A NEW EVOLUTIONARY ALGORITHM FOR THE MULTIOBJECTIVE 0/1 KNAPSACK PROBLEM

by
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Abstract. The 0/1 knapsack problem is a well-known and it appears in many real domains with practical importance. The problem is NP-complete. The multiobjective 0/1 knapsack problem is a generalization of the 0/1 knapsack problem in which many knapsacks are considered. Many algorithms have been proposed in the past four decades for both single and multiobjective knapsack problem. A new evolutionary algorithm for solving multiobjective 0/1 knapsack problem is proposed in this paper. This algorithm used a ε-dominance relation for direct comparison of two solutions. Some numerical experiments are realized using the best and recent algorithms proposed for this problem. Experimental results show that the new proposed algorithm outperforms the existing evolutionary approaches for this problem.
Keywords: 0/1 Knapsack Problem, Evolutionary Multiobjective Optimization, ε-dominance, NP- Completeness

1. Introduction

The 0/1 knapsack problem is a widely studied problem due its practical importance. In the last years the generalization of this problem has been very much studied. Many algorithms have been proposed. Evolutionary approaches for solving the multiobjective 0/1 knapsack problem are of great interest. Many papers can be founded in the literature about multiobjective knapsack problem and about the algorithms proposed for solving them ([1], [2], [3], [6], [7], [8], [9], [10], [11]). In this paper we propose a new evolutionary approach for multiobjective 0/1 knapsack problem. We use the ε-dominance concept in order to determinate the solution quality.

In section 2 of the paper both single and multiobjective 0/1 knapsack problems are presented. The new proposed algorithm description is given in section 3 of the paper. The definition of ε-dominance notion is also given in section 3. Some comparisons with the most recent algorithms for this problem are realized in section 4. A set of conclusions are given in section 5 of the paper.

2. Problem formulation

The classical 0/1 knapsack problem can be formulated as follow: a set of n items and a knapsack of capacity c are considered. Each item has a profit p_j and a weight w_j. The problem is to select a subset of the items whose total weight does not exceed knapsack capacity c and whose total profit is maximum. Using the variables x_j (with $x_j = 1$ if the item j is selected and $x_j = 0$ otherwise) the problem can be written:

$$\text{maximize } \sum_{j=1}^{n} p_j x_j$$

$$\text{subject to } \sum_{j=1}^{n} w_j x_j \le c,\ x_j \in \{0, 1\},\ j = \{1, \ldots, n\}.$$

The problem can be extended for an arbitrary number of knapsacks. The multiobjective 0/1 knapsack problem is defined in the following way: A set of n items and a set of k knapsacks are considered. For each item are known:

$p_{i,j}$ – profit of item j according to knapsack i;

$w_{i,j}$ – weight of item j according to knapsack i.

The capacity of each knapsack i is c_i.

The problem is to find a vector $x = (x_1, x_2, \ldots, x_n) \in \{0, 1\}^n$ such that the capacity constraints

$$e_i(x) = \sum_{j=1}^{n} w_{ij} \cdot x_j \le c_i, \quad (1 \le i \le k)$$

are satisfied and for which $f(x) = (f_1(x), f_2(x), \ldots, f_k(x))$ is maximum, where

$$f_i(x) = \sum_{j=1}^{n} p_{ij} \cdot x_j$$

and $x_j = 1$ if the item j is selected and $x_j = 0$ otherwise.

3. Proposed algorithm

We will give here the definition of ε-dominance concept used instead of Pareto dominance.

Definition

Consider a maximization problem. Let x, y be two decision vectors (solutions) from the search space.

Solution x *ε-dominate* y if and only if the following conditions are fulfilled:

(i) $f_i(x) \ge f_i(y),\ \forall\ i = 1, 2, \ldots, n,$

(ii) $\exists\ j \in \{1, 2, \ldots, n\}: f_j(x) > f_j(y) + \varepsilon.$

The proposed algorithm can be described as follows: Each individual consists of a string of zero and one. The value one for the position *j* of the chromosome means that the item *j* was selected. The algorithm starts with a population of such individuals. For each chromosome the total items weight for each knapsack is computed. If there are knapsacks for which the allowed capacity is overdone the items beginning with one for which the proportion utility/weight is lowest are eliminated. This process continues until there are no knapsacks for which the capacity is overdone. All nondominated solutions are founded. The *ε*-dominance concept is used. Selection is applied by randomly chosing of two solutions. Convex crossover operator and mutation are applied. Two offspring are obtained. For each offspring the procedure for eliminating items if the capacity of one of knapsacks is overhead is used. The offspring enter the population of the new generation.

This algorithm is called *ε* Multiobjective Knapsack Problem Algorithm (*ε*-MOKPA).

The *ε*-MOKPA technique is described in what follows:

ε-MOKPA technique

```
begin
   Initialize population;
   for  nr = 1 to iterations_number do
     begin
       for i = 1 to popsize do
         for j = 1 to knapsaks_number do
            if exceed(pop[i], j)
            than eliminate(pop[i], j)
       Find nondominated solutions from population;
       for selection_number = 1 to popsize / 2 do
         begin
           Random select two individuals pop[ind1] and pop[ind2];
            Crossover(pop[ind1], pop[ind2]);
            Two offspring off1 and off2 are obtained;
       Mutate the offspring;
           for j = 1 to knapsaks_number do
              if exceed(off1, j)
                than eliminate(off1, j)
           for j = 1 to knapsaks_number do
              if exceed(off2, j)
                than eliminate(off2, j)
           The offspring enter the population of the new generation;
         end;
     end.
```

4. Experimental results

We will test our algorithm considering two knapsacks and 100 items. The results obtained are compared with the results obtained by SPEA ([12], [13]). For this comparison distance metric proposed in [4] and [5] is used. This metric is defined in what follows:

Assume that the Pareto front is known. Let us denote by P a set of Pareto optimal solutions. For each individual from the final population distance (Euclidian distance or other suitable distance) to the all points of P is computed. The minimum distance is kept for each individual. The average of these distances represents the measure of convergence to the Pareto front.

The results obtained in 30 runs are averaged.
The number of iterations is 25000.
The result obtained by SPEA is 19.2622.
The result obtained by ε-MOKPA considering the value 200 for ε is 8.25632.
The value of ε after each 1000 iterations is $\varepsilon / 2$.

If the value of ε is 5 the result obtained is 10.4092. For different values of ε different results are obtained. If $\varepsilon = 20$ the value 22.9026 is obtained. For $\varepsilon = 10$ the value is 13.6487. If $\varepsilon = 2$ the value is 8.40312. If $\varepsilon = 50$ and after each 500 iterations $\varepsilon = \varepsilon - 1$ the result is 9.83373. If $\varepsilon = 5$ and after each 5000 iterations $\varepsilon = \varepsilon - 1$ the result is 7.20486.

5. Conclusions

In this paper we have presented a highly effective evolutionary algorithm for solving 0/1 multiobjective knapsack problem. This algorithm uses the ε – dominance concept instead Pareto dominance. Some numerical experiments are realized. For these experiments two knapsacks and 100 items are considered. A comparison to SPEA is also realized. Experimental results shown that the proposed algorithm strongly outperforms SPEA.

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