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ON I_λ AND I_λ^*-CONVERGENCE IN RANDOM 2-NORMED SPACES

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Abstract. An ideal I is a family of subsets of positive integers $\mathbb{N}$ which is closed under taking finite unions and subsets of its elements. In [10], Kostyrko et. al introduced the concept of ideal convergence as a sequence (x_k) of real numbers is said to be I-convergent to a real number ℓ, if for each $\varepsilon > 0$ the set $\{k \in \mathbb{N} : |x_k - \ell| \geq \varepsilon\}$ belongs to I. In [18], Mursaleen and Alotaibi introduced the concept of I-convergence of sequences in random 2-normed spaces. In this paper, we define and study the notion of I_λ-convergence as a variant of the notion of ideal convergence. Also $I_\lambda-$limit points and $I_\lambda-$cluster points have been defined and the relation between them has been established. Furthermore, I_λ^*-convergence and $I_\lambda-$Cauchy sequences are introduced and studied, where $\lambda = (\lambda_n)$ is a nondecreasing sequence of positive real numbers such that $\lambda_{n+1} \leq \lambda_n + 1, \lambda_1 = 1, \lambda_n \to \infty (n \to \infty)$.

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1. Introduction

The probabilistic metric space was introduced by Menger [15] which is an interesting and important generalization of the notion of a metric space. Karakus [9] studied the concept of statistical convergence in probabilistic normed spaces. The theory of probabilistic normed spaces was initiated and developed in [2, 22, 23, 24, 25] and further it was extended to random/probabilistic 2-normed spaces by Goleţ [5] using the concept of 2-norm which is defined by Gähler [4].

The notion of statistical convergence depends on the density (natural or asympototic) of subsets of $\mathbb{N}$. A subset E of $\mathbb{N}$ is said to have natural density $\delta(E)$

if

$$\delta(E) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \chi_E(k) \quad \text{exists.}$$

Definition 1. *A sequence $x = (x_k)$ is said to be statistically convergent to ℓ if for every $\varepsilon > 0$*

$$\delta(\{k \in \mathbb{N} : |x_k - \ell| \geq \varepsilon\}) = 0.$$

In this case, we write $S - \lim x = \ell$ or $x_k \to \ell(S)$ and S denotes the set of all statistically convergent sequences.

The notion of I-convergence was initially introduced by Kostyrko et al., [10] as a generalization of statistical convergence (see [3, 21]) which is based on the structure of the ideal I of subset of natural numbers $\mathbb{N}$. Kostyrko, et. al [11] gave some of basic properties of I-convergence and dealt with extremal I-limit points. Although an ideal is defined as a hereditary and additive family of subsets of a non-empty arbitrary set X, here in our study it suffices to take I as a family of subsets of $\mathbb{N}$, positive integers, i.e. $I \subset 2^{\mathbb{N}}$, such that $A \cup B \in I$ for each $A, B \in I$, and each subset of an element of I is an element of I.

A non-empty family of sets $F \subset 2^{\mathbb{N}}$ is a filter on $\mathbb{N}$ if and only if $\phi \notin F$, $A \cap B \in F$ for each $A, B \in F$, and any superset of an element of F is in F. An ideal I is called *non-trivial* if $I \neq \phi$ and $\mathbb{N} \notin I$. Clearly I is a non-trivial ideal if and only if $F = F(I) = \{\mathbb{N} - A : A \in I\}$ is a filter in $\mathbb{N}$, called the filter associated with the ideal I. A non-trivial ideal I is called *admissible* if and only if $\{\{n\} : n \in \mathbb{N}\} \subset I$. A non-trivial ideal I is maximal if there cannot exist any non-trivial ideal $J \neq I$ containing I as a subset. Further details on ideals can be found in Kostyrko et al., (see [10]). Recall that a sequence $x = (x_k)$ of points in $\mathbb{R}$ is said to be I-convergent to a real number ℓ if $\{k \in \mathbb{N} : |x_k - \ell| \geq \varepsilon\} \in I$ for every $\varepsilon > 0$ (see [10]). In this case we write $I - \lim x_k = \ell$.

If we take $I = I_f = \{A \subseteq \mathbb{N} : A \text{ is a finite subset }\}$. Then I_f is a non-trivial admissible ideal of $\mathbb{N}$ and the corresponding convergence coincides with the usual convergence. If we take $I = I_\delta = \{A \subseteq \mathbb{N} : \delta(A) = 0\}$ where $\delta(A)$ denote the asymptotic density of the set A. Then I_δ is a non-trivial admissible ideal of $\mathbb{N}$ and the corresponding convergence coincides with the statistical convergence.

Definition 2. *[10] An admissible ideal $I \subset 2^{\mathbb{N}}$ is said to satisfy the condition (AP) if for every countable family of mutually disjoints sets $\{A_1, A_2, ...\}$ belonging to I there exists a countable family of sets $\{B_1, B_2, ...\}$ such that $A_j \Delta B_j$ is a finite set for $j \in \mathbb{N}$ and $B = \bigcup_{j=1}^{\infty} B_j \in I$.*

Definition 3. *[10] A sequence $x = (x_k)$ of points in $\mathbb{R}$ is said to be I^*-convergent to a real number ℓ if there exists a set $M \in F(I)$ (i.e. $\mathbb{N} - M \in I$), $M = \{k_m : k_1 <$*

$k_2 < ... < k_m < ...\}$ *such that* $\lim_m x_{k_m} = \ell$. *In this case we write* $I^* - \lim x_k = \ell$ *and* ℓ *is called the* $I^* - $*limit of* x.

Definition 4. *[14] Let* $\lambda = (\lambda_n)$ *be a non-decreasing sequence of positive numbers tending to infinity such that* $\lambda_{n+1} \leq \lambda_n + 1,\ \lambda_1 = 1$. *The generalized de la Vallée-Poussin mean is defined by*

$$t_n(x) = \frac{1}{\lambda_n} \sum_{k \in J_n} x_k$$

where $J_n = [n - \lambda_n + 1, n]$. *A sequence* $x = (x_k)$ *is said to be* (V,λ)*-summable to number* L *if* $t_n(x) \to L$ *as* $n \to \infty$. *In this case we write* L *is the* λ*-limit of* x. *If* $\lambda_n = n$, *then* (V,λ)*-summability reduces to* $(C,1)$*-summability.*

Definition 5. *[20] Let* $I \subset 2^{\mathbb{N}}$ *be a non-trivial ideal. A sequence* $x = (x_k)$ *is said to be* I-$[V,\lambda]$*-summable to a number* L *if, for every* $\varepsilon > 0$

$$\left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} |x_k - L| \geq \varepsilon \right\} \in I.$$

In this case we write I-$[V,\lambda]$-$\lim x = L$. *If* $I = I_f$, *then* I-$[V,\lambda]$*-summabilily becomes* $[V,\lambda]$*-summabilily (see [14]).*

Throughout the paper, we shall denote by I and λ are admissible ideal of subsets of $\mathbb{N}$ and $\lambda = (\lambda_n)$ sequence as in Definition 1.4., respectively, unless otherwise stated.

The existing literature on ideal convergence and its generalizations appears to have been restricted to real or complex sequences, but in recent years these ideas have been also extended to the sequences of fuzzy real numbers ([6, 7]), in fuzzy normed spaces [13] and intutionistic fuzzy normed spaces [12, 16], n-normed spaces [8].

2. Preliminaries

Definition 6. *A function* $f : \mathbb{R} \to \mathbb{R}_0^+$ *is called a distribution function if it is a non-decreasing and left continuous with* $\inf_{t\in\mathbb{R}} f(t) = 0$ *and* $\sup_{t\in\mathbb{R}} f(t) = 1$. *By* D^+, *we denote the set of all distribution functions such that* $f(0) = 0$.

If $a \in \mathbb{R}_0^+$, then $H_a \in D^+$, where

$$H_a(t) = \begin{cases} 1, & \text{if } t > a; \\ 0, & \text{if } t \leq a \end{cases}$$

It is obvious that $H_0 \geq f$ for all $f \in D^+$.

A *t-norm* is a continuous mapping $* : [0,1] \times [0,1] \to [0,1]$ such that $([0,1], *)$ is abelian monoid with unit one and $c*d \geq a*b$ if $c \geq a$ and $d \geq b$ for all $a, b, c \in [0,1]$. A triangle function τ is a binary operation on D^+, which is commutative, associative and $\tau(f, H_0) = f$ for every $f \in D^+$.

In [4],Gähler introduced the following concept of 2-normed space.

Definition 7. *Let X be a linear space of dimension $d > 1$ (d may be infinite). A real-valued function $||.,.||$ from X^2 into $\mathbb{R}$ satisfying the following conditions:*

(1) $||x_1, x_2|| = 0$ *if and only if x_1, x_2 are linearly dependent,*

(2) $||x_1, x_2||$ *is invariant under permutation,*

(3) $||\alpha x_1, x_2|| = |\alpha|||x_1, x_2||$, *for any $\alpha \in \mathbb{R}$,*

(4) $||x + \overline{x}, x_2|| \leq ||x, x_2|| + ||\overline{x}, x_2||$

is called an 2-norm on X and the pair $(X, ||.,.||)$ is called an 2-normed space.

A trivial example of an 2-normed space is $X = \mathbb{R}^2$, equipped with the Euclidean 2-norm $||x_1, x_2||_E =$ the volume of the parallellogram spanned by the vectors x_1, x_2 which may be given expicitly by the formula

$$||x_1, x_2||_E = |det(x_{ij})| = abs\left(\det(< x_i, x_j >)\right)$$

where $x_i = (x_{i1}, x_{i2}) \in \mathbb{R}^2$ for each $i = 1, 2$.

Recently, Goleţ [5] used the idea of 2-normed space to define the random 2-normed space.

Definition 8. *Let X be a linear space of dimension $d > 1$ (d may be infinite), τ a triangle, and $\mathcal{F} : X \times X \to D^+$. Then $\mathcal{F}$ is called a probabilistic 2-norm and $(X, \mathcal{F}, \tau)$ a probabilistic 2-normed space if the following conditions are satisfied:*

(P2N1) $\mathcal{F}(x, y; t) = H_0(t)$ *if x and y are linearly dependent, where $\mathcal{F}(x, y; t)$ denotes the value of $\mathcal{F}(x, y)$ at $t \in \mathbb{R}$,*

(P2N2) $\mathcal{F}(x, y; t) \neq H_0(t)$ *if x and y are linearly independent,*

(P2N3) $\mathcal{F}(x, y; t) = \mathcal{F}(y, x; t)$, *for all $x, y \in X$,*

(P2N4) $\mathcal{F}(\alpha x, y; t) = \mathcal{F}(x, y; \frac{t}{|\alpha|})$, *for every $t > 0, \alpha \neq 0$ and $x, y \in X$,*

(P2N5) $\mathcal{F}(x + y, z; t) \geq \tau\left(\mathcal{F}(x, z; t), \mathcal{F}(y, z; t)\right)$, *whenever $x, y, z \in X$.*
If (P2N5) is replaced by

(P2N6) $\mathcal{F}(x+y,z;t_1+t_2) \geq \mathcal{F}(x,z;t_1)*\mathcal{F}(y,z;t_2)$, *for all* $x,y,z \in X$ *and* $t_1,t_2 \in \mathbb{R}_0^+$;

then $(X,\mathcal{F},*)$ *is called a random 2-normed space (for short, R2NS).*

Remark 1. *Every 2-normed space* $(X,||.,.||)$ *can be made a random 2-normed space in a natural way, by setting*
(*i*) $\mathcal{F}(x,y;t) = H_0(t-||x,y||)$, *for every* $x,y \in X, t>0$ *and* $a*b=\min\{a,b\}, a,b \in [0,1]$;
(*ii*) $\mathcal{F}(x,y;t) = \frac{t}{t+||x,y||}$, *for every* $x,y \in X, t>0$ *and* $a*b=ab, a,b \in [0,1]$.

Definition 9. *A sequence* $x=(x_k)$ *in a random 2-normed space* $(X,\mathcal{F},*)$ *is said to be convergent (or* $\mathcal{F}$*-convergent) to* $\ell \in X$ *with respect to* $\mathcal{F}$ *if for each* $\varepsilon>0$, $\eta \in (0,1)$ *and non zero* $z \in X$ *there exists a positive integer* n_0 *such that* $\mathcal{F}(x_k-\ell,z;\varepsilon) > 1-\eta$, *whenever* $k \geq n_0$. *In this case we write* $\mathcal{F}-\lim_k x_k = \ell$, *and* ℓ *is called the* $\mathcal{F}$*-limit of* x.

Definition 10. *A sequence* $x=(x_k)$ *in a random 2-normed space* $(X,\mathcal{F},*)$ *is said to be Cauchy with respect to* $\mathcal{F}$ *if for each* $\varepsilon>0$, $\eta \in (0,1)$ *and non zero* $z \in X$ *there exists a positive integer* $n_0 = n_0(\varepsilon,z)$ *such that* $\mathcal{F}(x_k-x_m,z;\varepsilon) > 1-\eta$, *whenever* $k,m \geq n_0$.

Definition 11. *Let* $(X,\mathcal{F},*)$ *be a random 2-normed space. For* $t>0$, *the open ball* $B(x,r;t)$ *with center* $x \in X$ *and radius* $r \in (0,1)$ *is defined as*

$$B(x,r;t) = \{y \in X : \mathcal{F}(x-y,z;t) > 1-r, \ for \ \ all \ \ z \in X\}.$$

Definition 12. *Let* $(X,\mathcal{F},*)$ *be a random 2-normed space. A subset* F *of* X *is said to be closed if any sequence* (x_k) *in* X *converging to some* $x \in X$ *with respect to* $\mathcal{F}$ *implies that* $x \in F$.

A subset Y of X is said to be the closure of $A \subset X$ if, for any $x \in Y$, there exists a sequence (x_k) in A converging to x with respect to $\mathcal{F}$. We denote the set Y by $\overline{A}$.

In, [19] Mursaleen and Mohiuddine studied the concept of ideal convergence in probabilistic normed spaces. In [17], Mursaleen studied the concept of statistical convergence of sequences in random 2-normed spaces. In [18], Mursaleen and Alotaibi introduced the concept of I-convergence of sequences in random 2-normed spaces.

Definition 13. *[18] A sequence* $x=(x_k)$ *in a random 2-normed space* $(X,\mathcal{F},*)$ *is said to be* I*-convergent or* I^{R2N}*-convergent to some* $\ell \in X$ *with respect to* $\mathcal{F}$ *if for each* $\varepsilon>0$, $\eta \in (0,1)$ *and non zero* $z \in X$

$$\{k \in \mathbb{N} : \mathcal{F}(x_k-\ell,z;\varepsilon) \leq 1-\eta\} \in I$$

or equivalently

$$\{k \in \mathbb{N} : \mathcal{F}(x_k - \ell, z; \varepsilon) > 1 - \eta\} \in F(I).$$

In this case we write $I^{R2N} - \lim x = \ell$ *and* ℓ *is called the* $I^{R2N} -$*limit of* x. *Let* I^{R2N} *denotes the set of all* I*-convergent sequences in random 2-normed space* $(X, \mathcal{F}, *)$.

In this paper we define and study I_λ-convergence in random 2-normed space which is quite a new and interesting idea to work on. We prove some properties of I_λ-convergence in random 2-normed spaces. Also the notions, I_λ-limit points and I_λ-cluster points have been defined and the relation between them has been established. Finaly we find the relation between the I_λ-convergent and I_λ-Cauchy sequences in random 2-normed spaces.

3. I_λ-convergence in random 2-normed spaces

In this section we define I_λ-convergence in random 2-normed spaces. Also we obtained some basic properties of this notion in random 2-normed space.

Definition 14. *A sequence* $x = (x_k)$ *in a random 2-normed space* $(X, \mathcal{F}, *)$ *is said to be* λ*-convergent to* $\ell \in X$ *with respect to* $\mathcal{F}$ *if for every* $t > 0$, $\eta \in (0, 1)$ *and non zero* $z \in X$ *there exist positive integers* $k_0 = k_0(t, z)$

$$\frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - \ell, z; t) > 1 - \eta \quad \textit{for all } n \geq k_0.$$

Definition 15. *A sequence* $x = (x_k)$ *in a random 2-normed space* $(X, \mathcal{F}, *)$ *is said to be* I_λ*-convergent to* $\ell \in X$ *with respect to* $\mathcal{F}$ *if for every* $t > 0$, $\eta \in (0, 1)$ *and non zero* $z \in X$

$$\left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - \ell, z; t) \leq 1 - \eta \right\} \in I.$$

or equivalently

$$\left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - \ell, z; t) > 1 - \eta \right\} \in F(I).$$

In this case we write $I_\lambda^{R2N} - \lim x = \ell$ *or* $x_k \to \ell(I_\lambda^{R2N})$. *Let* I_λ^{R2N} *denotes the set of all* I_λ*-convergent sequences in random 2-normed space* $(X, \mathcal{F}, *)$.

The above definition, immediately implies the following Lemma.

Lemma 1. *Let $(X, \mathcal{F}, *)$ be a random 2-normed space. If $x = (x_k)$ is a sequence in X, then for every $\varepsilon > 0$, $\eta \in (0,1)$ and non zero $z \in X$, then the following statements are equivalent.*

(i) $I_\lambda^{R2N} - \lim_{k\to\infty} x_k = \ell.$

(ii) $\{n \in \mathbb{N} : \frac{1}{\lambda_n} \sum\limits_{k\in J_n} \mathcal{F}(x_k - \ell, z; \varepsilon) \leq 1 - \eta\} \in I.$

(iii) $I_\lambda - \lim_{k\to\infty} \mathcal{F}(x_k - \ell, z; \varepsilon) = 1.$

Theorem 2. *Let $(X, \mathcal{F}, *)$ be a random 2-normed space. If $x = (x_k)$ is a sequence in X such that $I_\lambda^{R2N} - \lim x_k = \ell$ exists, then it is unique.*

Proof. Suppose that there exist elements ℓ_1, ℓ_2 ($\ell_1 \neq \ell_2$) in X such that

$$I_\lambda^{R2N} - \lim_{k\to\infty} x_k = \ell_1; I_\lambda^{R2N} - \lim_{k\to\infty} x_k = \ell_2.$$

Let $\varepsilon > 0$ be given. Choose $a > 0$ such that

$$(1-a) * (1-a) > 1 - \varepsilon. \tag{1}$$

Then, for any $t > 0$ and for non zero $z \in X$ we define

$$K_1(a,t) = \left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k\in J_n} \mathcal{F}\left(x_k - \ell_1, z; \frac{t}{2}\right) \leq 1 - a \right\};$$

$$K_2(a,t) = \left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k\in J_n} \mathcal{F}\left(x_k - \ell_2, z; \frac{t}{2}\right) \leq 1 - a \right\}.$$

Since $I_\lambda^{R2N} - \lim_{k\to\infty} x_k = \ell_1$ and $I_\lambda^{R2N} - \lim_{k\to\infty} x_k = \ell_2$, we have $K_1(a,t) \in I$ and $K_2(a,t) \in I$ for all $t > 0$. Now let $K(a,t) = K_1(a,t) \cup K_2(a,t)$, then it is easy to observe that $K(a,t) \in I$. But $K^c(a,t) \in F(I)$.

Now , if $k \in K^c(a,t)$, then $k \in K_1^c(a,t) \cap K_2^c(a,t)$, we have

$$\frac{1}{\lambda_n} \sum_{k\in J_n} \mathcal{F}\left(x_k - \ell_1, z; \frac{t}{2}\right) > 1 - a \text{ and } \frac{1}{\lambda_n} \sum_{k\in J_n} \mathcal{F}\left(x_k - \ell_2, z; \frac{t}{2}\right) > 1 - a.$$

Now clearly we get $p \in \mathbf{N}$ such that

$$\mathcal{F}(x_p - \ell_1, z; t) > \frac{1}{\lambda_n} \sum_{k\in J_n} \mathcal{F}\left(x_k - \ell_1, z; \frac{t}{2}\right) > 1 - a;$$

and

$$\mathcal{F}\left(x_p - \ell_2, z; \frac{t}{2}\right) > \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}\left(x_k - \ell_2, z; \frac{t}{2}\right) > 1 - a$$

Then, we have

$$\mathcal{F}(\ell_1 - \ell_2, z; t) \geq \mathcal{F}\left(x_p - \ell_1, z; \frac{t}{2}\right) * \mathcal{F}\left(x_p - \ell_2, z; \frac{t}{2}\right) > (1-a) * (1-a).$$

It follows by (1) that

$$\mathcal{F}(\ell_1 - \ell_2, z; t) > (1 - \varepsilon).$$

Since $\varepsilon > 0$ was arbitrary, we get $\mathcal{F}(\ell_1 - \ell_2, z; t) = 1$ for all $t > 0$ and non zero $z \in X$. Hence $\ell_1 = \ell_2$.

Next theorem gives the algebraic characterization of I_λ-convergence in random 2-normed spaces.

Theorem 3. *Let $(X, \mathcal{F}, *)$ be a random 2-normed space and $x = (x_k)$ and $y = (y_k)$ be two sequences in X.*

(a) If $I^{R2N}_\lambda - \lim x_k = \ell$ and $c(\neq 0) \in \mathbb{R}$, then $I^{R2N}_\lambda - \lim cx_k = c\ell$.

(b) If $I^{R2N}_\lambda - \lim x_k = \ell_1$ and $I^{R2N}_\lambda - \lim y_k = \ell_2$, then $I^{R2N}_\lambda - \lim(x_k + y_k) = \ell_1 + \ell_2$.

Proof. The proof of the theorem is straightforward, thus omitted.

4. λ-Cauchy and I_λ-Cauchy sequences

Definition 16. *A sequence $x = (x_k)$ in a random 2-normed space $(X, \mathcal{F}, *)$ is said to be λ-Cauchy with respect to $\mathcal{F}$ if for every $t > 0$, $\eta \in (0, 1)$ and non zero $z \in X$ there exist positive integers $k_0 = k_0(t, z)$*

$$\frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - x_m, z; t) > 1 - \eta \quad \text{for all } n, m \geq k_0.$$

Definition 17. *A sequence $x = (x_k)$ in a random 2-normed space $(X, \mathcal{F}, *)$ is said to be I_λ-Cauchy with respect to $\mathcal{F}$ if for every $t > 0$, $\eta \in (0, 1)$ and non zero $z \in X$ there exists a positive integer $m = m(t, z)$*

$$\left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - x_m, z; t) \leq 1 - \eta \right\} \in I$$

or equivalently

$$\left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - x_m, z; t) > 1 - \eta \right\} \in F(I).$$

Theorem 4. *Let $(X, \mathcal{F}, *)$ be a random 2-normed space. If $x = (x_k)$ be a sequence in X, then $I_\lambda^{R2N} - \lim x_k = \ell$ if and only if there exists a subset $K = \{k_1 < k_2 < \cdots < k_n < \dots\} \subseteq \mathbb{N}$ such that $K \in F(I)$ and $\lambda - \lim x_{k_n} = \ell$.*

Proof. Suppose first that $I_\lambda^{R2N} - \lim x_k = \ell$. Then for any $t > 0$, $a = 1, 2, 3, ...$ and non zero $z \in X$, let

$$A_a(\lambda, t) = \left\{ n \in \mathbb{N} : \mathcal{F}(t_{k_n}(x) - \ell, z; t) > 1 - \frac{1}{a} \right\}$$

and

$$B_a(\lambda, t) = \left\{ n \in \mathbb{N} : \mathcal{F}(t_{k_n}(x) - \ell, z; t) \leq 1 - \frac{1}{a} \right\}.$$

Since $I_\lambda^{R2N} - \lim x_k = \ell$ it follows that $B_a(\lambda, t) \in I$.

Now for $t > 0$ and $a = 1, 2, 3, ...$, we observe that $A_a(\lambda, t) \supset A_{a+1}(\lambda, t)$ and

$$A_a(\lambda, t) \in F(I). \tag{2}$$

Now we have to show that, for $n \in A_a(\lambda, t), \lambda - \lim x_k = \ell$. Suppose that for $n \in A_a(\lambda, t), (x_k)$ not λ-convergent to ℓ with respect to $\mathcal{F}$. Then there exists some $s > 0$ such that

$$\left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \mathcal{F}(t_{k_n}(x) - \ell, z; t) \leq 1 - s \right\}.$$

Let

$$A_s(\lambda, t) = \{ n \in \mathbb{N} : \mathcal{F}(t_{k_n}(x) - \ell, z; t) > 1 - s \}$$

and $s > \frac{1}{a}, a = 1, 2, 3, ...$ Then we have $A_s(\lambda, t) \in I$. Furthermore, $A_a(\lambda, t) \subset A_s(\lambda, t)$ implies that $A_a(\lambda, t) \in I$, which contradicts (2) as $A_a(\lambda, t) \in F(I)$. Hence $\lambda - \lim x_{k_n} = \ell$.

Conversely, suppose that there exists a subset $K \subseteq \mathbb{N}$ such that $K \in F(I)$ and $\lambda - \lim_{k \in K} x_k = \ell$. Then for every $\varepsilon \in (0, 1)$, $t > 0$ and non zero $z \in X$, we can find out a positive integer $m = m(t, z)$ such that

$$\frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - \ell, z; t) > 1 - \varepsilon$$

for all $n \geq m$. If we take

$$K(\varepsilon,t) = \left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - \ell, z; t) \leq 1 - \varepsilon \right\}$$

then it is easy to see that

$$K(\varepsilon,t) \subset \mathbb{N} - \{n_{m+1}, n_{m+2}, ...\}$$

and consequently $K(\varepsilon,t) \in I$. Hence $I_\lambda^{R2N} - \lim x_k = \ell$.

Now, we establish the Cauchy convergence criteria in random 2-normed spaces.

Theorem 5. *Let $(X, \mathcal{F}, *)$ be a random 2-normed space. Then a sequence (x_k) in X is I_λ-convergent if and only if it is I_λ-Cauchy.*

Proof. Let (x_k) be I_λ-convergent sequence in X. We assume that $I_\lambda^{R2N} - \lim x_k = \ell$. Let $\varepsilon > 0$ be given. Choose $a > 0$ such that (1) is satisfied. For $t > 0$ and for non zero $z \in X$ define

$$A(a,t) = \left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - \ell, z; \frac{t}{2}) \leq 1 - a \right\},$$

i.e.

$$A^c(a,t) = \left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - \ell, z; \frac{t}{2}) > 1 - a \right\}.$$

Since $I_\lambda^{R2N} - \lim x_k = \ell$ it follows that $A(a,t) \in I$ and consequently $A^c(a,t) \in F(I)$. Then we will get $p \in \mathbb{N}$ such that

$$\mathcal{F}(x_p - \ell, z; \frac{t}{2}) > \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - \ell, z; \frac{t}{2}) > 1 - a. \tag{3}$$

If we take

$$B(\varepsilon,t) = \left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - x_p, z; t) \leq 1 - \varepsilon \right\}$$

then to prove the result it is sufficient to prove that $B(\varepsilon,t) \subseteq A(a,t)$. Let $m \in B(\varepsilon,t)$ then for non zero $z \in X$

$$\frac{1}{\lambda_n} \sum_{m \in J_n} \mathcal{F}(x_m - x_p, z; t) \leq 1 - \varepsilon \tag{4}$$

and using (3) we will get $m \in \mathbb{N}$ such that

$$\mathcal{F}(x_m - \ell, z; \frac{t}{2}) > \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - \ell, z; \frac{t}{2}) > 1 - a. \tag{5}$$

Then from (1), (3) (4 and (5) we have

$$1 - \varepsilon \geq \mathcal{F}(x_m - x_p, z; t) \geq \mathcal{F}(x_m - \ell, z; \frac{t}{2}) * \mathcal{F}(x_p - \ell, z; \frac{t}{2})$$
$$> (1-a) * (1-a) > 1 - \varepsilon$$

which is not possible. Thus $B(\varepsilon, t) \subset A(a, t)$. Since $A(a,t) \in I$, it follows that $B(\varepsilon, t) \in I$. This shows that (x_k) is I_λ-Cauchy.

Conversely, suppose (x_k) is I_λ-Cauchy but not I_λ-convergent. Then there exists positive integer p and for non zero $z \in X$ such that if we take

$$A(\varepsilon, t) = \left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - x_p, z; t) \leq 1 - \varepsilon \right\}$$

then $A(\varepsilon, t) \in I$ and consequently

$$A^c(\varepsilon, t) \in F(I). \tag{6}$$

For $a > 0$ such that (1) is satisfied and we take

$$B(a, t) = \left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - \ell, z; \frac{t}{2}) > 1 - a \right\}.$$

Then we will get $p \in \mathbb{N}$ such that

$$\mathcal{F}(x_p - \ell, z; \frac{t}{2}) > \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - \ell, z; \frac{t}{2}) > 1 - a.$$

Since

$$\mathcal{F}(x_k - x_p, z; t) \geq \mathcal{F}(x_p - \ell, z; \frac{t}{2}) * \mathcal{F}(x_k - \ell, z; \frac{t}{2}) > (1-a) * (1-a) > 1 - \varepsilon,$$

then we have

$$\left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - x_p, z; t) > 1 - \varepsilon \right\} \in I$$

i.e. $A^c(\varepsilon, t) \in I$, which contradicts (6) as $A^c(\varepsilon, t) \in F(I)$. Hence (x_k) is I_λ-convergent.

Combining Theorem 4 and Theorem 5 we get the following corollary.

Corollary 6. *Let $(X, \mathcal{F}, *)$ be a random 2-normed space and and $x = (x_k)$ be a sequence in X. Then the following statements are equivalent:*

(a) x is I_λ-convergent.

(b) x is I_λ-Cauchy.

(c) there exists a subset $K = \{n \in \mathbb{N} : k \in J_n\} \subseteq \mathbb{N}$ such that $K \in F(I)$ and $\lambda - \lim_{k \in K} x_k = \ell$.

5. I_λ-limit points and I_λ-cluster points

Definition 18. *Let $(X, \mathcal{F}, *)$ be a random 2-normed space and $x = (x_k)$ be a sequence in X.*

(i) An element $\ell \in X$ is said to be a I_λ-limit point of x if there is a set $M = \{m_1 < m_2 < ... < m_k < ....\} \subset \mathbb{N}$ such that the set $M^c = \{n \in \mathbb{N} : m_k \in J_n\} \notin I$ and $\lambda - \lim x_{m_k} = \ell$.

(ii) An element $\ell \in X$ is said to be a I_λ-cluster point of x if, for each $t > 0$, $\eta \in (0, 1)$ and non zero $z \in X$,

$$\left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - \ell, z; t) > 1 - \eta \right\} \notin I.$$

Let $I_\lambda(\Lambda_x^{R2N})$ denote the set of all I_λ-limit points and $I_\lambda(\Gamma_x^{R2N})$ denote the set of all I_λ-cluster points in X, respectively.

Theorem 7. *Let $(X, \mathcal{F}, *)$ be a random 2-normed space. Then for any sequence $x = (x_k)$ in X, $I_\lambda(\Lambda_x^{R2N}) \subset I_\lambda(\Gamma_x^{R2N})$.*

Proof. Let $\ell \in I_\lambda(\Lambda_x^{R2N})$. Then there exists a set $M = \{m_1 < m_2 < ... < m_k < ....\} \subset \mathbb{N}$ such that the set $M^c = \{k \in \mathbb{N} : m_k \in J_n\} \notin I$ and $\mathcal{F}_\lambda - \lim x_{m_k} = \ell$. Thus, for each $t > 0$, $\eta \in (0, 1)$ and non zero $z \in X$, there eixsts a positive integer k_0 such that

$$\frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_{m_k} - \ell, z; t) > 1 - \eta \text{ for } \quad \text{all} \quad n \geq k_0.$$

Therefore

$$A(\eta,t)=\left\{n\in\mathbb{N}:\frac{1}{\lambda_n}\sum_{k\in J_n}\mathcal{F}(x_k-\ell,z;t)>1-\eta\right\}\supset M^c-\{m_1,m_2,...,m_{k_0}\}.$$

and the ideal I is admissible, we have $M^c-\{m_1,m_2,...,m_{k_0}\}\notin I$, and as such $A(\eta,t)\notin I$. This shows that $\ell\in I_\lambda(\Gamma_x^{R2N})$. This completes the proof of the theorem.

Theorem 8. *Let $(X,\mathcal{F},*)$ be a random 2-normed space. Then for any sequence $x=(x_k)$ in X, the set $I_\lambda(\Gamma_x^{R2N})$ is closed in X with respect to the usual topology induced by $\mathcal{F}$.*

Proof. Let $y\in\overline{I_\lambda(\Gamma_x^{R2N})}$. Take $t>0$, $\varepsilon\in(0,1)$ and non zero $z\in X$, there exists $\ell_0\in I_\lambda(\Gamma_x^{R2N})\cap B(y,\varepsilon;t)$. Choose $\delta>0$ such that $B(\ell_0,\delta;t)\subset B(y,\varepsilon;t)$. Then we have

$$G(\varepsilon,t)=\left\{n\in\mathbb{N}:\frac{1}{\lambda_n}\sum_{k\in J_n}\mathcal{F}(x_k-y,z;t)>1-\varepsilon\right\}$$

$$\supset\left\{n\in\mathbb{N}:\frac{1}{\lambda_n}\sum_{k\in J_n}\mathcal{F}(x_k-\ell_0,z;t)>1-\delta\right\}=H(\delta,t).$$

Thus $H(\delta,t)\notin I$, and so $G(\varepsilon,t)\notin I$. This shows that $y\in I_\lambda(\Gamma_x^{R2N})$. This completes the proof of the theorem.

Theorem 9. *Let $(X,\mathcal{F},*)$ be a random 2-normed space and let $x=(x_k)$ be sequence in X. Then the following conditions are equivalent.*

(i) *ℓ is an I_λ-limit point of x.*

(ii) *There exists two sequences $y=(y_k)$ and $z=(z_k)$ in X such that $x=y+z;\lambda-\lim y=\ell$ and the set $\{n\in\mathbb{N}:k\in I_n,z_k\neq\overline{0}\}\in I$ where $\overline{0}$ denotes the zero element in X.*

Proof. Suppose that the condition (i) holds. Then there exists a sets M and M^c as above such that

$$M^c\notin I\quad\text{and}\quad\lambda-\lim_k x_{m_k}=\ell. \tag{7}$$

We define the sequences $y=(y_k)$ and $z=(z_k)$ as follows:

$$y_k=\begin{cases}x_k, & \text{if } k\in J_n, \text{ such that } n\in M^c;\\ \ell, & \text{otherwise}\end{cases}$$

and

$$z_k = \begin{cases} \ell, & \text{if } k \in J_n, \text{ such that } n \in M^c; \\ x_k - \ell & \text{otherwise} \end{cases}$$

It suffices to consider the case $k \in J_n$ such that $n \in \mathbb{N} - M^c$. Then for each $\varepsilon > 0, t > 0$ and for non zero $z \in X$,

$$\mathcal{F}(y_k - \ell, z; t) = 1 > 1 - \varepsilon.$$

Thus, in this case,

$$\frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(y_k - \ell, z; t) = 1 > 1 - \varepsilon.$$

Using (7) we have $\lambda - \lim y = \ell$. Now $\{n \in \mathbb{N} : k \in J_n, z_k \neq \overline{0}\} \subset \mathbb{N} - M^c$. But $\mathbb{N} - M^c \in I$, so we have $\{n \in \mathbb{N} : k \in I_n, z_k \neq \overline{0}\} \in I$.

Next, suppose that the condition (ii) holds. If we take $M^c = \{n \in \mathbb{N} : k \in J_n, z_k = \overline{0}\}$, then obviously $M^c \in F(I)$ is an infinite set, because I is an admissible ideal of $\mathbb{N}$. Let $M = \{m_1 < m_2 < ...\} \subset \mathbb{N}$ such that $m_k \in J_n, z_k = \overline{0}$. As $x_{m_k} = y_{m_k}$ and $\lambda - \lim y = \ell$, $\lambda - \lim_k x_{m_k} = \ell$. This completes the proof of the theorem.

6. I^*_λ-convergence in random 2-normed spaces

In this section, we introduce the concept of I^*_λ-convergence in random 2-normed space.

Definition 19. *A sequence $x = (x_k)$ in a random 2-normed space $(X, \mathcal{F}, *)$ is said to be I^*_λ-convergent to a number ℓ with respect to $\mathcal{F}$ if there exists a subset $K = \{m_k : m_1 < m_2 < ... < m_k < ...\}$ of $\mathbb{N}$ such that $K^c = \{n \in \mathbb{N} : m_n \in J_n\} \in F(I)$ and $\lambda - \lim_k x_{m_k} = \ell$, i.e. for any $\varepsilon \in (0,1), t > 0$ and non zero $z \in X$, there exists a positive integer $N = N(t, z)$ such that*

$$\frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_{m_k} - \ell, z; t) > 1 - \varepsilon \textit{ for all } n \geq N.$$

In this case we write $I^{*,R2N}_\lambda - \lim x = \ell$ and ℓ is called the I^*_λ-limit of x in $(X, \mathcal{F}, *)$.

Theorem 10. *Let $(X, \mathcal{F}, *)$ be a random 2-normed space and I be an admissible ideal, and $x = (x_k)$ be a sequence in X. If $I^{*,R2N}_\lambda - \lim x_k = \ell$ then $I^{R2N}_\lambda - \lim x_k = \ell$.*

Proof. Suppose that $I^{*,R2N}_\lambda - \lim x_k = \ell$. Then there exists asubset $K = \{m_k : m_1 < m_2 < ... < m_k < ...\}$ of $\mathbb{N}$ such that $K \in F(I)$ and $\mathcal{F}_\lambda - \lim_k x_{m_k} = \ell$. But then for any $\varepsilon \in (0,1), t > 0$ there exists a positive integer N such that for non zero $z \in X$,

$$\frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_{m_k} - \ell, z; t) > 1 - \varepsilon \text{ for all } n \geq N.$$

Since $\{m_n \in K : \frac{1}{\lambda_n} \sum\limits_{k\in J_n} \mathcal{F}(x_{m_k} - \ell, z; t) \leq 1-\varepsilon\}$ is contained in $\{m_1 < m_2 < ... < m_{N-1}\}$ and the ideal I is admissible, we have

$$\left\{ m_n \in K : \frac{1}{\lambda_n} \sum_{k\in J_n} \mathcal{F}(x_{m_k} - \ell, z; t) \leq 1-\varepsilon \right\} \in I.$$

Hence

$$\left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k\in J_n} \mathcal{F}(x_k - \ell, z; t) \leq 1-\varepsilon \right\} \subseteq (\mathbb{N} - K) \cup \{m_1 < m_2 < ... < m_{N-1}\} \in I,$$

for all $\varepsilon \in (0,1)$ and $t > 0$, and for non zero $z \in X$. This implies that $I_2^{R2N} - \lim x_k = \ell$.

Remark 2. *The converse of the above theorem is not true in general. It follows from the following theorem.*

Theorem 11. *Let $(X, \mathcal{F}, *)$ be a random 2-normed space and the ideal I satisfy the condition (AP). If $x = (x_k)$ be a sequence in X such that $I_\lambda^{R2N} - \lim x_k = \ell$, then $I_\lambda^{*,R2N} - \lim x_k = \ell$.*

Proof. Suppose I satisfies condition (AP) and $I_\lambda^{R2N} - \lim x_k = \ell$. Then for every $\varepsilon > 0$, $\varepsilon \in (0,1)$ and for non zero $z \in X$,

$$\left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k\in J_n} \mathcal{F}(x_k - \ell, z; t) \leq 1-\varepsilon \right\} \in I.$$

We define the set K_s for $s \in \mathbb{N}$, $t > 0$ and for non zero $z \in X$ as

$$K_s = \left\{ n \in \mathbb{N} : 1 - \frac{1}{s} \leq \frac{1}{\lambda_n} \sum_{k\in J_n} \mathcal{F}(x_k - \ell, z; t) < 1 - \frac{1}{s+1} \right\}.$$

Obviously, $\{K_1, K_2, ...\}$ is countable and belongs to I, and $K_i \cap K_j = \phi$ for $i \neq j$. By condition (AP), there is a countable family of sets $\{M_1, M_2, ...\}$ such that $K_i \Delta M_i$ is a finite set for each $i \in \mathbb{N}$ and $M = \bigcup_{i=1}^{\infty} M_i \in I$. Using the definition of the associate filter $F(I)$ there exists a set $A \in F(I)$ such that $A = \mathbb{N} - M \in I$. To prove the theorem it is sufficient to show that $\mathcal{F}_\lambda - \lim_{k\in A, k\to\infty} x_k = \ell$. Let $\delta > 0, t > 0$ and for non zero $z \in X$, choose $p \in \mathbb{N}$ such that $\frac{1}{p} < \delta$. Then

$$\left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k\in J_n} \mathcal{F}(x_k - \ell, z; t) \leq 1-\delta \right\}$$

$$\subset \left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mathcal{F}(x_k - \ell, z; t) \leq 1 - \frac{1}{p} \right\} \subset \bigcup_{i=1}^{p-1} K_i.$$

Since $K_i \Delta M_i,\ i = 1, 2, ..., p+1$ are finite, there exists $k_0 \in \mathbf{N}$ such that

$$\left(\bigcup_{i=1}^{p+1} M_i \right) \cap \{k : k \geq k_0\ \} = \left(\bigcup_{i=1}^{p+1} K_i \right) \cap \{k : k \geq k_0\ \}. \tag{8}$$

If $k \geq k_0$ and $k \in A$ then $k \notin \bigcup_{i=1}^{p+1} M_i$. Therefore by (8), we have $k \notin \bigcup_{i=1}^{p+1} K_i$. Hence for every $k \geq k_0$ and $k \in A$, we have

$$\frac{1}{\lambda_n} \sum_{j \in J_n} \mathcal{F}(x_k - \ell, z; t) > 1 - \delta.$$

Since $\delta > 0$ is arbitrary, we have $I_\lambda^{*,R2N} - \lim x_k = \ell$. This completes the proof of the theorem.

Definition 20. *Let $(X, \mathcal{F}, *)$ be a random 2-normed space. A sequence $x = (x_k)$ in X is said to be I_λ^*-Cauchy with respect to $\mathcal{F}$ if there exists a set $M = \{m_1 < m_2 < ... < m_k < ...\} \subset \mathbb{N}$ such that the set $M^c = \{n \in \mathbb{N} : m_k \in J_n\} \in F(I)$ and the subsequence (x_{m_k}) is a λ-Cauchy sequence with respect to $\mathcal{F}$.*

The proof of the following theorem is easy, so omitted.

Theorem 12. *Let $(X, \mathcal{F}, *)$ be a random 2-normed space. If a sequence $x = (x_k)$ in X is λ-Cauchy with respect to $\mathcal{F}$, then there is a subsequence of x which is ordinary Cauchy with respect to the same.*

The following is an analogue of Theorem 5.

Theorem 13. *Let $(X, \mathcal{F}, *)$ be a random 2-normed space. Then a sequence (x_k) in X is I_λ^*-convergent if and only if it is I_λ^*-Cauchy.*

The following can be proved easily using similar techniques as in the proof of Theorem 7.

Theorem 14. *Let $(X, \mathcal{F}, *)$ be a random 2-normed space. If a sequence $x = (x_k)$ in X is I_λ-Cauchy with respect to $\mathcal{F}$, then it is I_λ-Cauchy as well.*

Problem 1. For further study, we suggest to investigate I_λ-convergence for the fuzzy points. However due to the change in settings, the definitions and methods of proofs will not always be analogous to these of the present work(for example see [1]).

Problem 2. For another further study we suggest to introduce a new concept in dynamical systems using I_λ-convergence.

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