

## **BIPOLAR CONVERGENCE IN GENETIC ALGORITHM FOR MULTIMODAL OPTIMIZATION**

by  
**Corina Rotar**

**Abstract:** Regarding multimodal optimization problems, the genetic algorithm with clonal selection proposed by De Castro & Von Zuben offers a satisfying solution for providing multiple optima (either maximum optima or minima) of the test functions. In our proposed algorithm based on clonal selection principle, we made a modification to force the population converges in two direction, toward maximums and also, minima of the multimodal function. An interesting phenomenon occurs and we called it - bipolar convergence. We noticed that even if the population is leaded toward the maximums, the final population accumulates the minima as well. Thus, the population is split in two: a part of it improves the quality of the maximum points and the other one comports itself as a memory population, which accumulates the minima.

**Keywords:** genetic algorithms, multi-modal optimization, bipolar convergence.

### **1. Introduction**

Evolutionary techniques, particularly Genetic Algorithms, have known a significant development lately. Inspired by natural phenomena, due to their simple implementation and comprehension, and having surprising results, these algorithms expand their field of applicability toward different types of problems. Without insisting on GA's behavior or their way to produce the solutions, we are going to take into consideration a specific kind of problems, as multi-modal optimization. Next section surveys several successful techniques existing by now, suggesting their advantages and disadvantages.

The idea of this study started during an experiment, which had been made with an algorithm based on Clonal Selection Principle, on several multi-modal functions. De Castro & Von Zuben developed the Clonal Selection Algorithm for multimodal optimization problems, which was proved to be a successful evolutionary method. Our study describes in a special paragraph this algorithm for multi-modal optimization and continues illustrating how an apparent minor modification of this procedure could change the population's behavior. We developed an algorithm capable of finding not only multiple maximums of the considerate function. Multiple minimums are also detected. Because the population is somehow split in two and converges toward minimums and maximums as well, we called our developed algorithm as Bipolar Convergence Genetic Algorithm.

Different test functions had been taken into consideration for sustaining our theory. Numerical experiments reveal the fact that Bipolar Convergence Genetic Algorithm works satisfactory only for a specific kind of the test functions.

## 2. GA-Based Multimodal Optimization Techniques

Real problems often have more than a global optimum. The main advantage of finding multiple optima of a problem involves the possibility of finally choosing of an appropriate solution among those, according to the real necessities. The existing evolutionary techniques deal with this fact, also. Nevertheless, the major inconvenience of the genetic algorithms relies on the premature convergence toward a single solution (mainly, the global optimum). In the cases of multimodal optimization problems, the standard genetic algorithm is not capable of satisfying the problem's request. Because of this fact, new evolutionary mechanisms for detecting multiple optima are welcome in the landscape of *Evolutionary Computation*. Several successful evolutionary techniques for multi-modal optimization are presented next.

### **Crowding techniques**

De Jong proposed crowding techniques in 1975. The basic idea consists in provoking the individuals of the population to settle around the most promising regions of the searching space, corresponding to multiple optimum points. Every optimal region imprints the belonging individuals with a specific probability of selection for crossover. The number of individuals concentrated around a point of optimum is proportional with the power of attraction of that point. The diversity of the population is maintained by a mechanism of replacing the similar individuals. The degree of similitude between any two coves is measured through a function of distance, determined in agreement to the context of the respective problem.

### **Genetic Chromodynamics**

Although developed initially for solving the classification problems, by Dumitrescu, Genetic Chromodynamics proves to be a technique used successfully in problems of multi-modal optimization. Essentially, genetic chromodynamics is an evolutionary strategy for searching and optimizing, which uses a population of a variable size and a special scheme of chromosome interaction. Genetic Chromodynamics (GC) provokes the formation and maintaining of some subpopulations of possible solutions, corresponding to many optimum points. Each subpopulation will evolve towards the optimum of the region. A characteristic of the method is that the number of individuals decreases: very similar/closed individuals are unified, thus, a decrease of dimension of the population results. Using a local scheme of chromosomal crossover and a unification mechanism of similar individuals, the strategy makes every subpopulation to contain finally just one chromosome, which represents the corresponding local optimum. The advantage of this method is the fact that finally both multiple optima and their number in the searching space will be determined.

### **Island Model**

The island model proposed by Cohoon, Hedge, Martin and Richardson in 1987 considers the population as divided in small disjoint subpopulations with isolated

evolution. These subpopulations are called “islands”. Every island represents a group of semi-independent individuals having the liberty of binding to the other islands by the mechanism of *migration* between islands. The separate evolution of the subpopulations corresponds to the process of exploring the space of possible solutions, while every subpopulation exploits the corresponding region of the occupied space for finding the local optimum. The mechanism of migration of the individuals from an island to another maintains the diversity inside the population and guarantees the efficient exploration of the space.

#### **Artificial Endocrine System**

The natural endocrine system proves to be an adaptable and complex system. These observations lead us to the idea of suitable artificial development of an endocrine system for approaching some complex problems as multimodal optimization. The principle of this method consists in regular control of the proportion of individuals among the search space. The artificial endocrine system developed encloses a mechanism for diversity preservation inspired from the manner in which the correspondent natural system controls the concentration of different types of hormones.

#### **Artificial Immune Systems**

Biological metaphor of immune systems represents the base for developing new computational techniques and constructing various evolutionary systems, called artificial immune systems. To define and develop an artificial immune system the following aspects are taken into consideration:

- immune mechanism represents the source of inspiration for the constructions of the algorithms and the structures involved
- by implementing the immune principles for describing some complex processes as optimization, learning, pattern recognition, unlimited potential applications results
- understanding and exploiting of the natural phenomenon as the immune response of an organism opens new directions of research for obtaining new computational techniques for solving complex problems

### **3. Clonal Selection Principle**

The principle of clonal selection represents the algorithm applied by the immune system to generate an immune response to antigenic stimulus. The central idea of this theory is that only the cells that have the quality to recognize the antigens are selected for cloning. The main characteristics of this theory are:

- the new immune cells are clones of the parents` cells and are subject to a high rate mutation mechanism with (somatic hypermutation)
- the newest different lymphocytes, which carry auto-reactive receptors are eliminated

- the cloning and differentiation in memory cells or plasma cells at the contact of mature cells with the antigens
- the persistence of forbidden clones, resistant to early elimination

Leandro Nunes de Castro and Fernando Jose Von Zuben [3,4,5] propose an algorithm based on the clonal selection theory with good results regarding multimodal optimization problems.

#### **Clonal Selection Algorithm (CSA)**

1. Generating a set  $P$  of  $n$  candidate solutions, composed of a subset of memory cells  $M$  added to the remained population  $P_r$ :  $P = M \cup P_r$ .
2. Determine the best  $n$  individuals of  $P$  population on the basis of measuring their affinity. Let them be the best  $n$  individuals retained in  $P_r$  population.
3. Individuals from  $P_r$  are cloned and their clones go to the intermediary population of clones,  $C$ .
4. Individuals from  $C$  population suffer mutations, thus, a population of mature clones  $C^*$  results.
5. Improved individuals from  $C^*$  population are reselected for recompose  $M$  memory set.
6. Replace  $d$  individuals with low affinities from  $P$  population, for keeping its diversity.

The CSA causes the reproduction of those individuals, which codify the best possible solutions. Also, the algorithm facilitates selection of the best individuals among the improved offspring. The final results of the CSA algorithm, previously described, are the maximum solutions of the function.

The next paragraph presents a similar technique for multimodal optimization and a surprising behavior of the population.

#### **4. Bipolar Convergence Genetic Algorithm based on Clonal Selection Principle**

In this section, an evolutionary technique for optimization and a different kind of convergence of the population are described. The algorithm itself is essentially built on the clonal selection algorithm's scheme. Only one modification is produced within clonal selection algorithm, but the effects are interesting.

The standard algorithm for multi-modal optimization proposed by Leandro Nunes de Castro and Fernando José Von Zuben detects the maximum points. The fifth line of the algorithm is modified as follow: the best parents from current population are replaced with the best mature clones. As we noticed, for a faster convergence, the standard algorithm would replace the worst individuals of the memory population with

mature clones. Contrarily, our modification eliminates the best chromosomes and includes the mature improved clones.

Numerical experiments show that the population splits in two parts: first part contains the best individuals according to performance function and converges toward the maximum solutions of the test function; second part behaves as a collection of weaker individuals according to performance function. After several generations, even if the population continues to settle in those zones from the search space, which contains the maxima, the population also retains the minimal solution.

Paradoxically, the population evolves toward the maximum solutions increasing the offspring's performance according to objective function, but finally the minima are found and offered.

**Remark:** The quality (performance) of each chromosome is given by the value of the objective function in that point, which is codified by the chromosome.

As Michalewicz (1996) remarked, any evolutionary technique should have five main components:

1. A genetic representation for potential solutions
2. A method for generating initial population of potential solutions
3. A function for solution's evaluations
4. Genetic operators which change the individuals and lead the population toward the best possible solutions.
5. Values for specific parameters of the algorithm

From this point of view, the next algorithm will be described following those five enunciated issues. Firstly, we used a suitable genetic representation for function optimization problems. Thus, each individual codifies a potential solution in a natural manner: due to the fact that we took into consideration test functions defined in the real space,  $\mathbf{R}$ , a real representation fits to our needs. The chromosome's structure appears as follow: let be  $f : D \rightarrow R$ , where  $D \subseteq R^n$ , the objective function. Each  $c$  chromosome is an  $n$ -couple of real values:  $c = (x_1, x_2, \dots, x_n)$ .

Secondly, the initial population is randomly generated among the search space. For evaluation of the performance of the candidate solutions, the objective function has the role of the fitness function.

We used two genetic operators: proportional selection and mutation. The selection operator favors the best individuals. So, a better individual has a higher probability for generating descendents than a weaker one. In order to ensure a selection with uniform probability distribution, we use in our algorithm a selection operator based on *Monte Carlo* method (roulette wheel selection).

The mutation operator induces small modifications into the composition of the selected individuals in order to generate the next generation. Our option was a non-uniform mutation operator given by Janikow and Michalewicz, in 1991. This approach provokes an accentuated increase of the modifications in the first generations.

Therefore, the progress of search is significant in the initial phases. Further, the search's strength decreases in the last generations.

Finally, the next algorithm doesn't have more parameters than the regular genetic algorithm. We enumerate below several of the algorithm's parameters that could alter in a way or another, the progress of the population:

- $n$  - the size of the main population
- $n_{\text{clones}}$  - the number of clones
- $n_{\text{best}}$  - the size of the intermediate population  $P_{\text{best}}$
- the number of final generation
- the number of individuals which are replaced from the current population with improved individuals

The *BCGA* algorithm is given below:

### **Bipolar Convergence Genetic Algorithm for Multi-modal Optimization (BCGA)**

1. Initial population of  $n$  individuals is generated:  $P$ .
2. While (condition) do:
  - 2.1. The best  $n_{\text{best}}$  individuals are extracted from  $P$  population. A second population  $P_{\text{best}}$  results.
  - 2.2. The individuals from  $P_{\text{best}}$  are cloned. A population of clones,  $C$ , results. Its size is given by the  $n_{\text{clones}}$  parameter.
  - 2.3. The clones of  $C$  population suffer modifications through mutation operator. The mutated clones are collected in the  $CM$  population.
  - 2.4. Population  $P$  is rebuilt by replacing best individuals from the current population with maturated clones (improved individuals) from  $CM$ .

**Remark:** The stopping condition became true when the established maximum number of generations was achieved.

Although the previous algorithm seems to lead the population toward the maxima, during the evolution, the population accumulates the minima of the test function. Numerical tests reveal the fact that the described phenomenon appears only in the cases of particular type of multi-modal functions.

## **5. Experiments**

We choose several multi-modal functions for numerical experiments. First one is known as Schaffer's F6 function:  $f : D \rightarrow R$ ,  $D \subseteq R$  given by the next formula:

$$f(x) = 0.5 + \frac{\sin^2(x) - 0.5}{(1 + 0.001 \cdot (x^2))^2} \quad (1)$$

The next two figures show how the chromosomes settle on the minima. We considered firstly that  $D$  domain is  $[-20, 20]$  and the size of the population is 70.

Secondly, the population size is 100 and domain D is larger:  $[-30,30]$ . The pictures show only that part of the population that corresponds to the minima. In both cases, we stopped the algorithm after 20 generations.

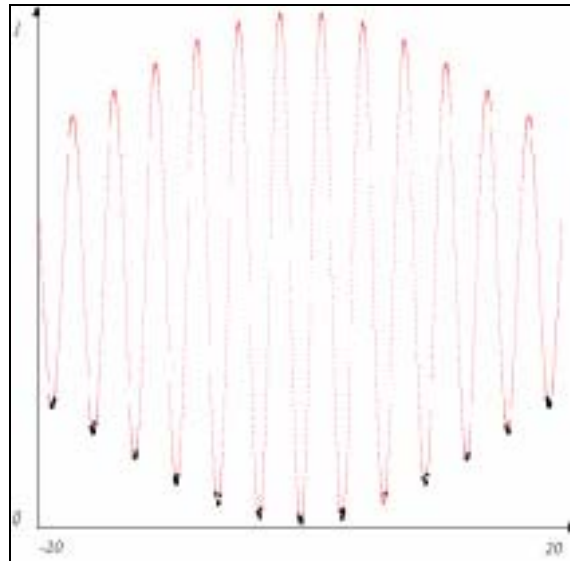


Figure 1: Schaffer's F6. Population after 20 generation

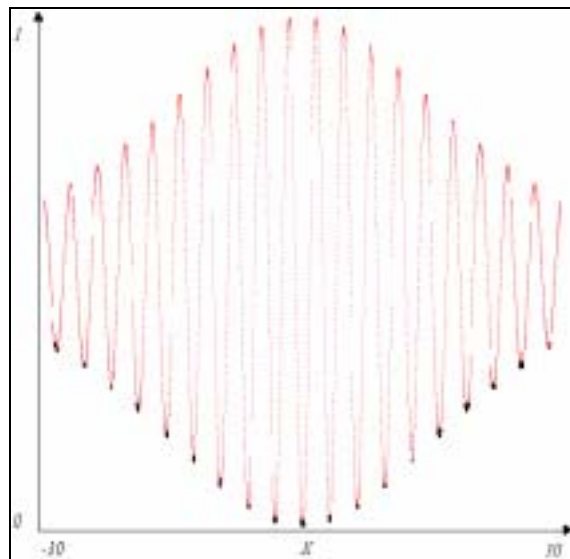


Figure 2: Schaffer's F6. Larger domain

Another test function is described next.  $g : D \rightarrow R$ ,  $D \subseteq R$ , given by the following formula:

$$g(x) = e^{-2 \cdot \log(2) \cdot \left(\frac{x-0.1}{0.8}\right)^2} \cdot \sin(5 \cdot \pi \cdot x)^6 \quad (2)$$

The next figure presents how the chromosomes crowd along the minima after 20 generations. The population size was fixed at 100 individuals.

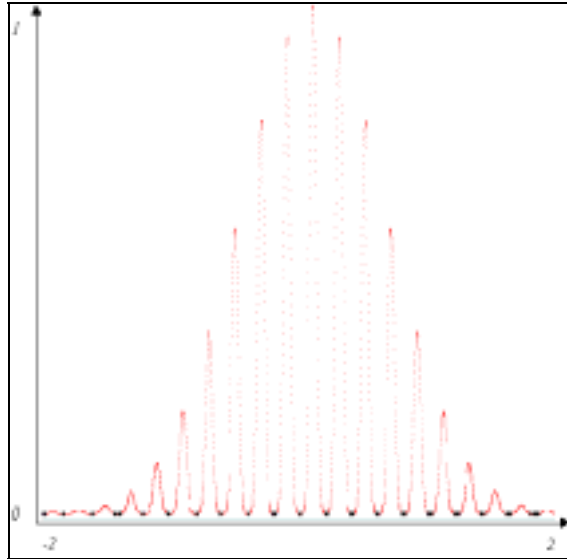


Figure 3: Minima are detected in 20 generations

For a better comprehension of the occurred phenomenon, we analyzed mostly in our tests the most uncomplicated case: one-variable optimization problem.

Moreover, we noticed experimentally that the *BCGA* technique gives satisfactory solutions for this kind of functions, which have multiple optima and these optima are distributed somehow uniformly among the search space. We analyzed only the cases of function maximization. We have no doubts that the *BCGA* algorithm could be easily reformulated for function minimization. In fact, paradoxically, the demand for minima solutions makes us firstly to reconsider the problem as maximization problem and only then to apply the *BCGA* in order to detect the minima.



## 6. Conclusions

Our research reveals the fact that Genetic Algorithms prove to be useful tools for various problems, including multimodal optimization problems. Present landscape of evolutionary computation regarding multimodal optimization, abounds with inspired techniques, which are more or less successfully applicable. Our study focuses on developing a new technique, which gives us the opportunity to observe a different behavior of the population. The investigation's result mainly consists of developing *BCGA* algorithm for multimodal optimization. Beyond its limitation concerning the fact that it gives satisfactory results only in cases of some particular functions, this technique signs us an interesting phenomenon, which we called it as *bipolar convergence*.

The lack of predetermination in the manner in which the genetic algorithms lead the population toward the wanted solutions, and their "strength to guide the chaotic and the random events" make these techniques suitable for complex, hard problems. Thus, the main advantage of genetic algorithms for optimization relies on the fact that they don't require any additional properties of the objective function (as continuity, derivability, convexity, and so on).

Another concluding remark comes from the present work: simple artifices could imprint different behavior among the population. Thus, *BCGA* stand for previous remark. For example, while the set of candidate-solutions is led toward maxima, an almost neglected part of the population acts as if it would be led toward the opposite optima (minima, in this case).

### References:

1. Bäck T., *Evolutionary Algorithms in Theory and Practice*, Oxford University Press, 1996.
2. Bessaou M., Pétrowski A., Siarry P., *Island Model Cooperating with Speciation for Multimodal Optimization*, Parallel Problem Solving from Nature (PPSN 2000), Paris, France, September 16-20, 2000.- <http://www-citi.int-evry.fr/~petro/EC-pub/>.
3. De Castro L. N., Von Zuben F. J., *Artificial Immune Systems*, Technical Report, TR – DCA 01/99 December, 1999.
4. De Castro, L. N., Von Zuben, F. J., *Artificial Immune Systems: Part I – Basis Theory and Applications*, Technical Report – RT DCA 01/99, 1999 - <http://www.dca.fee.unicamp.br/~lnunes/immune.html>
5. De Castro, L. N., Von Zuben, F. J., *Learning and Optimization Using the Clonal Selection Principle*, accepted for publication at the IEEE Transactions on Evolutionary Computation, Special Issue on Artificial Immune Systems, 6(3), pp. 239-251., 2002 - <http://www.dca.fee.unicamp.br/~lnunes/immune.html>

6. Dumitrescu D., *Algoritmi Genetici și Strategii Evolutive – aplicații în Inteligența Artificială și în domenii conexe*, Editura Albastră, Cluj Napoca, 2000.
7. Dumitrescu D., *Genetic Chromodynamics*, Studia, seria Informatica, volum XLV, Număr 1, 2000.
8. Dumitrescu D., Lazzerini B., Jain L.C., Dumitrescu A., *Evolutionary Computation*, CRC Press, New York, 2000.
9. Goldberg D. E., Richardson J., *Genetic algorithms with sharing for multimodal function optimization.*, Proc. 2nd Int. Conf. on Genetic Algorithms (Cambridge, MA), ed J. Greffenstette, pp. 41-49, 1987.
10. Goldberg D.E., *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison-Wesley Publishing Company, Inc., 1989.
11. Martin W. N., Lienig J., Cohoon J. P., *Handbook of Evolutionary Computation: C6.3 - Island (migration) models: evolutionary algorithms based on punctuated equilibria* - [http://www.iop.org/Books/CIL/HEC/pdf/ECC6\\_3.PDF](http://www.iop.org/Books/CIL/HEC/pdf/ECC6_3.PDF)
12. Michalewicz, Z., *Genetic Algorithms + Data Structures = Evolution programs*, Springer-Verlag, 3<sup>rd</sup> Ed.
13. Rotar C., Ileană I., *Models of Population for Multimodal Optimization. A New Evolutionary Approach*, Proc. 8<sup>th</sup> Int. Conf. on Soft Computing, Mendel2002, p.51 – 57, Czech Republic, 2002.

**Author:**

**Corina Rotar**, “1 Decembrie 1918” University, Alba Iulia, Computer Science Department, [crotar@lmm.uab.ro](mailto:crotar@lmm.uab.ro)