Proceedings of the *International Conference on Theory and Application of Mathematics and Informatics ICTAMI 2005* - Alba Iulia, Romania

CONTROL PROBLEM FOR EVENT-SWITCHED PROCESSES

Andrey M. Valuev

Abstract.A class of models is introduced to represent production processes in complex industrial systems where elements of the system change discrete characteristics of their states subsequently or cyclically; after every such a state change, or an event (such as changes of equipment units work modes, origination and termination of partial production processes, switches of materials flows destination and so on) the set of relationships between production system variables alter. Events subdivide the entire process period into stages, or scenes; the succession of events, or a process scenario, within a given period is fixed neither in order nor in the number and depends on the process control. In format, the proposed model generalize the model of catastrophe control recently put forward by V.V.Velichenko in which the scene of the process terminates when the process state reaches its boundary.

The paper presents the results of the study of general properties of the introduced models. The conditions of finiteness of the events number within a given period are established as well as the conditions under which two nearby subsequent events may be done simultaneous, or, v.v., two simultaneous events may be separated in time. The necessary conditions of the process optimality are formulated as well, both with a given scenario and irrespective to the scenario. A hybrid method of the process optimization is proposed that combines a branch-and-bound method to seek the optimum scenario and a decomposition method with features of feasible directions and gradient-restoration methods for optimization within a fixed scenario.

2000 *Mathematics Subject Classification*:90B30, 34K35, 93C65, 49K15

1. The origin of the problem and some related topics

In the USSR the study of transforming systems control was stimulated mainly with the problems of space flight. It is well known that space flights usually have several stages and the event of the stage end means the separation of used parts from the spacecraft. So, three features must be taken into account: first, the parameters of the spacecraft, such as mass, length etc (and even motion equations) change instantaneously due to these events, second, the instant of the transformation is not fixed and depends on the control; third, there are relationships between values of parameters before and after the transformation. On the other hand, the number and succession of events for a normal flight is fixed. Models of such processes were studied by V.V.Velichenko, L.T.Ashchepkov and recently by A.S.Filatyev.

And what about an abnormal, catastrophic flight? As the recent space shuttle flight has shown the succession of them is arbitrary, but may be predicted with the adequate model of its dynamics. So we go to the more general model introduced by V.V.Velichenko [1, 2] as the catastrophe control problem. In the model the succession of events is not fixed and these events are treated as shifts between two adjacent scenes of the process; the scene is characterised with a certain domain in the space of the system state and the set of relationships between the initial and final state within the scene. The shift between the two adjacent scenes is characterised with the relationships between the final state within the former scene and initial state within the latter one. Unfortunately, the model does not determine to which scene the process proceeds after reaching the bounds of a definite scene, so the study in [1,2] is concentrated on the process with a given succession of scenes, or a process *scenario*.

In fact, the term “catastrophe” is an attractive label only and really means some kind of transformation of both parameters and the behaviour of the dynamic system. Such a kind of “catastrophes” exists in normal performance of many complex production systems. Models of controlled discrete-time processes with continuous variables were first put forward due to problems of chemical industry. They happened to be adequate to the conditions of mineral and petroleum industry, agriculture and probably to many cases of construction and use of terrestrial technological objects. To be more adequate it is necessary to take into account not only dynamics of parameters but also qualitative changes of the state of the production system elements and the environment. So the proposed approach to production planning or regulation problems formulation explicitly takes into consideration the fact that the qualitative state of the production system changes after some events, so instants of these events are treated as ends of plan period stages. To find instants of events of a certain type corresponding relationships are introduced into the model. For this reason the author [3, 4] put forward the general model of a controlled process with both continuous- and discrete-valued state and control variables in which state variables change their values after events depending on controls. This form of models was substantiated first for surface mining but it seems very likely to be relevant to most cases of systems usually described with models of discrete-time processes.

The general formulation of the new models class is given in the paper. These models give the possibility to apply exact optimization techniques for determination of values of some parameters that earlier might be appointed only by experts and to embrace in the sole problem statement a lot of plan problem variants that traditionally may be regarded only separately. Such kind of models may be treated either as deterministic or stochastic that lead to broad possibilities of controlled processes modeling in the context of planning as well as regulation due to various disturbances, but now we concentrate the study on deterministic models only and present the general results pertaining to them in a semi-formal way. The exact formulations will be issued soon in the author's paper in the Russian journal Doklady Akademii Nauk in the section "Mathematics" translated into English under the title Doklady Mathematics.

2. Some examples of transformations in production systems performance

Elements of mathematical models of event-switched processes may be illustrated for medium- and short-term open pit production planning problems. For big open pits the maximum efficiency of production is achieved with the combination of direct ore dispatch from excavators faces with blending stockpiles formation and unloading; this technology of consumer production formation is however very difficult for planning. So the production system may be represented as the set of primary (ore blocks) and secondary (stockpiles) reservoirs and material flows between them; it must be emphasized that the current state of each reservoir is characterized not only with a continuous-valued variables (the amount and average quality of the ore) but with a discrete-valued variables of their qualitative state (the kind of the operation — boring, blasting, excavating etc — for ore blocks, the state of loading or unloading for a stockpile section), the latter changing their values at non-fixed instants, not simultaneously.

The open pit production system is represented as the oriented graph in which vertices ($i \in I$) correspond to reservoirs and arcs ($j \in J$) to material flows, J_i^-, J_i^+ are sets of arcs terminating and originating in the vertex i, respectively. The current state of an ore or waste block is described with a scalar state variable x_i meaning the amount of the material excavated from it. The relationship between instantaneous values of ore quality indices and x_i is $a_{il}(x_i)$. Current characteristic of a material flow is its intensity (cubic meters or tons per second) q_{jl} and values of quality indices a_{jl}. For the set of ore flows initiating in the i-th block the following equations take place

$$\dot{x}_i = \sum_{j \in J_i^+} q_j; \tag{1}$$

$$a_{jl} = a_{il}(x_i). \tag{2}$$

The condition of the m-th switch of the i-th block at the instant t is

$$x_i(t) = x_{im}; \tag{3}$$

after such an event we have for its qualitative state $m+1$ instead of m and for its quantitative state x_i=0 instead of $x_i = x_{im}$.

For other reservoirs state variables are M_i(material quantity in the i-th reservoir) and a_{il} (the mean value of the l-th quality index). In general the relationship between M_i, a_{il} and the values of initiating and terminating flows is expressed with the balance equations

$$\begin{gathered} \dot{M}_i = \sum_{j \in J_i^-} q_j - \sum_{j \in J_i^+} q_j, \\ d(M_i a_{il})/dt = \sum_{j \in J_i^-} q_j a_{jl} - a_{il} \sum_{j \in J_i^+} q_j, \quad a_{jl} = a_{il}, j \in J_i^+. \end{gathered} \tag{4}$$

For two-sectioned stockpiles it is necessary to introduce the state variables for both sections (M_{i1}, a_{i1l},M_{i2}, a_{i2l}) where the index value 1 pertains to a loaded section and 2 to an unloaded one:

$$\dot{M}_{i1} = \sum_{j \in J_i^-} q_j,$$

$$\dot{M}_{i2} = -\sum_{j \in J_i^+} q_j, \quad d(M_{i1}a_{i1l})/dt = \sum_{j \in I_i^-} q_j a_{jl}, \dot{a}_{i2l} = 0, \quad a_{jl} = a_{i2l}, j \in J_i^+. \tag{5}$$

A stockpile switch (change of its sections roles) occurs when one the following conditions is valid:

$$M_{i1}(t) = M_{imax}, M_{i2}(t) = 0, \tag{6}$$

and the result of the switch at the instant t is expressed with relationships:

$$M_{ir}(t+0) = M_{i3-r}(t), a_{irl}(t+0) = a_{i3-rl}(t), l = 1, \ldots, L_i, r = 1, 2. \tag{7}$$

In fact, such a form of the model of a controlled production process may not be limited to the realm of mining industry. It must be emphasized that the switch condition in the form (3) pertains to any kind of works which state may be measured quantitatively. On this way lies the generalization of models of project management with PERT/CPM.

To incorporate the search of maintenances terms into the entire planning problem it is necessary to express conditions on them in the proposed form. In fact, possible terms of maintenance for the j-th machine may be expressed related to the total net time of its operation elapsed after the termination of the precedent maintenance (T_{wj}); T_{wj} must be treated as a state variable. Let I_{wj} denotes the type of work (or stay) and I_{wj}=1 denotes operation. Difference equations for T_{wj} are (for the k-th stage of the process)

$$T_{wj1}(k) = \begin{cases} T_{wj0}(k) + \Delta T(k), \text{ if } I_{wj}(k) = 1, \\ T_{wj0}(k), \text{ otherwise.} \end{cases} \tag{8}$$

At the moment of the next maintenance beginning the condition must be satisfied:

$$T_{wj1}(k) = \Delta T_{mj} \tag{9}$$

where $\Delta T_{m\ j}$is an interval between maintenances for the j-th machine (measured as the net operation time).

3. The general formulation of the problem

In the above examples the characteristic features of a event-switched process were shown, namely two kinds of state variables, i.e. continuous-valued and discrete-valued, the process period subdivision into stages having a constant quality state and terminating with events of switching, differential (1), (4), (5) or difference (8) equations for their dynamics within a stage alongside with some functional relations (2), (4), (5) conditions for a definite type of events (3), (6), (9) and equations for the system state transformation as a result of an event (7). It must be underline that such a transformation changes the limited set of state values linked to this kind of events.

An event-switched process is an N-staged process in which instants of stages ends are moments of the advent of one or more events (for an arbitrary k-th stage the set of these events is $S(k) \subseteq \{1, \ldots, L\}$ where L denotes the number of events types). For an arbitrary k-th stage, i.e. for the flowing time interval $[T(k),\ T(k+1))$ vectors of qualitative state $d(k) \in A_D (A_D$ is a finite set) and control $u(k) \in R^m$ are constant and the relationship between the final ($x^1(k) \in R^n$) and initial ($x^0(k) \in R^n$) state vectors and the stage duration $t(k)$ has a form of difference equations

$$x^1(k) = Y(d(k),\ x^0(k),\ u(k),\ t(k)), \tag{10}$$

where $Y(d(k),\ x^0(k),\ u(k),\ t)$ denotes the solution of the Cauchy problem for the ODE system

$$dx(t,\ k)/dt = f(d(k), x(t,\ k), u(k)) \tag{11}$$

with the initial conditions t=0, $x(0,\ k) = x^0(k)$. For the s-th event type there are the sets of components of $I_{Xs},\ I_{Ds}$ of $x(t,\ k),\ d(k)$ (the latter forming vectors $x^{(s)}(t,\ k),\ d^{(s)}(k)$, respectively), so that $I_{Xs'} \cap I_{Xs} = I_{Ds'} \cap I_{Ds} = \emptyset$ for $s' \neq s$ and $i(s) \in I_{Xs}$ exists for which

$$f_{i(s)}(d(k), x(t,\ k), u(k)) \geq f_{min} > 0. \tag{12}$$

The conditions for the stage termination are

$$r^Y_{i(s)}(d^{(s)}(k), x^{1(s)}(k)) \equiv x^1_{i(s)}(k) - x_{s0}(d^{(s)}(k)) = 0, s \in S(k), \tag{13}$$

$$r^Y_{i(s)}(d^{(s)}(k), x^{1(s)}(k)) < 0, s \notin S(k), \tag{14}$$

resulting in no events within the stage. The values of some components of both state vectors change as a result of the above events, so that:

$$d_i(k+1) = D_{is}(d^{(s)}(k)), i \in I_{Ds}, s \in S(k),$$

$$d_i(k+1) = d_i(k), i \notin I_{Ds}, s \in S(k), \tag{15}$$

$$x_i^0(k+1) = X_{is}(d^{(s)}(k),\ x^{1\ (s)}(k)), i \in I_{Xs}, s \in S(k), \tag{16}$$

$$x_i^0(k+1) = x_i^1(k), i \notin I_{Xs}, s \in S(k). \tag{17}$$

Equations (15)–(17) may be denoted as

$$d(k+1) = D(S(k),\ d(k)), x^0(k+1) = X(S(k),\ d(k),\ x^1(k)).$$

The number of the process stages N is determined from the process termination condition

$$T(N+1) = T(0) + T_1. \tag{18}$$

Constraints on the process have two types: the constraints for any stage

$$r_j^U(d(k), u(k)) \le 0, j \in J_1(d(k)), \tag{19}$$

$$r_j^U(d(k), u(k)) = 0, j \in J_2(d(k)), \tag{20}$$

and the constraints for a definite event (including terminal constraints)

$$r_j^Y(d^{(s)}(k), x^{1\,(s)}(k)) \le 0, j \in K_1(s), s \in S(k), \tag{21}$$

$$r_j^Y(d^{(s)}(k), x^{1\,(s)}(k)) = 0, j \in K_2(s), s \in S(k). \tag{22}$$

It is supposed that for any $d(k) \in A_D$ the set $U_0(d(k))$ of $u(k)$ satisfying (19), (20) is non-empty and bounded.

The problem consists in the determination of the process scenario S=($S(1)$, ..., $S(N)$) and control (i.e., the succession v=($v(1)$,...,$v(N)$) of vectors $v(k) =$ $= (u(k),\ t(k))$ with trajectories in continuous- and discrete-valued state variables $d = (d(1),\dots,\ d(N))$, x=($x^0(1)$, $x^1(1)$,..., $x^0(N)$, $x^1(N)$) corresponding to S, v due to (10), (15)–(17) so that restrictions (13), (14), (18)–(22) are satisfied and the target functional

$$F_0(x^1(N)). \tag{23}$$

has the minimum value. We assume that for every $d' \in A_D$, $x' \in R^n$, $u' \in U^0_\Delta(d')$ (where the constant $\Delta > 0$) all the functions $f_i(d', x', u')$, $r^U_j(d', u')$, $r^Y_j(d'^{(s)}, x'^{(s)})$ are determined and continuously differentiated with respect to x', u' and for all their first partial derivatives the generalized Lipschitz condition $|g(y')\text{-}g(y)| \leq K||y'–y||^\beta$ is valid (here $y=(x', u')$ and the constants $K > 0$, $\beta \in (0, 1]$ do not depend on a function $g(y)$).

4. Finiteness of the number of switches

For the class of models introduced here boundedness of the set of trajectories and the question finiteness of the number of switches are closely interrelated. Even the functional space for trajectories of the system where finiteness of the number of switches is not guaranteed is a very unusual mathematical object. Fortunately, real production systems with their limited resources cannot display unlimited values of their state variables (if they are reasonably determined), as well as there are non-negative limits for least time intervals between changes of a definite control variable. In this section the conditions are established for the model that guarantee both properties.

First of all, the amount of any particular work has obviously lower non-negative limit. So we formulate

Condition 1. There are the constants K_{X1}, $K_{X2} > 0$ so that for an arbitrary s=1,. . . , L the inequalities are satisfied

$$K_{X2} \leq x_{s0}(D_{is}(d^{(s)}(k)) - X_{i(s)\ s}(d^{(s)}(k),\ x^{1\ (s)}(k)),$$
$$|X_{is}(d^{(s)}(k),\ x^{1\ (s)}(k))| \leq K_{X1}, i \in I_{Xs},$$

that expresses both the existence of the lower non-negative bound for the amount of arbitrary work and the fact that values of state variables after transformation are bounded. If we can state that values of state variables are bounded ever, then $f_{i(s)}(d(k),\ x(t,\ k),\ u(k))$ would be bounded with a certain K_{X3} as well, and the least time interval between switches of the s-th type would be not less than K_{X2}/K_{X3}. But for an arbitrary ODE system (13), even for linear one right side of the equations is not bounded despite the fact that $u(k)$ belongs to the bounded set $U_0(d(k))$.

The above assumptions for the properties of functions determining a model may guarantee only that the Cauchy problem has a bounded solution only on a small time interval initiating in $T(k)$. Due to (12) for the Chebyshev vector norm the equation takes place:

$$d\,\|x(t,k)\|_\infty /dt =$$
$$= \max\{f_i(d(k),x(t,k),u(k))\cdot \mathrm{sgn} x_i(t,k) |\; i\in Arg\max|x_i(t,k)|\} \equiv$$
$$\equiv f^\infty(d(k),x(t,k),u(k)).$$

Condition 2. There are the constant $T_{1max} > T_1$ and the function $f^\infty_{\max}(y) \geq$ ≥ 0, so that for an arbitrary $d' \in A_D$, $x' \in R^n$, $u' \in U_0(d')$ the inequality $f^\infty(d',x',u') \leq f^\infty_{\max}(\|x'\|_{\infty)}$ is valid and the solution of the Cauchy problem for the equation

$$\dot{y} = f^\infty_{\max}(y)$$

with the initial conditions t=0, $y(0)$=y_0 $\in$[0, ∞] exists and is unique on the time interval [0, T_{1max}].

One can note that an arbitrary linear ODE system of the form

$$dx(t,\ k)/dt = B(d(k))x_i(t,\ k) + C(d(k))u(k)$$

satisfies condition 2. In fact, the vector $C(d(k))u(k)$ is bounded for an arbitrary $d' \in A_D$ and $u' \in U^0(d')$ and so (taking into account finiteness of the set A_D) for all pairs $d(k)$, $u(k)$. We have

$$|dx_i(t,\ k)/dt| = |B_i(d(k))\ x(t,\ k) + C_i(d(k))\ u(k)| \leq$$
$$\leq ||B_i(d(k))||_\infty ||x(t,\ k)||_\infty + ||C_i(d(k))||_1 ||u(k)||_\infty \leq$$
$$\leq b_i||x(t,\ k)||_\infty + c_i,\ i=1,\ldots,n;$$

$$d||x(t,\ k)/dt||_\infty \leq ||b||_\infty ||x(t,\ k)||_\infty + ||c||_\infty = K_b||x(t,\ k)||_\infty + K_c,$$

and the scalar linear ODE $\dot{y} = K_b y + K_c$ satisfies the condition 2.

The combination of conditions 1 and 2 guarantee the desired finiteness of the number of switches.

5. Changes in a succession of events

The further study assumes some degree of the model regularity. Given a certain scenario of N-staged process we have dependencies between the vector v and the residuals in constraints (13), (14), (18)–(22). All these constraints may be represented as $F_j(v,\ S)$; to be more exact, all the residuals for constraints related to stages $1,\ldots,k$, depend on $v(1{:}k) \equiv (v(1),\ldots,v(k))$. Let $V(S)$ be the set of all controls v determining a feasible process with a scenario S. For any $v \in V(S)$ satisfaction of constraints (14) results from satisfaction of (13) and (12), so residuals of almost active constraints (14) as functions of v are not independent of residuals of other active and almost active constraints. So let $J_1(k,\ S)$ and $J_2(k,\ S)$ be the set of constraints (19), (21) and (13), (18), (20), (22), respectively and let $J_{1\varepsilon}(k,\ v,\ S)=\{j \in J_1(k,\ S) \mid F_j(v) \geq -\varepsilon\}$ for $\varepsilon > 0$,

$$J_\varepsilon(k,\ v,\ S) = J_{1\varepsilon}(k,\ v,\ S) \cup J_2(k,\ S),$$
$$J_{1\varepsilon}(1{:}k,\ v,\ S) = J_{1\varepsilon}(1,\ v,\ S)\cup\ldots\cup J_{1\varepsilon}(k,\ v,\ S),$$

$$J_2(1{:}k,\ S) = J_2(1,\ S)\cup\ldots\cup J_2(k,\ S),\ J_\varepsilon(1{:}k,\ v,\ S) = J_\varepsilon(1,\ v,\ S)\cup\ldots\cup J_\varepsilon(k,\ v,\ S).$$

The *regularity condition* is that 1) for any $v \in V(S)$ and $k{=}1,\ldots,\ N$ gradients of $F_j(v(1{:}k),\ S)$, $j \in J_\varepsilon(1{:}k,\ v,\ S)$, are linearly independent and 2) for $d' \in A_D$, $u' \in U^0(d')$ vectors $r^U_{ju}(d',u')$, $j \in J_{1\ 0}(d',u') \cup J_2(d')$, are linearly independent.

Let us slacken the condition (14) admitting controls, which satisfy

$$r^Y_{i(s)}(d^{(s)}(k), x^{1(s)}(k)) \leq 0, s \notin S(k),$$

and denote $V^*(S)$ the corresponding set of feasible controls. It is established that if the regularity condition is valid for $V(S)$ it is valid for $V^*(S)$. Moreover, one may assert that there exists the constant δ so that for all $v \in V^*(S)$ and $k{=}1,\ldots,\ N$ $\dim(J_\delta(1{:}k,\ v,\ S)) \leq \dim(v(1{:}k))$.

Two aims concern the question of changes in a succession of events: first, to make simultaneous two events separated with a short time interval, second, to separate two simultaneous events with a short time interval. For changing the order of two events separated with a short time interval it is sufficient to reach first aim, then the second one. Both aims may be represented with a set of equations (13), (18), (20), (22) and a subset of inequalities (19), (21) transformed into equations. Let a number of a stage separating two events is k_{INS}.

To reach the both aim we seek for a slightly altered control v_A resolving the proper set of equations with respect to $v_A(1)$,...,$v_A(k_{INS}$-1),$v_A(k_{INS}$+1),..., $v_A(N)$; we suppose for the first aim that $t_A(k_{INS})$=0 and for second aim that $t_A(k_{INS}) > 0$ and $u(k_{INS}) \in U^0(d(k_{INS}))$ are given. Under such a regularity condition both sets of equations have their only solutions if the value of $t_A(k_{INS})$ is sufficiently small. It is proved too that the constant K_v exists, so that $||v_A(k) - v(k)|| \leq K_v t_A(k_{INS})$.

6. Necessary optimality conditions and numerical solution of the optimization problem

The problem of optimization of $v \in V^*(S)$ with a given scenario is a particular form of the well-known optimization problem for discrete-time processes with mixed constraints and a fixed scenario. So the principal formulations of necessary optimality conditions of the first type may be found in [5, 6], or, more close to problem (10)–(23), in [2]. Specific form of necessary optimality conditions occurs when for a certain k $\dim(S(k)) > 1$ and so events from $S(k)$ may be separated with a short stage.

Given a certain subdivision $S(k)$ into two sets, $S_A(k)$ and $S_A(k$+1), one can found that the increment of the value of the target functional (12) may be assessed as

$$F_0(v_A) - F_0(v) =$$
$$= (q_{A,}\ f(d_A(k+1),\ x^0(k+1),\ u(k+1))t_A(k+1) + o(t_A(k+1)). \qquad (24)$$

¿From the formula (24) one can come to the conclusion that if the pair $(v,\ S)$ gives the optimum, then for all the adjacent scenarios for all the $u_{INS} \in U^0(d_A(k$+1)) the condition must be satisfied that

$$(q_{A,}\ f(d_A(k+1),\ x^0(k+1),\ u_{INS}) \geq 0. \qquad (25)$$

For numerical solution of optimization problems for event-switched processes the two-level method is constructed in which the choice of the optimum scenarios fulfilled by the branch-and-bound scheme [7], and the optimum within a fixed scenario is calculated with the decomposition method similar to the method proposed in [8]. In solving the auxiliary optimization problem to test satisfaction of conditions (25) a perspective adjacent scenario is being found. The decomposition technique enables the simple procedure of restrictions restoration when a new vertex is generated due to such a scenario that differs from the parent vertex scenario in the order of two subsequent adjacent events.

References

[1] Velichenko, V.V. Catastrophe Control Problem, Fuzzy Logic and Intelligent Technology in Nuclear Science, Proceedings 1st Internat. FLINS Workshop, Mol (Belgium), 14–16 September 1994, World Scientific, 1994, pp. 117–121.

[2] Velichenko, V.V. Variational analysis and control of catastrophic dynamical systems, Nonlinear analysis, Theory, Methods & Applications, Vol. 30, No 4, 1997, pp. 2065–2074.

[3] Valuev, A.M. On the substantiation of technological solutions for open pits via production planning simulation, Mine Planning and Equipment Selection: Proceedings 5th Internat. Symposium. Sao Paulo, 22-26 October 1996. Balkema, Rotterdam, pp. 91–95.

[4] Valuev, A.M. Concept Of Time-Event Controlled Processes — A Way To The Most General Formulations Of Production Planning And Regulation Problems, Proceedings of the International Conference "Mathematical Modelling Of Social And Economical Dynamics" (MMSED-2004), June 23–25, 2004, Moscow, Russia, pp. 373–376.

[5] Propoy, A.I. Elementy teorii optimal'nykh diskretnykh protsessov (Elements of the theory of optimum discrete-time processes), Nauka, Moscow, 1973. (In Russian).

[6] Boltyanski, V.G. Optimal'noye upravleniye diskretnymi sistemami (Optimum control of discrete-time systems), Nauka, Moscow, 1973. (In Russian).

[7] Valuev, A.M. and Velichenko, V.V. On the Problem of Planning a Civil Aircraft Flight along a Free Route, Journal of Computer and Systems Sciences International, Vol. 41, No. 6, 2002, pp. 979–987.

[8] Valuev, A.M. Numerical method for multistage optimization problems with a stage-wise computation of descent directions, USSR Comput. Mathematics, Vol. 27, No. 10, 1987.

Andrey M. Valuev
Department of Management in Mineral Industries
Moscow State Mining University
MSMU, Leninsky prospect 6, Moscow 119991 Russia
e-mail: *amvaluev@online.ru*