SIGNAL IDENTIFICATION USING A MODIFY STRUCTURED NONLINEAR TOTAL LEAST NORM ALGORITHM

by
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Abstract. This paper proceeds series of papers devoted for solving overdetermined systems (A E_{error})x = b + β_{error} where errors occur as in the matrix A as in the vector b.
A problem is describe of the SNTLN modification (Structured Nonlinear Total Least Norm algorithm) for solving nonlinear system A(α)x = b. Numerical experiments have showed that performed modification is well founded.

Keywords: Total least squares, total least norm, nonlinear total least norm, overdetermined linear system, signal identification.

1. Introduction

There are some effective methods for solving nonlinear overdetermined system A(α) ≈ b. The SNTLN [3]-[4] is one of these. In this case the matrix A has Vandermond structure. The essence of the SNTLN algorithm is the optimal criterion which is next:

$$\left\| \begin{matrix} r(\alpha, x) \\ \alpha - \hat{\alpha} \end{matrix} \right\|_p \to \min_{\alpha, x} \qquad (1)$$

where r is the residual vector, $\hat{\alpha}$ is an initial estimate of the vector α which is equivalent to the matrix A, α - $\hat{\alpha}$ is a correction vector. The minimum solution search is done by using a linearization procedure.

Although numerical experiments which have been done by authors of [3]-[4] show that the SNTLN is effective algorithm nevertheless in the applied problems errors occur not only in A but in b too. In fact this approach does not use the information about the system there is in vector the b (does not correct it). That is why we propose to add extra term β and reconstruct the vector b at each iteration.

2. SNTLN algorithm modification

The kernel of the parameter and frequency estimation problems which occur in practice is using functional where t and y are input and output vectors in the next form:

$$y(t) = \sum_{j=1}^{n} x_j f_j(\alpha, t)$$

where $f_j(\alpha, t)$ are respective functions. The essence of identification such objects is estimated α and x using measurements $\{t_i, y_i\}$ i = 0, … , m-1 and known functions $f_j(\alpha, t)$ where m is number of measurements. This problem equivalent to the next problem of solving overdetermined system:

$$\begin{bmatrix} f_1(\alpha,t_0) & f_2(\alpha,t_0) & \dots & f_n(\alpha,t_0) \\ f_1(\alpha,t_1) & f_2(\alpha,t_1) & \dots & f_n(\alpha,t_1) \\ \dots & \dots & \dots & \dots \\ f_1(\alpha,t_{m-1}) & f_2(\alpha,t_{m-1}) & \dots & f_n(\alpha,t_{m-1}) \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_{m-1} \end{bmatrix}$$

But in fact vector b = y(t) + ε are registered where ε represent the noise random or normal mostly.

The SNTLN modification consists of adding extra correction vector β. In this case the residual vector r = b(β) – A(α)x is a function depended on (α, x, β). The optimal criterion of the modified algorithm have to be stated as:

$$\left\| \begin{matrix} r(\alpha, x, \beta) \\ \alpha - \hat{\alpha} \\ \beta \end{matrix} \right\|_p \to \min_{\alpha,x,\beta} \qquad (2)$$

This problem could be solved by using a linearization procedure conformable to the residual vector r(α, x, β).

$$r(\alpha+\Delta\alpha, x+\Delta x, \beta+\Delta\beta) = r(\alpha, x, \beta) - A(\alpha)\Delta x - J(\alpha, x)\Delta\alpha + \Delta\beta \qquad (3)$$

where J(α, x) is the Jacobian of the expression A(α)x. If $a_i(\alpha)$ represents the i-th column of matrix A(α) then:

$$J(\alpha, x) = \sum_{i=1}^{n} x_i \nabla_\alpha a_i(\alpha) = \nabla_\alpha [A(\alpha)x]$$

The matrix form of the optimal criterion (2) using a linearization (3) is next:

$$\left\| \begin{bmatrix} J(\alpha,x) & A(\alpha) & -I_m \\ I_n & 0 & 0 \\ 0 & 0 & I_m \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta x \\ \Delta\beta \end{bmatrix} + \begin{bmatrix} -r \\ \alpha \\ \beta \end{bmatrix} \right\|_p \rightarrow \min_{\Delta\alpha,\Delta x,\Delta\beta}$$

For clarity, it is useful to make a next notation:

$$\varphi(x, \alpha, \beta) = \left\| \begin{matrix} r(\alpha, x, \beta) \\ \alpha - \hat{\alpha} \\ \beta \end{matrix} \right\|_p$$

SNTLN algorithm modification

Input – $A(\hat{\alpha})$, $\hat{x}$, b and ε
Output –, x, β and residual vector r

1. Set $\alpha = \hat{\alpha}$, $x = \hat{x}$, $\beta = 0$. Compute $A(\alpha)$, $J(\alpha, x)$ set $r = b - Ax$
2. **repeat**
   a) $$\min_{\Delta\alpha,\Delta x,\Delta\beta} \left\| \begin{bmatrix} J(\alpha,x) & A(\alpha) & -I_m \\ I_n & 0 & 0 \\ 0 & 0 & I_m \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta x \\ \Delta\beta \end{bmatrix} + \begin{bmatrix} -r \\ \alpha \\ \beta \end{bmatrix} \right\|_p$$
   b) set $\delta = r(\alpha, x, \beta) - r(\alpha+\theta\Delta\alpha, x+\theta\Delta x, \beta+\theta\Delta\beta)$
   c) $\min_{0\le\theta\le1} = \varphi(\alpha+\theta\Delta\alpha, x+\theta\Delta x, \beta+\theta\Delta\beta)$
   d) $x := x + \Delta x$, $\alpha := \alpha + \Delta\alpha$, $\beta := \beta + \Delta\beta$
   e) Compute $A(\alpha)$, $J(\alpha, x)$ set $r = b - Ax$.

   until $\delta \ge \varepsilon$

Numerical tests have shoved that $\hat{x} = \min\|Ax - b\|_2$ is a good initial estimate for the vector x. It is due to underline that the SNTLN and its modification do not require uniformly dispersion of t_i.

SNTLN modification optimality conditions for p = 2

If p = 2 step 2(a) is equivalent of minimizing the differentiable function $\delta(\alpha, x, \beta)$:

$$\delta(\alpha, x, \beta) = \frac{1}{2} r^T r + \frac{1}{2}(\alpha - \hat{\alpha})^T(\alpha - \hat{\alpha}) + \frac{1}{2}\beta^T\beta$$

The first-order optimal conditions for a local optimum δ using the relations presented above become:

$$\begin{aligned} \nabla_\alpha \varphi &= -J^T r + I_n(\alpha - \hat{\alpha}) = 0 \\ \nabla_x \varphi &= -A(\alpha)^T r = 0 \\ \nabla_\beta \varphi &= -I_m r + I_m \beta = 0 \end{aligned} \quad (4)$$

In fact we need to solve over-determined system at each iteration

$$\begin{bmatrix} J(\alpha, x) & A(\alpha) & -I_m \\ I_n & 0 & 0 \\ 0 & 0 & I_m \end{bmatrix} \times \begin{bmatrix} \Delta\alpha \\ \Delta x \\ \Delta\beta \end{bmatrix} = \begin{bmatrix} r \\ -\alpha \\ -\beta \end{bmatrix} \quad (5)$$

Let M be a matrix at the (5). Using the optimality conditions (4) normal equations for system (5) are:

$$M^T M \begin{bmatrix} \Delta\alpha \\ \Delta x \\ \Delta\beta \end{bmatrix} = M^T \begin{bmatrix} r \\ -\alpha \\ -\beta \end{bmatrix} = -\begin{bmatrix} \nabla_\alpha \varphi \\ \nabla_x \varphi \\ \nabla_\beta \varphi \end{bmatrix} \quad (6)$$

Therefore normal solution of the system (5) can be find by pseudoinverse of M or QR decomposition of $M^T M$. These approaches are equivalent to the minimum solution search (4).

When M - has full rank then $M^T M$ - positive definite and there is a unique solution for the vector ($\Delta\alpha^T$, Δx^T, $\Delta\beta^T$).

This vector equals to zeros vector if and only if when right-hand-side (6) equals to zero. In fact step 2a is, in effect Gauss-Newton method which uses $M^T M$ as positive definite approximation to Hessian [1]. And the converge to optimum solution at step 2a SNTLNM is the same as SNTLN (at the second order rate).

3. Computational experiments

Osborne's signal [3] which is an exponential sum was chosen for computational test ($t_i \in [0, 1]$, i = 1…30):

$$y(t) = 0.5 \exp(0t) + 2 \exp(-4t) - 1.5 \exp(-7t).$$

Random noise $\varepsilon \in [-10^{-3}; 10^{-3}]$ was added to the correct values. For comparison the result the problem of fitting have been solved by using a standard lsqcurvefit function from Matlab. The computed results are tabulated at the tab 2. Additional characteristic such as ||r||- norm of the residual vector and the relative error of the vectors α and x are tabulated at the tab2.

Alg	α_1	α_2	α_3	x_1	x_2	x_3
Model	0	4.0000	7.0000	0.5000	2.0000	-1.5000
Initial	0.3000	5.0000	6.5000	0.6771	3.1300	-2.8020
lsqcurvefit	0.2034	4.9912	6.6508	0.6263	3.1577	-2.7862
SNTLN	0.1989	4.9908	6.6582	0.6241	3.1583	-2.7849
SNTLNM	0.1569	4.9866	6.7254	0.6034	3.1665	-2.7756

Table1: Obtained results

Obtained results show that performed modification is well founded. And that standard lsqcurvefit result can be improved.

Alg	$\|\|r\|\|$	ε_α	ε_x
lsqcurvefit	0.0110	0.1287	0.6806
SNTLN	0.0110	0.1323	0.6803
SNTLNM	0.0096	0.1285	0.6792

Table2: Additional characteristics

A modification of the Structured Total Least Norm algorithm has been presented for solving a class of problem related to SNTLN. Both theoretical and computational analysis show that this modification is well-founded when right-hand-side is subject on error. Research SNTLNM algorithm in different norms p = 1, ∞ is planning as future work.

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