,m)$and f, g share (∞,k), where $0 \le m < \infty, 0 \le k < \infty$, then*

$$\begin{aligned}[(n-2)k+n-3]\overline{N}(r,\infty;f \mid\ge k+1) &= [(n-2)k+n-3]\overline{N}(r,\infty;g \mid\ge k+1)\\ &\le \frac{m+2}{m+1}[\overline{N}(r,0;f)+\overline{N}(r,0;g)]\\ &+ \frac{2}{m+1}\overline{N}(r,\infty;f)+S(r,f)+S(r,g).\end{aligned}$$

Lemma 4.[2] *Let F and G be given by (3). Also let S be given as in Theorem 1, where $n \ge 3$ is an integer. If $E_f(S,0)=E_g(S,0)$ then $S(r,f)=S(r,g)$.*

Lemma 5. *If f and g share $(1,0)$ then*
$N(r,1;g)-\overline{N}(r,1;g)$
$\ge 2\overline{N}_L(r,1;g)+\overline{N}_L(r,1;f)+\overline{N}_E^{(2}(r,1;f)+\overline{N}_E^{(3}(r,1;f)-\overline{N}_{f>1}(r,1;g)-\overline{N}_{g>1}(r,1;f)$.

Proof. Let z_0 be a 1-point of f and g of respective multiplicities p and q.We denote by $N_2(r)$and $N_3(r)$the counting functions of those 1-points of f and g when $2 \le q = p$ and $1 \le p < q$ respectively where each point in these counting functions is counted $q-2$ times. Since f,g share $(1,0)$ we have
$N(r,1;g)-\overline{N}(r,1;g) \ge \overline{N}_L(r,1;g)+N_3(r)+N_2(r)+\overline{N}_E^{(2}(r,1;f)+\overline{N}_L(r,1;f)-\overline{N}_{f>1}(r,1;g)$.

Now observing $N_2(r) \ge \overline{N}_E^{(3}(r,1;f)$ and $N_3(r) \ge \overline{N}_L(r,1;g)-\overline{N}_{g>1}(r,1;f)$our lemma follows from above.

Lemma 6.[2] *Let F, G be given by (3). If F, G share $(1,m)$,where $0 \le m < \infty$,then*
(i) $\overline{N}_L(r,1;F) \le \frac{1}{m+1}[\overline{N}(r,0;f)+\overline{N}(r,\infty;f)]+S(r,f)$,
(ii) $\overline{N}_L(r,1;G) \le \frac{1}{m+1}[\overline{N}(r,0;g)+\overline{N}(r,\infty;g)]+S(r,g)$

Lemma 7.[1] *Let F, G be given by (3)and $H \not\equiv 0$. If F, G share $(1,m)$ and f, g share (∞,k),where $0 \le k \le \infty$,then*
$N_{E}^{1)}(r,1;F) \le \overline{N}_L(r,1;F)+\overline{N}_L(r,1;G)+\overline{N}(r,0;f)+\overline{N}(r,b;f)+\overline{N}_*(r,\infty;f,g)$ $+\overline{N}(r,0;g)+\overline{N}(r,b;g)+\overline{N}_0(r,0;f')+\overline{N}_0(r,0;g')+S(r,F)+S(r,G)$

where $\overline{N}_0(r,0;f')$ *denotes the reduced counting function corresponding to the zeros of* f'*which are not the zeros of* $f(f-b)$ *and* $F-1$, $\overline{N}_0(r,0;g')$*is defined similarly.*

Lemma 8. *Let* F *and* G *be given by (3). If* F, G *share* $(1,0)$ *and* f, g *share* (∞,k) *and* $H\not\equiv 0$ *then*
$(n+1)T(r,f)+T(r,g)\leq 2\{\overline{N}(r,0;f)+\overline{N}(r,b;f)+\overline{N}(r,0;g)+\overline{N}(r,b;g)+\overline{N}(r,\infty;f)\}$
$+\overline{N}(r,\infty;f\mid\geq k+1)+2\overline{N}_L(r,1;F)+S(r,f)+S(r,g).$

Proof. We denote by $N_0(r,0;f')$ the counting function of those zeros of f' which are not the zeros of $f(f-1)$ and $F-1$. $N_0(r,0;g')$ is defined similarly. By the second fundamental theorem we get

$(n+1)T(r,f)+(n+1)T(r,g)$
$\leq \overline{N}(r,1;F)+\overline{N}(r,0;f)+\overline{N}(r,b;f)+\overline{N}(r,\infty;f)+\overline{N}(r,1;G)+\overline{N}(r,0;g)+\overline{N}(r,b;g)+\overline{N}(r,\infty;g)-N_0(r,0;f')-N_0(r,0;g')+S(r,g)+S(r,f)$
$=\{\overline{N}(r,0;f)+\overline{N}(r,b;f)+\overline{N}(r,\infty;f)+\overline{N}(r,0;g)+\overline{N}(r,b;g)+\overline{N}(r,\infty;g)\}+N_E^{1)}(r,1;F)+\overline{N}(r,1;F\mid\geq 2)+\overline{N}(r,1;G)-N_0(r,0;f')-N_0(r,0;g')+S(r,g)+S(r,f)$

Note that since F, G share $(1,0)$ we have

$\overline{N}(r,1;F\mid\geq 2)=\overline{N}_E^{(2}(r,1;F)+\overline{N}_L(r,1;F)+\overline{N}_L(r,1;G)-\overline{N}_{G>1}(r,1;F).$

Since f,g share (∞,k), $\overline{N}_*(r,\infty;f,g)\leq\overline{N}(r,\infty;f\mid\geq k+1)$, and hence using Lemma 7 with $m=0$ and Lemma 5 we obtain from above

$(n+1)T(r,f)+(n+1)T(r,g)$
$\leq\{\overline{N}(r,0;f)+\overline{N}(r,b;f)+\overline{N}(r,\infty;f)+\overline{N}(r,0;g)+\overline{N}(r,b;g)+\overline{N}(r,\infty;g)\}$
$+\overline{N}_L(r,1;F)+\overline{N}_L(r,1;G)+\overline{N}(r,0;f)+\overline{N}(r,b;f)$
$+\overline{N}_*(r,\infty;f,g)+\overline{N}(r,0;g)+\overline{N}(r,b;g)+\overline{N}_0(r,0;f')+\overline{N}_0(r,0;g')$
$+\overline{N}_E^{(2}(r,1;F)+\overline{N}_L(r,1;F)+\overline{N}_L(r,1;G)-\overline{N}_{G>1}(r,1;F)$
$+\overline{N}(r,1;G)-N_0(r,0;f')-N_0(r,0;g')+S(r,g)+S(r,f)$
$\leq 2\{\overline{N}(r,0;f)+\overline{N}(r,b;f)+\overline{N}(r,0;g)+\overline{N}(r,b;g)\}$
$+\overline{N}(r,\infty;f)+\overline{N}(r,\infty;g)+\overline{N}(r,\infty;f\mid\geq k+1)+\overline{N}_L(r,1;F)+N(r,1;G)$
$+\overline{N}_{F>1}(r,1;G)+S(r,f)+S(r,g)$
$\leq 2\{\overline{N}(r,0;f)+\overline{N}(r,b;f)+\overline{N}(r,0;g)+\overline{N}(r,b;g)+\overline{N}(r,\infty;f)\}$
$+\overline{N}(r,\infty;f\mid\geq k+1)+2\overline{N}_L(r,1;F)+nT(r,g)-m(r,1;G)+S(r,f)+S(r,g).$

Therefore:

$(n+1)T(r,f)+T(r,g)$
$\leq 2\{\overline{N}(r,0;f)+\overline{N}(r,b;f)+\overline{N}(r,0;g)+\overline{N}(r,b;g)+\overline{N}(r,\infty;f)\}$
$+\overline{N}(r,\infty;f\mid\geq k+1)+2\overline{N}_L(r,1;F)+S(r,f)+S(r,g).$
This completes the proof.

Lemma 9.[2] *Let f, g be two non-constant meromorphic functions sharing $(\infty, 0)$ and suppose that α_1 and α_2 are two distinct roots of the equation*

$$n(n-1)w^2 - 2n(n-2)bw + (n-1)(n-2)b^2 = 0.$$

Then

$$\frac{f^n}{(f-\alpha_1)(f-\alpha_2)}.\frac{g^n}{(g-\alpha_1)(g-\alpha_2)} \not\equiv \frac{n^2(n-1)^2}{a^2},$$

where $n \geq 3$ is an integer.

Lemma 10.[7] *Let*

$$Q(w) = (n-1)^2(w^n - 1)(w^{n-2} - 1) - n(n-2)(w^{n-1} - 1)^2,$$

then

$$Q(w) = (w-1)^4(w-\beta_1)(w-\beta_2)..(w-\beta_{2n-6})$$

where $\beta_j \in \mathbb{C} \setminus \{0, 1\}$, $(j = 1, 2, .., 2n-6)$ which are pairwise distinct.

Lemma 11.*Let F, G be given by (5), where $n \geq 4$ is an integer. If f, g share $(\infty, 0)$ then $F \equiv G \Rightarrow f \equiv g$.*

Proof. From the definitions of F, G we observe that

$$F \equiv G \Rightarrow \frac{f^n}{(f-\alpha_1)(f-\alpha_2)} \equiv \frac{g^n}{(g-\alpha_1)(g-\alpha_2)}.$$

Therefore f, g share $(0, \infty)$ and (∞, ∞). Then from above and in view of the definition of $R(w)$ we obtain

$$n(n-1)f^2g^2(f^{n-2}-g^{n-2})-2n(n-2)bfg(f^{n-1}-g^{n-1})+(n-1)(n-2)b^2(f^n-g^n) = 0. \tag{5}$$

Let $h = \frac{f}{g}$ that is $f = gh$ which on substitution in (5) yields

$$n(n-1)h^2g^2(h^{n-2}-1)-2n(n-2)bhg(h^{n-1}-1)+(n-1)(n-2)b^2(h^n-1) = 0. \tag{6}$$

Note that since f and g share $(0, \infty)$ and (∞, ∞),$0, \infty$ are the exceptional values of Picard of h. If h is non-constant then from Lemma 2.10 and (6) we have

$$\{n(n-1)h(h^{n-2}-1)g - n(n-2)b(h^{n-1}-1)\}^2 = -n(n-2)b^2Q(h) \tag{7}$$

where $Q(h) = (h-1)^4(h-\beta_1)(h-\beta_2)...(h-\beta_{2n-6})$, $\beta_j \in \mathbb{C} \setminus \{0, 1\}$,$j = 1, 2, .., 2n-6$ which are pairwise distinct. From (7) we observe that each zero of $h - \beta_j$,$j = 1, 2, .., 2n-6$ is of order at least two.Therefore by the second main theorem we obtain

$$(2n-6)T(r,h) \leq \overline{N}(r,\infty;h) + \overline{N}(r,0;h) + \sum_{j=1}^{2n-6} \overline{N}(r,\beta_j;h) + S(r,h)$$
$$\leq \tfrac{1}{2}(2n-6)T(r,h) + S(r,h),$$

which is a contradiction for $n \geq 4$. Thus h must be a constant. From (7) it follows that $h^{n-2} - 1 = 0$ and $h^{n-1} - 1 = 0$ which implies that $h \equiv 1$. Therefore $f \equiv g$. This completes the proof.

Lemma 12.[2] *Let F, G be given by (3) and S be defined as in Theorem 1, where $n \geq 4$. If $E_f(S,0) = E_g(S,0)$ then $S(r,f) = S(r,g)$.*

3. Proof of theorems

Proof of Theorem 1. Since $E_f(S,0) = E_g(S,0)$, we see that F, G share $(1,0)$. We first suppose that $H \not\equiv 0$. From Lemma 3 we obtain for $m = 0$ and $k = 0$,

$$\overline{N}(r,\infty;f) \leq \frac{2}{n-5}\{\overline{N}(r,0;f) + \overline{N}(r,0;g)\}$$

and for $m = 0$ and $k = k$,

$$\overline{N}(r,\infty;f \mid\geq k+1) \leq \frac{2n-6}{(n-5)[(n-2)k+(n-3)]}\{\overline{N}(r,0;f) + \overline{N}(r,0;g)\}.$$

Hence using the above inequalities we obtain from Lemma 8 and Lemma 6 with $m = 0$

$$\begin{aligned}(n+1)T(r,f) + T(r,g) \quad &\leq \quad 4\overline{N}(r,0;f) + 4\overline{N}(r,\infty;f) + 2\overline{N}(r,b;f) + 2\overline{N}(r,0;g) \\ +2\overline{N}(r,b;g) + \overline{N}(r,\infty;f \quad &\mid \quad \geq k+1) + S(r,f) + S(r,g)\end{aligned} \tag{8}$$

Similarly we obtain

$$\begin{aligned}(n+1)T(r,g) + T(r,f) \quad &\leq \quad 4\overline{N}(r,0;g) + 4\overline{N}(r,\infty;f) + 2\overline{N}(r,b;g) + 2\overline{N}(r,0;f) \\ +2\overline{N}(r,b;f) + \overline{N}(r,\infty;f \quad &\mid \quad \geq k+1) + S(r,f) + S(r,g)\end{aligned} \tag{9}$$

Combining (8) and (9)we obtain from above for $\epsilon > 0$

$$\begin{aligned}&(n+2)\{T(r,f) + T(r,g)\} \\ &\quad \leq 6\overline{N}(r,0;f) + 8\overline{N}(r,\infty;f) + 4\overline{N}(r,b;f) \\ &\quad\quad + 6\overline{N}(r,0;g) + 4\overline{N}(r,b;g) + 2\overline{N}(r,\infty;f \mid\geq k+1) + S(r,f) + S(r,g) \\ &\quad \leq 6\overline{N}(r,0;f) + 4\overline{N}(r,b;f) + 6\overline{N}(r,0;g) + 4\overline{N}(r,b;g) \\ &\quad\quad + \tfrac{16}{n-5}\{\overline{N}(r,0;f) + \overline{N}(r,0;g)\} + \tfrac{4n-12}{(n-5)[(n-2)k+(n-3)]}\{\overline{N}(r,0;f) + \overline{N}(r,0;g)\} \\ &\quad\quad + S(r,f) + S(r,g)\end{aligned}$$

$\leq \{10 - 6\Theta(0;f) - 4\Theta(b,f) + \epsilon\}T(r,f) + \{10 - 6\Theta(0;g) - 4\Theta(b,g) + \epsilon\}T(r,g)$
$+ \{\frac{16}{n-5} + \frac{4n-12}{(n-5)[(n-2)k+(n-3)]}\}\{T(r,f) + T(r,g)\}.$

and hence

$\{3\Theta(0;f) + 2\Theta(b,f) - 4 - \frac{8}{n-5} - \frac{2n-6}{(n-5)[(n-2)k+(n-3)]} + \frac{n}{2} - \frac{\epsilon}{2}\}T(r,f)$
$+ \{3\Theta(0;g) + 2\Theta(b,g) - 4 - \frac{8}{n-5} - \frac{2n-6}{(n-5)[(n-2)k+(n-3)]} + \frac{n}{2} - \frac{\epsilon}{2}\}T(r,g) \leq S(r,f) + S(r,g), r \notin E.$

This leads to a contradiction for arbitrary $\epsilon > 0$. Hence $H \equiv 0$. We do not prove the rest of the part of the

Theorem as it is same as the proof of the corresponding part of Theorem 2.

Proof of Theorem 2. Since $E_f(S,2) = E_g(S,2)$ according to the definitions of F and G we observe that F, G share $(1,2)$. If possible suppose that $H \not\equiv 0$. Since $n \geq 6$, using Lemma 1 for $m = 2$ and Lemma 2 for

$k = 0$ and Lemma 3 for $m = 2$ we obtain for $\epsilon > 0$

$$\begin{aligned}
&(\tfrac{n}{2}+1)\{T(r,f)+T(r,g)\}\\
&\leq 2\{\overline{N}(r,0;f)+\overline{N}(r,0;g)+\overline{N}(r,b;f)+\overline{N}(r,b;g)\}+\overline{N}(r,\infty;f)\\
&+\overline{N}(r,\infty;g)+\overline{N}_*(r,\infty;f,g)-\tfrac{1}{2}\overline{N}_*(r,1;F,G)+S(r,f)+S(r,g)\\
&\leq 2\{\overline{N}(r,0;f)+\overline{N}(r,0;g)+\overline{N}(r,b;f)+\overline{N}(r,b;g)\}+\overline{N}(r,\infty;f)\\
&+\overline{N}(r,\infty;g)+\overline{N}(r,\infty;f\mid\geq k+1)-\tfrac{1}{2}\overline{N}_*(r,1;F,G)+S(r,f)+S(r,g)\\
&\leq 2\{\overline{N}(r,0;f)+\overline{N}(r,0;g)+\overline{N}(r,b;f)+\overline{N}(r,b;g)\}+\overline{N}(r,\infty;f)\\
&+\overline{N}(r,\infty;g)+\tfrac{4n-12}{(3n-11)\{(n-2)k+n-3\}}[\overline{N}(r,0;f)+\overline{N}(r,0;g)]-\tfrac{1}{2}\overline{N}_*(r,1;F,G)\\
&+S(r,f)+S(r,g)\\
&\leq 2\{\overline{N}(r,0;f)+\overline{N}(r,0;g)+\overline{N}(r,b;f)+\overline{N}(r,b;g)\}+\tfrac{1}{2}\{\overline{N}(r,\infty;f)+\overline{N}(r,\infty;g)\}\\
&+\tfrac{1}{n-3}\{\overline{N}(r,\infty;f)+\overline{N}(r,\infty;g)\}+\tfrac{4n-12}{(3n-11)\{(n-2)k+n-3\}}[\overline{N}(r,0;f)+\overline{N}(r,0;g)]\\
&+S(r,f)+S(r,g)\\
&\leq (\tfrac{9}{2}-2\Theta(0,f)-2\Theta(b,f)-\tfrac{1}{2}\Theta(\infty,f)+\tfrac{1}{n-3}+\tfrac{4n-12}{(3n-11)\{(n-2)k+n-3\}}+\epsilon)T(r,f)\\
&+(\tfrac{9}{2}-2\Theta(0,g)-2\Theta(b,g)-\tfrac{1}{2}\Theta(\infty,g)+\tfrac{1}{n-3}+\tfrac{4n-12}{(3n-11)\{(n-2)k+n-3\}}+\epsilon)T(r,g).
\end{aligned}$$

Thus

$$\begin{aligned}
&\{\Theta_f-(7+\tfrac{2}{n-3}+\tfrac{8n-24}{(3n-11)\{(n-2)k+n-3\}}-n)-2\epsilon\}T(r,f)\\
&+\{\Theta_g-(7+\tfrac{2}{n-3}+\tfrac{8n-24}{(3n-11)\{(n-2)k+n-3\}}-n)-2\epsilon\}T(r,g)\\
&\leq S(r,f)+S(r,g)
\end{aligned}$$

which is a contradiction. Hence $H \equiv 0$. Then

$$F \equiv \frac{AG+B}{CG+D} \tag{10}$$

where A, B, C, D are constants such that $AD - BC \neq 0$. Also $T(r, F) = T(r, G) + O(1)$, and hence from (4)

$$T(r, f) = T(r, g) + O(1) \ . \tag{11}$$

Since $R(w) - c = \frac{a(w-b)^3 Q_{n-3}(w)}{n(n-1)(w-\alpha_1)(w-\alpha_2)}$, where $c = \frac{ab^{n-2}}{2} \neq 1, \frac{1}{2}$ and $Q_{n-3}(w)$ is a polynomial in w of degree $n-3$, then in view of the definitions of F and G we notice that

$$\begin{aligned} \overline{N}(r, c; F) &\leq \overline{N}(r, b; f) + (n-3)T(r, f) \leq (n-2)T(r, f) + S(r, f), \\ \overline{N}(r, c; G) &\leq \overline{N}(r, b; g) + (n-3)T(r, g) \leq (n-2)T(r, g) + S(r, g). \end{aligned} \tag{12}$$

Now we consider the following cases. **Case 1.**$C \neq 0$. Since f, g share (∞, ∞) it follows from (10) that ∞ is an exceptional value of Picard of f and g. Therefore in view of the definitions of F and G it follows that

$$\begin{aligned} \overline{N}(r, \infty; F) &= \overline{N}(r, \alpha_1; f) + \overline{N}(r, \alpha_2; f) \\ \overline{N}(r, \infty; G) &= \overline{N}(r, \alpha_1; g) + \overline{N}(r, \alpha_2; g). \end{aligned} \tag{13}$$

Subcase 1.1 $A \neq 0$. Suppose $B \neq 0$. Then from (10)it follows that $\overline{N}(r, -\frac{B}{A}; G) = \overline{N}(r, 0; F)$. Thus from the second main theorem we have from (4) and (13)

$$\begin{aligned} nT(r, g) &\leq \overline{N}(r, 0; G) + \overline{N}(r, \infty; G) + \overline{N}(r, -\frac{B}{A}; G) + S(r, G) \\ &\leq \overline{N}(r, 0; g) + \overline{N}(r, \alpha_1; g) + \overline{N}(r, \alpha_2; g) + \overline{N}(r, 0; f) + S(r, g) \end{aligned} \tag{14}$$

Clearly (14) leads to a contradiction if $n \geq 5$. Therefore $B = 0$. Then $F \equiv \frac{\frac{A}{C} \cdot G}{G + \frac{D}{C}}$ and $\overline{N}(r, \frac{-D}{C}; G) = \overline{N}(r, \infty; F)$. We also note that $c = \frac{ab^{n-2}}{2} \neq 0$. If possible suppose $c = \frac{-D}{C}$. Also suppose that F has no 1-points. This amounts to saying that f has no w_i-points where $w_i \in S$ and $i = 1, 2, .., n(\geq 4)$, which is not possible. Therefore F must have some 1-points. Since F, G share 1-points, we have $A = C + D = C - cC$ and hence

$$F = \frac{(C - cC)G}{CG - cC} = \frac{(1 - c)G}{G - c},$$

since $C \neq 0$ by our assumption. Then since $c \neq \frac{1}{2}$, $\overline{N}(r, c; F) = \overline{N}(r, \frac{c^2}{2c-1}; G)$. Thus by the second main theorem and (12) we have

$$\begin{aligned} 2nT(r, g) &\leq \overline{N}(r, 0; G) + \overline{N}(r, \infty; G) + \overline{N}(r, c; G) + \overline{N}(r, \tfrac{c^2}{2c-1}; G) + S(r, g) \\ &\leq \overline{N}(r, 0; g) + \overline{N}(r, \alpha_1; g) + \overline{N}(r, \alpha_2; g) + \overline{N}(r, \alpha_1; f) + \overline{N}(r, \alpha_2; f) + (n - 2)T(r, f) + S(r, g) \end{aligned}$$

$\leq (5+n-2)T(r,g)+S(r,g)$ which leads to a contradiction for $n \geq 4$.

Next let $c \neq \frac{-D}{C}$. Hence as before by the second main theorem

$$2nT(r,g) \leq \overline{N}(r,0;G)+\overline{N}(r,\infty;G)+\overline{N}(r,\tfrac{-D}{C};G)+\overline{N}(r,c;G)+S(r,G)$$
$$\leq \overline{N}(r,0;g)+\overline{N}(r,\alpha_1;g)+\overline{N}(r,\alpha_2;g)+\overline{N}(r,\alpha_1;f)+\overline{N}(r,\alpha_2;f)+ (n-2)T(r,g)+S(r,g)$$
$$\leq (5+n-2)T(r,g)+S(r,g).$$

which leads to a contradiction for $n \geq 4$.

Subcase 1.2 $A=0$. Then clearly $B \neq 0$ and $F \equiv \frac{1}{\gamma G+\delta}$ where $\gamma = \frac{C}{B}$ and $\delta = \frac{D}{B}$.

Since F and G have some 1-points, then $\gamma+\delta = 1$ and so $F \equiv \frac{1}{\gamma G+1-\gamma}$. Suppose $\gamma \neq 1$. If $\frac{1}{1-\gamma} \neq c$ then by second main theorem

$$2nT(r,f) \leq \overline{N}(r,0;F)+\overline{N}(r,\tfrac{1}{1-\gamma};F)+\overline{N}(r,c;F)+\overline{N}(r,\infty;F)+S(r,F)$$
$$\leq \overline{N}(r,0;f)+(n-2)T(r,f)+\overline{N}(r,0;g)+\overline{N}(r,\alpha_1;f)+\overline{N}(r,\alpha_2;f)+S(r,f)$$
$$\Rightarrow (n+2)T(r,f) \leq \overline{N}(r,0;f)+\overline{N}(r,0;g)+\overline{N}(r,\alpha_1;f)+\overline{N}(r,\alpha_2;f)+S(r,f),$$

which is a contradiction for $n \geq 4$.

If $c = \frac{1}{1-\gamma}$, then $F \equiv \frac{c}{(c-1)G+1}$. If $c \neq \frac{1}{1-c}$, then by the second main theorem we obtain

$$2nT(r,g) \leq \overline{N}(r,0;G)+\overline{N}(r,c;G)+\overline{N}(r,\tfrac{1}{1-c};G)+\overline{N}(r,\infty;G)+S(r,g)$$
$$\leq \overline{N}(r,0;g)+(n-2)T(r,g)+\overline{N}(r,\infty;F)+\overline{N}(r,\alpha_1;g)+\overline{N}(r,\alpha_2;g)+ S(r,g)$$

$$\leq \overline{N}(r,0;g)+(n-2)T(r,g)+\overline{N}(r,\alpha_1;f)+\overline{N}(r,\alpha_2;f)+\overline{N}(r,\alpha_1;g)+ \overline{N}(r,\alpha_2;g)+S(r,g).$$

Thus $(n+2)T(r,g) \leq \overline{N}(r,0;g)+\overline{N}(r,\alpha_1;f)+\overline{N}(r,\alpha_2;f)+\overline{N}(r,\alpha_1;g)+\overline{N}(r,\alpha_2;g)+ S(r,g)$, which leads to a contradiction for $n \geq 4$.

If $c = \frac{1}{1-c}$ then $G \equiv \frac{c(F-c)}{F}$ and as above we obtain

$$nT(r,f) \leq \overline{N}(r,0;F)+\overline{N}(r,c;F)+\overline{N}(r,\infty;F)+S(r,f)$$
$$\leq \overline{N}(r,0;f)+\overline{N}(r,0;g)+\overline{N}(r,\alpha_1;f)+\overline{N}(r,\alpha_2;f)+S(r,f).$$

Above leads to a contradiction for $n \geq 5$. Therefore we must have $\gamma = 1$ and hence $FG \equiv 1$, which is impossible by lemma 9.

Case 2.$C=0$. Clearly $A \neq 0$ and $F \equiv \alpha G+\beta$, where $\alpha = \frac{A}{D}, \beta = \frac{B}{D}$. Since F and G must have some 1-points, $\alpha+\beta = 1$ and so $F \equiv \alpha G+1-\alpha$. Suppose $\alpha \neq 1$. If $1-\alpha \neq c$, then by the second main theorem and (12) we obtain:

$$2nT(r,f) \leq \overline{N}(r,0;F)+\overline{N}(r,c;F)+\overline{N}(r,\infty;F)+\overline{N}(r,1-\alpha;F)+S(r,f)$$

$\leq \overline{N}(r,0;f)+\overline{N}(r,\infty;f)+\overline{N}(r,\alpha_1;f)+\overline{N}(r,\alpha_2;f)+(n-2)T(r,f)+\overline{N}(r,0;G)+S(r,f)$.

Thus

$$(n+2)T(r,f) \leq \overline{N}(r,0;f)+\overline{N}(r,\infty;f)+\overline{N}(r,\alpha_1;f)+\overline{N}(r,\alpha_2;f)+\overline{N}(r,0;g)+S(r,f)$$

which leads to a contradiction for $n \geq 4$.If $1-\alpha = c$,then $F \equiv (1-c)G + c$.Since $c \neq 1$we obtain from the second main theorem and (12):

$2nT(r,g) \leq \overline{N}(r,0;G) + \overline{N}(r,c;G) + \overline{N}(r,\infty;G) + \overline{N}(r,\frac{c}{c-1};G) + S(r,g)$
$\leq \overline{N}(r,0;g) + (n-2)T(r,g) + \overline{N}(r,\infty;g) + \overline{N}(r,\alpha_1;g) + \overline{N}(r,\alpha_2;g) + \overline{N}(r,0;F) + S(r,g)$.

Thus

$$(n+2)T(r,g) \leq \overline{N}(r,0;g)+\overline{N}(r,\alpha_1;g)+\overline{N}(r,\alpha_2;g)+\overline{N}(r,\infty;g)+\overline{N}(r,0;f)+S(r,f)$$

which leads to a contradiction for $n \geq 4$.

So $\alpha = 1$. Hence $F \equiv G$ and therefore by Lemma 11, $f \equiv g$.

This completes the proof.

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Arindam Sarkar
Department of Mathematics,
Kandi Raj College,
West Bengal, India-742137.
email: *arindam_ku@rediffmail.com*

Paulomi Chattopadhyay
Department of Mathematics,
Academy of Technology,
West Bengal, India-712121.
email: *paulomi.chattopadhyay@rediffmail.com*