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ANALYSIS OF MORPHOLOGY-LIKE OPERATORS USED IN COLOR IMAGE CONTRAST ENHANCEMENT

by
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Abstract. Mathematical morphology is difficult to introduce for color images because in the color vector space, minimum and maximum cannot be easily defined. So, the necessary conditions, represented by the complete lattice structure induced by an ordering relation, cannot be accomplished. Instead, we can build a pseudo-morphology based on reduced color ordering. This approach uses geometrical shape invariants computed from equivalent two-dimensional star glyphs (triangles) associated to the *RGB* colors. It does not fulfill the exact conditions but we can, at least, define pseudo-morphological or morphological-like erosion and dilation. The color image processing applications presented in this paper indicate that the shape descriptors used for the triangle color representation (star glyph) insures an increased independence with respect to luminance changes, while preserving essential hue information. The comparative analysis of different geometric shape descriptors is based upon the study of the equivalency class set induced onto the color vectors set (or *factor set* induced by the *equivalence relation*).

1. Introduction

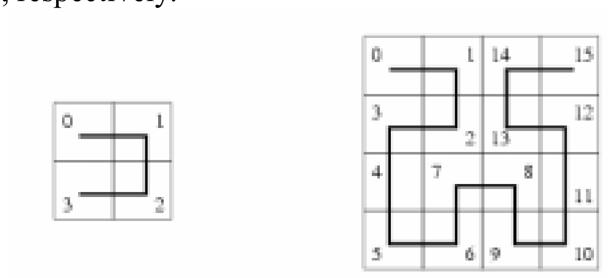
The processing of multichannel (multispectral, vector-valued) images, defined in each of their points by a vector of scalar values (color images being a particular common example), is a very important area of research. The major challenge in developing processing algorithms for such vector-valued images is the impossibility of the direct extension of the classical, scalar techniques. The difficulty of extension derives mainly from the correlation existing between the vector components that cannot be disregarded, or easily taken into account.

A significant part of the nonlinear processing techniques are based on rank-order statistics. Regardless of the particular filter type (the most usual being the median filter and the basic morphological operations by flat structuring elements), the common processing step is the ordering (sorting) in ascending order of the image samples within the filter window [5], [10]. An ordering relation must be reflexive, transitive and antisymmetric. Although the scalar ordering is obvious and very simple, there is no way to extend it to vector-valued signals for obtaining a mathematically-correct and topology-preserving ordering relation.

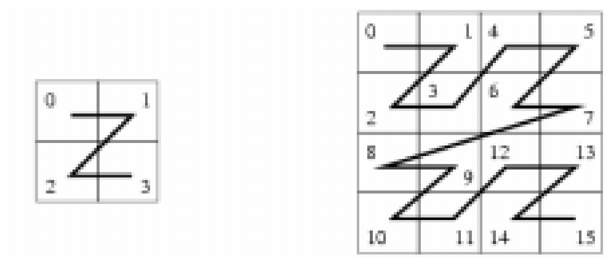
An obvious ordering relation in a vector space that satisfies the definition above is the lexicographic ordering. Furthermore, in [6] it was proven that lexicographic ordering is just a particular case of bit-mixing-based space filling curves, that can be used for ordering vectors (see figure 1). Two problems arise when it comes to apply this kind of ordering relations: the space topology is not preserved, and an importance criterion must be found for deciding the order of the vector components [6], [17].

This approach ends with significant implementations, for both color morphological operators and color median filtering.

The theoretical foundations of ***mathematical morphology*** have been constructed by ***J.Serra*** [13]**,** extending previously-defined set operators, known as the ***Minkowski*** set addition and subtraction. Primarily used as a processing and analysis tool for binary images (easily interpreted as sets), the mathematical morphology extends to multidimensional, scalar-valued signals by algebraic reformulations of the basic definitions. The fundamental operators, dilation and erosion, are defined as operators over a complete lattice, that commute with ***sup*** and ***inf***, respectively.



a)



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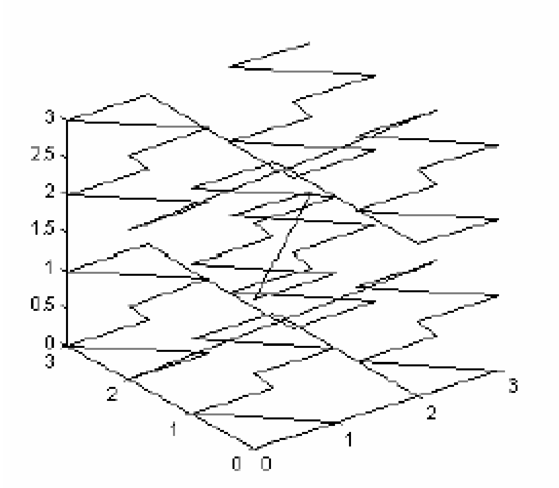


c)

Figure (1)

a) **Peano curve (Hilbert U-curve)** represented for a 2-D space, for which the coordinates quantization was made with 1 bit and 2 bits respectively;
b) **Morton curve (Hilbert Z-curve)** represented for a 2-D space, for which the coordinates quantization was made with 1 bit and 2 bits respectively;
c) **Morton curve (Hilbert Z-curve)** represented for a 3-D space (similar to color *RGB* space), for which the coordinates quantization was made with 2 bits.

According to this definition, the condition to be fulfilled for the existence of the morphological operators is the possibility of defining an ordering relation in the signal value space. For scalar signals the problem is trivial, but for vector valued signals we encounter the ordering problems already mentioned.

V.Barnett [3], [11] has investigated the possible use of incomplete ordering relations for multidimensional data and proposed four possibilities: the marginal ordering, which is equivalent to the separate ordering of each component; the partial ordering, which uses convex hull like sets; the conditional ordering, which is the scalar ordering according to a single component; the reduced ordering, which performs the ordering of vectors according to some scalars, computed from the components of each vector.

The marginal ordering is equivalent to the separate (independent) vector component processing. Although simple and direct, this technique may produce color artifacts (false colors) [2], [11]. The conditional ordering can be viewed as a reduced (or extreme) case of marginal ordering, since the processing involves a single vector component; its results are influenced by the choice of the component used for ordering [10]. Partial ordering can effectively determine the “core” of a set of vectors, by successively “peeling off” the outer convex-hull layers of data [9], [7]; still, extreme vectors cannot be further classified (as needed for defining dilation and erosion).

The reduced ordering offers the best results, being intensively used [2], [11], and [14]. The computed scalar is either the generalized distance from each data sample to some fixed reference point (the average, the marginal median), either a sum of distances from each data sample to some fixed characteristic points, either the sum of inter-sample distances or angles. In the case of defining color mathematical morphology operators based on reduced ordering, the exact conditions (complete lattice induced by an ordering relation) are obviously not accomplished. Thus, the obtained erosions and dilations must be called pseudo-morphologic, or morphological-like. An immediate approach is to define dilation as the color vector closest to the marginal maximum and erosion as the color vector closest to the marginal minimum [16]. A more complex, but still distance-based approach is based on combining several color vectors by a clustering technique in the color space [15].

The above mentioned approaches derive all from distance-based color filtering (i.e. scalars defined by distances in the color space). Still, different approaches can be considered, such as the mapping of initial data vectors (colors) into more “familiar” objects, such as one-dimensional functions (Andrews curve [1]), smileys (Chernoff faces [8]) or two-dimensional polygons (star glyphs [4]).

The triangle representation of colors is derived from the classical two-dimensional star glyph representation of multivariate data [4], [9]. In the general case, each component of a n-dimensional data vector is associated to one variation axis in the 2D plane, all axes having the same origin. The axes uniformly partition the space, being spaced by a $2\pi/n$ angle. The star glyph results by joining the points from each axis, corresponding to the given vector component values. In the particular case of the color ***RGB*** representation, the star glyph is a triangle (see figure 2), obtained by placing along three, ***120*** degrees spaced axes, the values of the three color components.

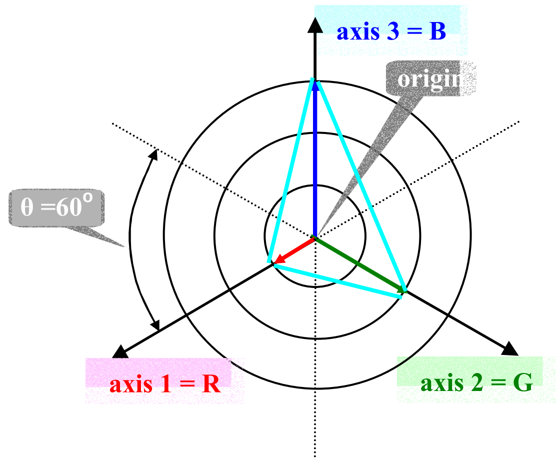


Figure (2). The star glyph (thick cyan triangle) associated to a three-component color vector.

In [17] we have shown that the shape of the triangle representing the color can be described by classical geometrical-based shape invariants [5], such as the surface-to-squared-perimeter ratio. From simple geometric considerations, this ratio can be easily expressed as function of the RGB color components as:

$$k = \frac{S}{P^2} \quad (1)$$

$$P = a + b + c \quad (2)$$

$$a = \sqrt{R^2 + G^2 + RG} \quad (3)$$

$$b = \sqrt{B^2 + G^2 + BG} \quad (4)$$

$$c = \sqrt{R^2 + B^2 + RB} \quad (5)$$

$$S = \frac{1}{4}(RG + GB + BR) \quad (6)$$

2. First geometric shape descriptor. The complete lattice induced over the factor set

The first geometric shape descriptor used for the color triangle was the *isoperimetric shape ratio.* This function is independent with respect to the scaling operations and is defined as the ratio between the triangle pseudo-surface and its squared perimeter:

$$\boldsymbol{k(v) = k(r,g,b) = \frac{S_\Delta(r,g,b)}{P_\Delta^{\,2}(r,g,b)} = \frac{(rg + gb + br)}{4\left(\sqrt{r^2 + g^2 + rg} + \sqrt{b^2 + g^2 + bg} + \sqrt{r^2 + b^2 + rb}\right)^2}} \quad (7)$$

The k-ratio increases as the shape of the triangle becomes more regular (and thus the color less saturated). k is maximum for a regular (equilateral) triangle (for achromatic colors or gray colors) and decreases as the triangle becomes more irregular (for saturated colors). Although the k-ratio is not a bijective function (all the proportional triangles are associated to the same value), this choice proved to be correct for reduced color ordering as it is dependent to the color hue and also luminance invariant by definition.

In this paper we will develop further comparative studies for different geometric shape descriptors. Finally this will be concluded by an optimal selecting algorithm for the most performant descriptor function.

The geometric shape descriptor functions representing the color triangle can induce a ***equivalence relation*** in the color vector space due to its symmetric properties as a scalar function defined over a 3 dimensions domain ($k: R^3 \rightarrow R$).

For example, ***isoperimetric shape ratio*** is symmetric by definition with respect to any of the three pairs of variables (the color vector components):

$$k(r,g,b) = k(g,r,b) = k(r,b,g) = k(b,g,r)$$

For the quantized color images, $k: E^3 \rightarrow R$ where $E=[0, M] \geq N$ and M is the maximum level of quantization.

Proposition 1: The ***binary relation*** $\rho \subseteq E^3 \times E^3$ defined as:

$$\forall v_1, v_2 \in E^3, \quad v_1 \ \rho \ v_2 \Leftrightarrow k(v_1) = k(v_2) \quad \textbf{(8)}$$

is an ***equivalence relation*** because it is, obviously, ***symmetric***, ***reflexive*** and ***transitive.***

We denote with K the equivalency class set induced onto the color vectors set E^3(otherwise named the ***factor set*** induced by the ***equivalence relation*** previously defined, E^3/ρ). Then we can define ***canonic surjective application*** $f: E^3 \rightarrow K$ which will associate to each and every vector its own equivalency class.

In the factor set, we can now define the known ordering relation, based upon the geometric shape descriptor:

Proposition 2: The ***binary relation*** $\rho_K \subseteq K \times K$ defined as:

$$\forall C_1, C_2 \in K, \ \ \forall v_1 \in C_1, \ \forall v_2 \in C_2, \quad C_1 \rho_K C_2 \Leftrightarrow k(v_1) \leq k(v_2) \quad \textbf{(9)}$$

is an ***ordering relation*** because it is, obviously, ***anti-symmetric***, ***reflexive*** and ***transitive.***

Thus we can associate a ***quasi-ordered*** set (the color vector space E^3) with its own ***factor set*** $E^3/\rho = K$, which is ***ordered.*** This association is made by a morphism between quasi-ordered sets (the canonic surjective application f). This construction allow us to reduce the study of the ***quasi-ordered*** set, E^3 to the study of its own factor set, $E^3/\rho = K$, which is an ***ordered*** set.

We can prove the ***factor set*** $E^3/\rho = K$ is ***total ordered set*** (i.e. ***lattice)*** and even more, a ***complete lattice*** because:

$$\forall K' \subseteq K,\ \ K' \neq \{\varnothing\},\ \exists \sup\{K'\} \in K \text{ and } \inf\{K'\} \in K$$

Subsequently, we will build a representative system for the equivalency classes based upon the following propositions:

Proposition 3: Let be a vector ***v,*** belonging to any equivalency class, then every vector obtained through a permutation of its color components will belong to the same equivalency class:

$$\forall C \in K,\ \ \forall v \in C \text{ cu } v = (x_1, x_2, x_3) \Rightarrow v' = (x_{\sigma(1)}, x_{\sigma(2)}, x_{\sigma(3)}) \in C \qquad \textbf{(10)}$$

where $\sigma \in S_3$ (i.e. the symmetric group of the 3-order permutations).

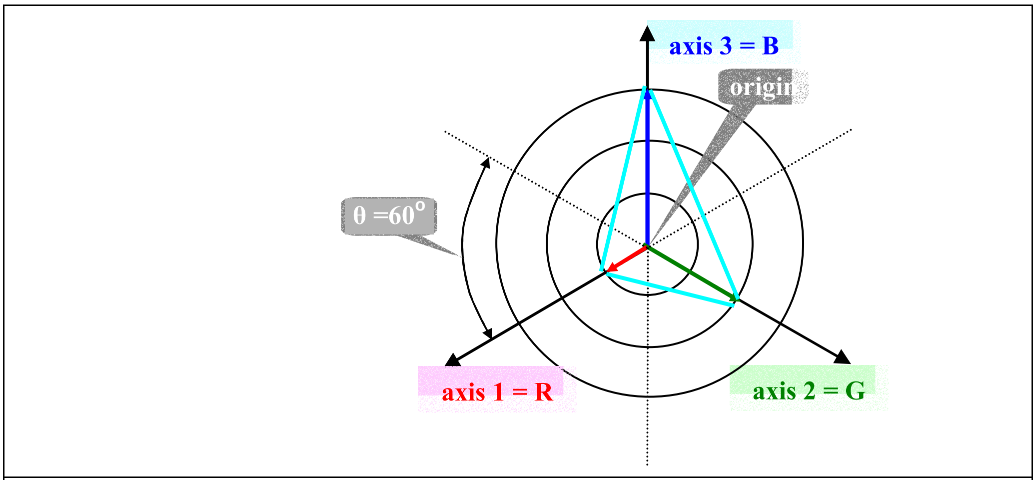
Observation: If $r \neq g \neq b$ then $Card(C) \geq P_3 = 3! = 6$:

$$\forall C \in K,\ \ \forall v \in C \text{ with } v = (r, g, b) \Rightarrow$$
$$(r, g, b)\ \rho\ (r, b, g)\ \rho\ (g, r, b)\ \rho\ (g, b, r)\ \rho\ (b, r, g)\ \rho\ (b, g, r) \qquad \textbf{(11)}$$

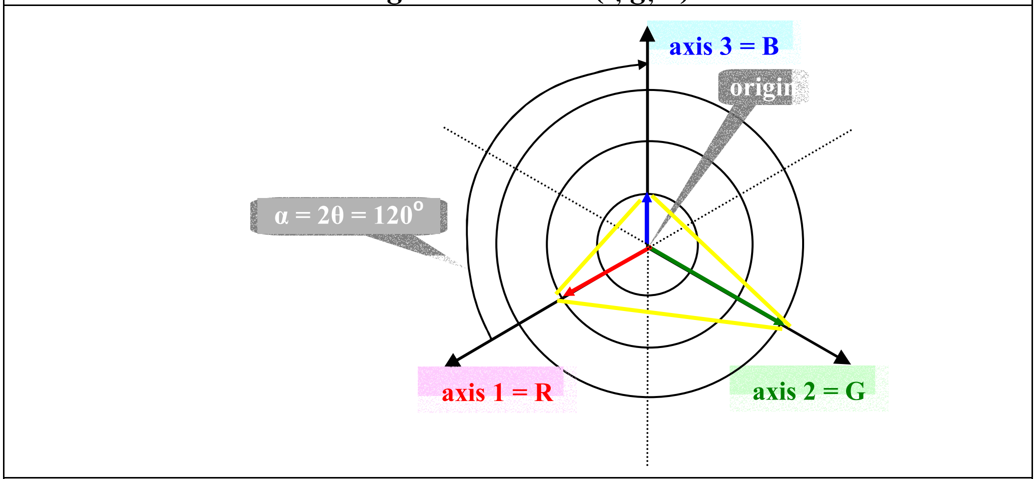
3. Geometric interpretation

Starting with any vector $v = (r, g, b)$ we can obtain **5** supplementary vectors represented by congruent color triangles with the original triangle:

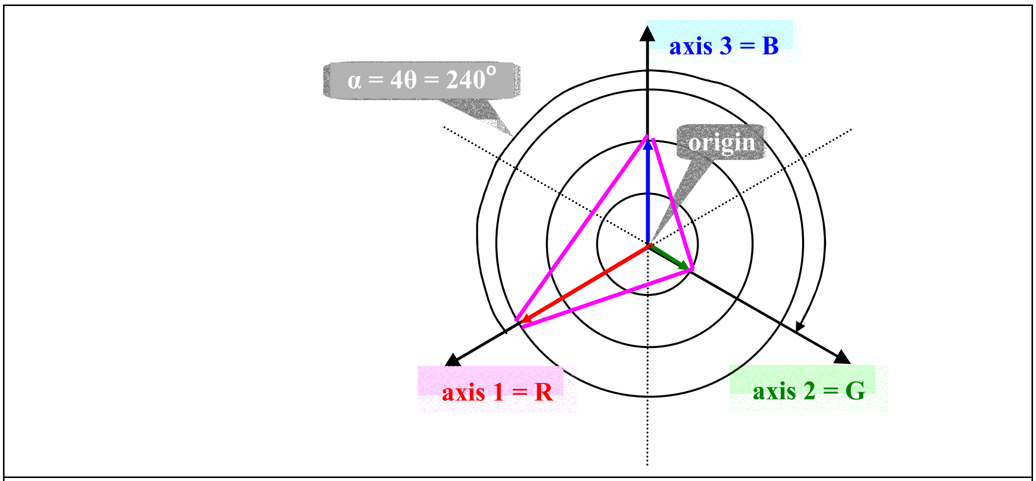
- If we associate rotary vectors to each color component, then the rotations by angle $\alpha = 120^o$ and $2\alpha = 240^o$ will produce congruent triangles in the geometric representation of colors:



Original vector v =(r, g, b)

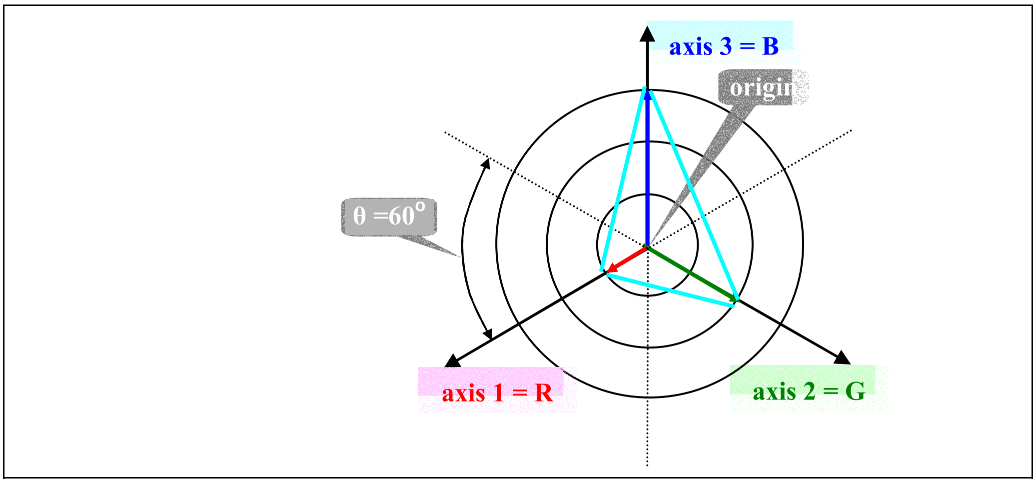


Original vector rotated by angle α=120°

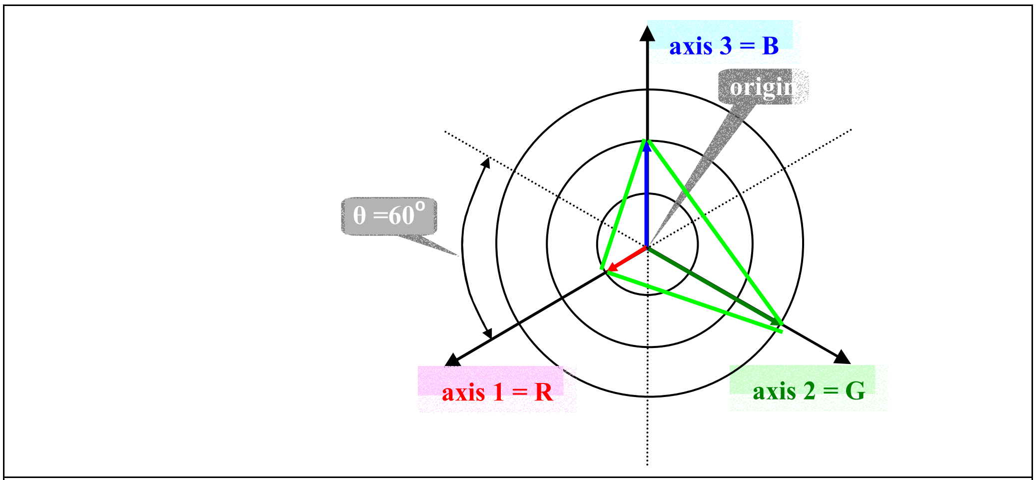


Original vector rotated by angle 2α=240°

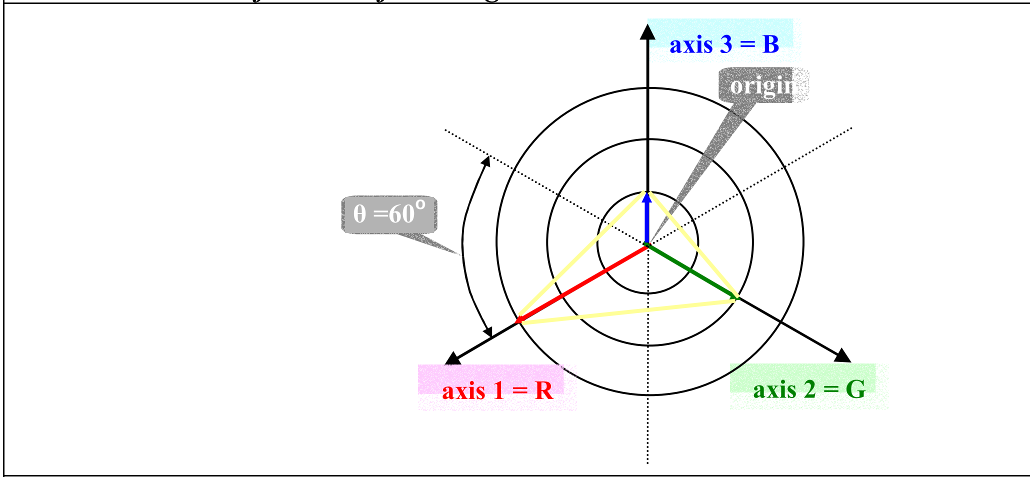
- The three reflections related to the ***3*** axe ***R,G*** and ***B*** will also generate congruent triangles in the geometric representation of colors:



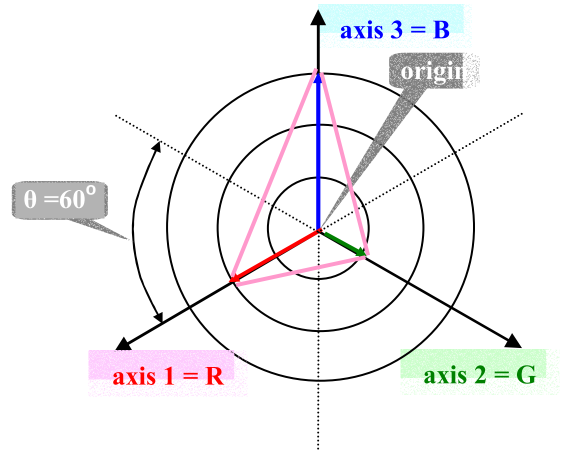
Original vector v =(r, g, b)



The reflection of the original vector related to axis R



The reflection of the original vector related to axis G



The reflection of the original vector related to axis B

4. Algorithm for chromatic correction

Observations:

- The members of the same equivalency class will have very different chromatic properties.
- This fact can produce artificial chromatic artifacts, if will define morphological-like operators based upon these symmetrical descriptors functions with respect to the color components.
- ***Solution:*** After the application of the morphological-like operators we must do a ***chromatic correction*** upon the new values of the vectors in a manner that will preserve the relative ratio between the color components.
- ***Chromatic correction*** is obtained by choosing another vector, from the same equivalency class. This new vector is constructed by a permutation of the color components in a manner that will preserve the relative order in the original vector.
- Because any ***3*** element-permutation can be achieved through the product of ***2*** transpositions then we can synthesize the next ***chromatic correction*** algorithm:

  1. We denote the vector in the original image with $\boldsymbol{v} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)$ and the vector in the morphologically processed image with $\boldsymbol{v'} = (\boldsymbol{x'}_1, \boldsymbol{x'}_2, \boldsymbol{x'}_3)$.

2. We sort ascending the color components of the two vectors and we obtain the next 2 permutations of the color components indexes:
   (i_1, i_2, i_3) for the vector in the original image, ***v*** and
   (k_1, k_2, k_3) for the vector in the morphologically processed image, ***v'***
3. We perform the ***2*** transpositions corresponding to the minimum and maximum components, preserving the original order:
   3.1. Interchange components x'_{i_1} with x'_{k_1} ***(minimum)***
   3.2. Interchange components x'_{i_3} with x'_{k_3} ***(maximum)***

5. The representative system for the equivalency classes

Proposition 4: Let be any vector ***v,*** in any equivalency class, then any other vector with color components multiples of the vector ***v,*** will belong to the same equivalency class:

$$\forall C \in K,\ \forall v \in C \text{ with } v = (x_1, x_2, x_3) \Rightarrow v' = (nx_1, nx_2, nx_3) \in C \tag{12}$$

where $n \leq \dfrac{M}{max(x_1, x_2, x_3)}$.

As a consequence of propositions ***4*** and ***5*** we can define an generating algorithm for a ***representative system*** for equivalency classes:

Proposition 5: A ***representative system*** for equivalency classes, as defined above, can be obtained by generating the complete set of ***relative prime*** numbers triplets, in the lexicographic order within the maximum quantization interval, $[0, M]$.

Proof:

Let be ***E'*** the ***representative system*** for equivalency classes obtained through the algorithm defined above. Then we have the following proposition:

- $\forall\ v = (x_1, x_2, x_3) \in E' \Rightarrow v' = (x_{\sigma(1)}, x_{\sigma(2)}, x_{\sigma(3)}) \notin E'$ with $\sigma \in S_3$, the symmetric group of the ***3^rd^ order permutations,*** because the generation was done in the lexicographic order.
- $\forall\ v = (x_1, x_2, x_3) \in E' \Rightarrow v' = (nx_1, nx_2, nx_3) \notin E'$ because the number triplets are selected to be ***relative prime numbers*** and with the above conditions we have $(nx_1, nx_2, nx_3) = n$,

Proposition 6: Let be ***v,*** a vector belonging to the ***representative system*** obtained by generating the complete set of ***relative prime*** numbers triplets, in the lexicographic order within the maximum quantization interval, **[*0, M*]**, then the cardinal of its equivalency classes less or equal with the expression:

$$\forall v \in E' \Rightarrow \mathbf{Card}(C_v) \le \frac{6 \cdot M}{max(x_1, x_2, x_3)} \tag{13}$$

6. Experimental results

The program has generated the ***representative system*** for equivalency classes as shown in the algorithm described above by proposition ***5*** and it has analyzed the distribution of the equivalency classes onto the co-domain for following three morphological descriptors functions:

1. Isoperimetric index for the triangle shape defined as the ratio between the triangle pseudo-surface and its squared-perimeter:

$$k(v) = k(r,g,b) = \frac{S_\Delta(r,g,b)}{P_\Delta^2(r,g,b)} = \frac{(rg+gb+br)}{4\left(\sqrt{r^2+g^2+rg}+\sqrt{b^2+g^2+bg}+\sqrt{r^2+b^2+rb}\right)^2}$$

2. Sum of the modified isoperimetric index for the 3 triangle shape composing the original triangle:

$$k_1(v) = k_1(r,g,b) = \frac{S_\Delta(r,g,rg)}{P_\Delta^2(r,g,rg)} + \frac{S_\Delta(b,g,bg)}{P_\Delta^2(b,g,bg)} + \frac{S_\Delta(r,b,rb)}{P_\Delta^2(r,b,rb)} = \frac{rg}{(r+g)^2} + \frac{bg}{(b+g)^2} + \frac{rb}{(b+g)^2}$$

3. Sum of the harmonic semi-average for the color components of the original vector:

$$k_2(v) = k_2(r,g,b) = \frac{rg}{r+g} + \frac{bg}{b+g} + \frac{rb}{b+g} = \frac{1}{\frac{1}{r}+\frac{1}{g}} + \frac{1}{\frac{1}{b}+\frac{1}{g}} + \frac{1}{\frac{1}{r}+\frac{1}{b}}$$

Observation:

- Only the last function has a quasi-normal distribution of the equivalency classes onto the function co-domain.
- The proposition *4* cannot be applied for this function**,** thus representative system is greater, in fact, which will determine a better

morphological discrimination. The effect can be observed in the contrast enhanced image associated.

- We can conclude that a histogram with a normal distribution is the correct selecting criteria for the morphological discrimination function.
- The program for generating relative prime number triplets has counted only ***2.342.503*** equivalency classes in the total color vector space, composed by ***16.777.216*** vectors. The histogram shows the distribution of the equivalency classes onto the morphological discrimination function co-domain.
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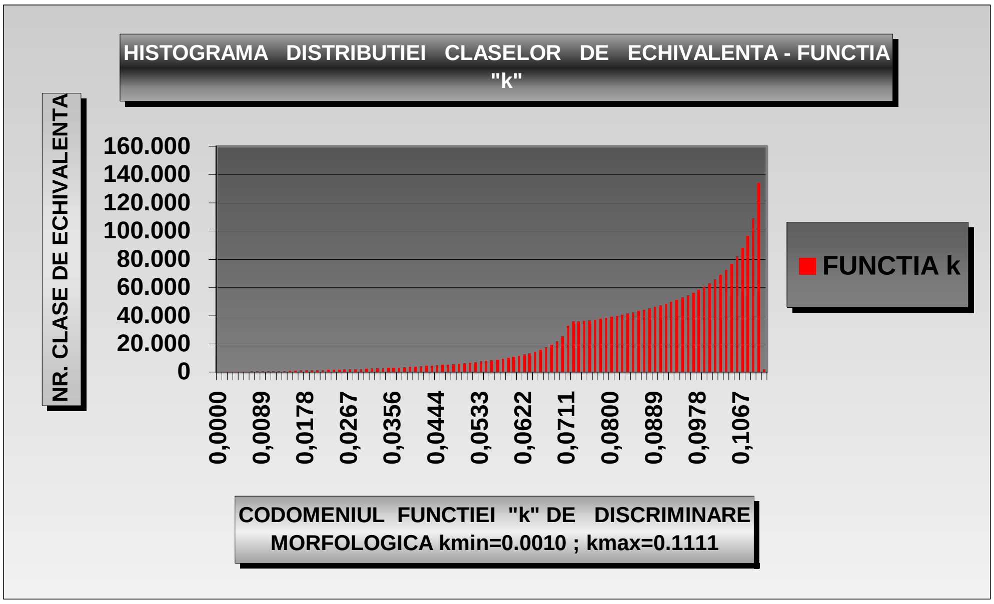

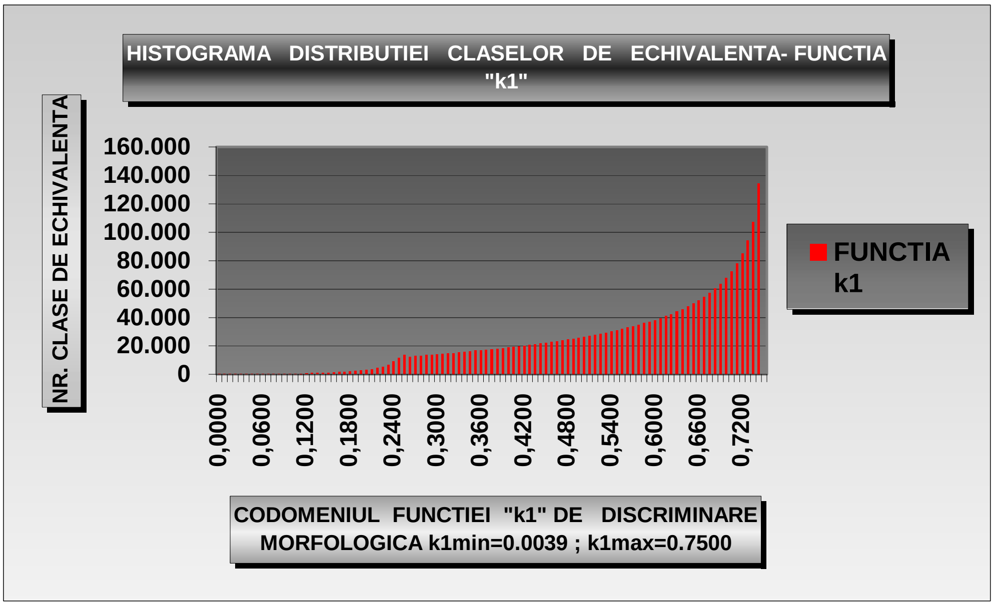
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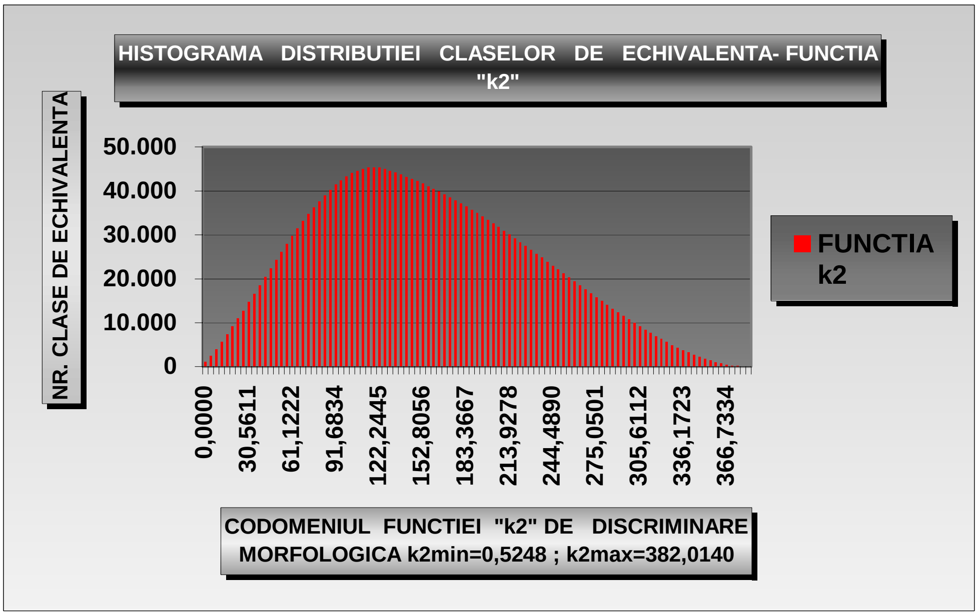
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Original image

Contrast enhanced image using function
k=S/(P^2)
by 3x3 structuring element

Contrast enhanced image using function
k_1=1/(R/G+G/R+2)+1/(B/G+G/B+2)+1/(R/B+B/R+2)
by 3x3 structuring element

Contrast enhanced image using function
k_2=1/(1/R+1/G)+1/(1/G+1/B)+1/(1/B+1/R)
by 3x3 structuring element

Original image
(zoom 400%)

Contrast enhanced image using function
k=S/(P^2)
(zoom 400%)

Contrast enhanced image using function
k_1=1/(R/G+G/R+2)+1/(B/G+G/B+2)+1/(R/B+B/R+2)
(zoom 400%)

Contrast enhanced image using function
k_2=1/(1/R+1/G)+1/(1/G+1/B)+1/(1/B+1/R)
(zoom 400%)

Original image
(zoom 400%)

Contrast enhanced image using function k=S/(P^2)
(zoom 400%)

Contrast enhanced image using function
k_1=1/(R/G+G/R+2)+1/(B/G+G/B+2)+1/(R/B+B/R+2)
(zoom 400%)

Contrast enhanced image using function
k_2=1/(1/R+1/G)+1/(1/G+1/B)+1/(1/B+1/R)
(zoom 400%)

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