

**$\aleph^2$  MINIMUM - ESTIMATION METHOD OF PARAMETERS IN  
AN ECONOMETRIC MODEL WITH QUALITATIVES  
VARIABLES**

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ABSTRACT. The article presents an estimate of parameters in an econometric model with qualitative variables; the article shows us how to work with multiple observations or with grouped data. The model proves us that we have a better estimation of parameters using this kind of estimation instead of using a model with individual data. The estimation method presented in this article is called minimum chi-square.

The estimation of parameters for an econometric model with qualitative variables in case that is working with multiple observations or with grouping data using the linear probabilistic model is better than we use the individual data.

We suppose that we have  $n_i$  observations for variable  $x_i$  and suppose too that for  $m_i$  observations we have the probe  $i$  and suppose that for  $n_i - m_i$  observations we don't have the probe. Then the empirical probability are:

$$\hat{p}_i = m_i/n_i$$

We suppose that we have the theoretical probability like:

$$p_i = \beta' x_i$$

This is a linear probability function. The name is because the probability  $p_i$  is a linear mixture of the regressing factors  $x_i$ . We can write the last equation in this way:

$$\hat{p}_i = \beta' x_i + u_i. \tag{1}$$

where  $u_i = \hat{p}_i - p_i$ .

When we discuss about large sample we have  $\hat{p}_i p_i$  and  $E(u_i) = 0$ . The following equality,  $\text{var}(\hat{p}_i) = p_i(1 - p_i)n_i$ , which can be estimated through  $\hat{p}_i(1 - \hat{p}_i)n_i$  if  $n_i$  is a large number. We can use the least squares weight for estimate method  $\beta$  in equality number (1), using as weights:

$$w_i = (\hat{p}_i(1 - \hat{p}_i)n_i)^{-\frac{1}{2}}$$

The method that is describe is call the minimum  $\chi^2$ . In a log-linear model we have:

$$\log p_i = \beta' x_i.$$

In this case we can write:

$$\log \hat{p}_i = \beta' x_i + u_i$$

where  $u_i = \log \hat{p}_i - \log p_i$ .

If we develop  $\log \hat{p}_i$  around the  $p_i$  in a Taylor series we have:

$$\log \hat{p}_i = \log p_i + (\hat{p}_i - p_i) \frac{1}{p_i} + \text{the terms with bigger orders}$$

because  $u_i = (\hat{p}_i - p_i) \frac{1}{p_i}$ . In large sample  $E(u_i) = 0$ , and then:

$$\text{var}(u_i) = \frac{1}{p_i^2} p_i(1 - p_i)n_i = (1 - p_i) \frac{1}{p_i} .$$

We can estimate again this through  $(1 - \hat{p}_i)n_i \hat{p}_i$ . Now in the estimation stage through least squares weight method we use  $[(1 - \hat{p}_i)\hat{p}_i]^{-\frac{1}{2}}$ . for weights. This is the minimum of  $\chi^2$  method in a log-linear model.

In the logit model we consider the following:

$$\log \frac{p_i}{1 - p_i} = \beta' x_i$$

We can write again:

$$\log \frac{\hat{p}_i}{1 - \hat{p}_i} = \beta' x_i + u_i$$

where

$$u_i = \log \frac{\hat{p}_i}{1 - \hat{p}_i} - \log \frac{p_i}{1 - p_i}.$$

Using the development in Taylor series of  $\log \hat{p}_i$  around  $p_i$  we obtain, if we leave the bigger orders terms, like before:

$$u_i(\hat{p}_i - p_i) (1 - p_i) = 1 - p_i (\hat{p}_i - p_i).$$

In big samples  $E(u_i) = 0$  and

$$\text{var}(u_i) = 1 - p_i (1 - p_i) n_i = 1 - p_i n_i (1 - p_i) .$$

We can estimate again  $\text{var}(u_i)$  through  $1 - p_i (1 - p_i) n_i$  and can use the least squares weight method for estimate the parameters.

The conclusion of the article is that the chi-square method is very useful in estimation of parameters specially when we worked with groups of data using the linear probabilistic model. How can be observe this method of estimation can be use with success in the log-linear model too and in the logit model because the work mode is not very complicated and not very hard.

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