ACTA UNIVERSITATIS APULENSIS No 11/2006

Proceedings of the *International Conference on Theory and Application of Mathematics and Informatics ICTAMI 2005* - Alba Iulia, Romania

THE BOUNDARY ELEMENT METHOD WITH LINEAR BOUNDARY ELEMENTS FOR THE STOKES FLOW

Grecu Luminiţa

Abstract. The aim of the paper is to apply the boundary element method to solve the problem of the Stokes flow, in fact to solve the second step in applying this method, the system of singular boundary integral equations the problem is reduced at, using linear isoparametric boundary elements of Lagrangean type.

2000 Mathematics Subject Classification:74S15

1.Introduction

The boundary integral method (BEM) is a powerful numerical technique used to solve boundary value problems for systems of partial differential equations. It has the ability to reduce the problem dimension by one, and it has two major steps. The first step is to formulate the problem in terms of a boundary integral equation, or a system of boundary integral equations and the second step is to solve the boundary integrals obtained (see[1]).

We apply the boundary element method for solving the problem of the steady slow viscous flow of an incompressible fluid over an obstacle, a fluid that has the uniform subsonic velocity $\bar{u}_0$at great distances. We consider that the uniform steady motion is perturbed by the presence of a fixed body of a known boundary, noted Σ, assumed to be smooth and closed. We want to find out the perturbed motion, and the fluid action on the body.

A boundary integral representation for the velocity is obtained in [2] ,using an indirect technique, so the fundamental solution of the mathematical model of the Stokes flow, and an integral representation of the pressure is obtained in [3]. The problem is reduced to a system of singular boundary integral equations. Because the singular integrals that appear exist only if we assume that the unknown function satisfies a hőlder condition on the boundary, so it is at least continuous on it, the system obtained is solved in this paper using linear isoparametric boundary elements of Lagrangean type.

Both, the geometry and the unknown, have local a linear behavior, and the solution obtained is continuous on the boundary. After the discretization of the boundary, we choose a linear model for approximating the unknown on each of the elements involved. We obtain a linear algebraic system whose solutions are the unknowns on the nodes chosen for the discretization.After solving this system we can find the velocity in any point of the boundary and everywhere in the fluid domain. Using this technique it can be also obtained the pressure on the boundary and the fluid action over the obstacle.

2.The system of boundary integral equations

The mathematical model of the problem is described by the following equations for the perturbed motion (ν is the coefficient of the cinematic viscosity)(see [4]):

$$\nabla p - \nu \Delta \overline{u} = 0, \quad \nabla \cdot \overline{u} = 0 \tag{1}$$

and the boundary conditions: $u_1(\overline{x}) = \nu_0, \overline{x} \in \Sigma,\ u_2(\overline{x}) = u_3(\overline{x}) = 0, \overline{x} \in \Sigma, \lim_{\infty}(\overline{u}, p) = 0$

Assimilating the body with a continuous distribution of sources on the boundary, having an unknown intensity $\overline{f}$, and assuming that $\overline{f}$satisfies a holder condition in any of the arguments we obtain a boundary representation for $u_i\left(\overline{\xi}_0\right)$, $Q_0\left(\overline{\xi}_0\right)$ being an arbitrary point situated on Σ. Introducing the boundary conditions, the problem can be reduced to the following system of boundary integral equations (see [2]):

$$\frac{1}{8\pi\nu}{}' \iint_{\Sigma} \frac{f_i(\xi_1,\xi_2,\xi_3)}{r^0} d\sigma + \frac{1}{8\pi\nu}{}' \iint_{\Sigma} \frac{\left(\overline{\xi} - \overline{\xi}^0\right)_i \left(\overline{\xi} - \overline{\xi}^0\right) \cdot \overline{f}(\xi_1,\xi_2,\xi_3)}{(r^0)^3} d\sigma = -\nu_{i0}, \tag{2}$$

for $i = 1, 2, 3$,

where $\nu_{20} = \nu_{30} = 0$, $r^0 = \left[(\xi_1 - \xi_1^0)^2 + (\xi_2 - \xi_2^0)^2 + (\xi_3 - \xi_3^0)^2\right]^{\frac{1}{2}}$.

For the pressure on the boundary we obtain(see [3]):

$$p\left(\overline{\xi}_0\right) = \frac{1}{4\pi}{}' \iint_{\Sigma} \frac{\overline{f}\left(\overline{\xi}\right)\left(\overline{\xi}^0 - \overline{\xi}\right)}{(r^0)^3} d\sigma - \frac{1}{2}\overline{f}\left(\overline{\xi}_0\right) \cdot \overline{n}_0. \tag{3}$$

The sign "$'$" denotes the principal value in Cauchy sense of an integral.

3. The discrete form of the equations

A collocation method is used for example in [7] for solving the system of integral equations to determine the perturbation velocity and pressure, but the unknown is there constant on every boundary element, so it is not even continuous on the boundary, as the representation asked. In the boundary element approach used herein, for solving the system of integral equations (2), the body surface, Σ , is approximated by N triangles, noted T_j , $j = 1, N$, with straight lines; the extremes $\overline{x}^i$, $i = \overline{1, M}$ of these triangles being on Σ. The system of integral equations becomes:

$$\frac{1}{8\pi\nu}\sum_{j=1}^{N}\int\int_{T_j}\frac{f_i(\xi_1,\xi_2,\xi_3)}{r^0}d\sigma + \tag{4}$$

$$+\frac{1}{8\pi\nu}\sum_{j=1}^{N}\int\int_{T_j}\frac{\left(\overline{\xi}-\overline{\xi}^0\right)_i\left(\overline{\xi}-\overline{\xi}^0\right)\cdot\overline{f}(\xi_1,\xi_2,\xi_3)}{(r^0)^3}d\sigma = -v_{i0}, i = 1,2,3. \tag{5}$$

Considering $\overline{\xi}^0 = \overline{x}^k, k \in \{1, 2, ..., M\}$, we have:

$$\frac{1}{8\pi\nu}\sum_{j=1}^{N}\int\int_{T_j}\frac{f_i(\xi_1,\xi_2,\xi_3)}{r^k}d\sigma + \tag{6}$$

$$+\frac{1}{8\pi\nu}\sum_{j=1}^{N}\int\int_{T_j}\frac{\left(\overline{\xi}-\overline{x}^k\right)_i\left(\overline{\xi}-\overline{x}^k\right)\cdot\overline{f}(\xi_1,\xi_2,\xi_3)}{(r^k)^3}d\sigma = -v_{i0}, i = \overline{1,3} \tag{7}$$

We have to calculate two types of integrals on T_j, with, and without singularities. If $\overline{x}^k$is a corner point of the triangle T_j, the integral calculated on T_j has a singularity, otherwise it doesn't. We calculate the integrals using a local system of coordinates. We consider that the corner points (nodes) of an arbitrary triangle T_j are noted with j_1, j_2, j_3. Denoting by $\lambda_1, \lambda_2, \lambda_3$ the intrinsic triangular coordinates $\left(\lambda_i \in [0, 1]\ ,\ i = \overline{1, 3}\ , \lambda_1 + \lambda_2 + \lambda_3 = 1\right)$,we have for an interior point of the triangle the relation:

$\overline{\xi} = \overline{x}^{j_1} + \left(\overline{x}^{j_2} - \overline{x}^{j_1}\right)\lambda_2 + \left(\overline{x}^{j_3} - \overline{x}^{j_1}\right)\lambda_3.$

Using the parametric representation: $\lambda_2 = r\cos\theta,\ \lambda_3 = r\sin\theta,\ \theta \in \left[0, \frac{\pi}{2}\right],\ r \in [0,\rho], \rho(\cos\theta + \sin\theta) = 1$we get: $\overline{\xi} = \overline{x}^{j_1} + (\overline{x}^{j_2} - \overline{x}^{j_3})\, r\cos\theta + (\overline{x}^{j_3} - \overline{x}^{j_1})\, r\sin\theta$. We also have $da = 2Sd\lambda_1 d\lambda_2 = 2Srdrd\theta$, where S is the area of the triangle. For describing the behavior of the unknown $\overline{f}$, we use a linear model:

$$\overline{f} = \overline{f}^{j_1} + \left(\overline{f}^{j_2} - \overline{f}^{j_1}\right) r\cos\theta + \left(\overline{f}^{j_3} - \overline{f}^{j_1}\right) r\sin\theta \tag{8}$$

where $\overline{f}^{j_l}, l = \overline{1,3}$ is the value of the unknown $\overline{f}$ for the node j_l .
The system (5) becomes:

$$\sum_{j=1}^{N} A_{ki}^{j} f_i^{j_1} + \sum_{j=1}^{N} B_{ki}^{j}\left(f_i^{j_2} - f_i^{j_1}\right) + \sum_{j=1}^{N} C_{ki}^{j}\left(f_i^{j_3} - f_i^{j_1}\right) + \sum_{j=1}^{N} \overline{D}_k^{j} \cdot \overline{f}^{j_1} +$$
$$+\overline{E}_k^{j} \cdot \left(\overline{f}^{j_2} - \overline{f}^{j_1}\right) + \sum_{j=1}^{N} \overline{F}_k^{j} \cdot \left(\overline{f}^{j_3} - \overline{f}^{j_1}\right) = -\nu_{i0} \tag{9}$$

The coefficients have the following expressions:

$$A_{ki}^{j} = \frac{1}{8\pi\nu}\int\int_{T_j} \frac{1}{r^k} d\sigma, B_{ki}^{j} = \frac{1}{8\pi\nu}\int\int_{T_j} \frac{r\cos\theta}{r^k} d\sigma, \tag{10}$$

$$C_{ki}^{j} = \frac{1}{8\pi\nu}\int\int_{T_j} \frac{r\sin\theta}{r^k} d\sigma, \overline{D}_k^{j} = \frac{1}{8\pi\nu}\int\int_{T_j} \frac{\left(\xi_i - x_i^k\right)\left(\overline{\xi} - \overline{x}^k\right)}{\left(r^k\right)^3} d\sigma, \tag{11}$$

$$\overline{E}_k^{j} = \frac{1}{8\pi\nu}\int\int_{T_j} \frac{\left(\xi_i - x_i^k\right)\left(\overline{\xi} - \overline{x}^k\right) r\cos\theta}{\left(r^k\right)^3} d\sigma, \tag{12}$$

$$\overline{F}_k^{j} = \frac{1}{8\pi\nu}\int\int_{T_j} \frac{\left(\xi_i - x_i^k\right)\left(\overline{\xi} - \overline{x}^k\right) r\sin\theta}{\left(r^k\right)^3} d\sigma \tag{13}$$

For evaluating the system' s coefficients we consider the two types of integrals that appear: the nonsingular and the singular ones. First we evaluate the nonsingular integrals, so we consider the triangles that have no corner point in $\overline{x}^k$. Denoting by $\overline{e}_j[\theta] = (\overline{x}^{j_2} - \overline{x}^{j_1})\cos\theta + (\overline{x}^{j_3} - \overline{x}^{j_1})\sin\theta,\ \overline{e}_j[\theta] = \left(e_1^j, e_2^j, e_3^j\right)$,we get: $r^k = \left\|\hat{\xi} - \overline{x}^k\right\| = \left\|\overline{x}^{j_1} - \overline{x}^k + r\overline{e}_j(\theta)\right\| = \sqrt{ar^2 + 2br + c}$, where $a = \|\overline{e}_{jk}(\theta)\|^2, b = \left(\overline{x}^{j_1} - \overline{x}^k\right)\cdot \overline{e}_j(\theta), c = \left\|\overline{x}^{j_1} - \overline{x}^k\right\|^2$

We also have the relations: $da = 2S_j d\lambda_1 d\lambda_2 = 2S_j r dr d\theta$, where S_j is the area of $T_j, j = \overline{1, N}$. We obtain: $\mathrm{A}_{ki}^{j} = \frac{S_j}{4\pi\nu} \int\limits_0^{\frac{\pi}{2}} J_1(\theta)\, d\theta,\ B_{ki}^{j} = \frac{S_j}{4\pi\nu} \int\limits_0^{\frac{\pi}{2}} J_2(\theta) \cos\theta d\theta$ $\mathrm{C}_{ki}^{j} = \frac{S_j}{4\pi\nu} \int\limits_0^{\frac{\pi}{2}} J_2(\theta) \sin\theta d\theta$,

$$\overline{D}_k^j = \frac{S_j}{4\pi\nu}\left(x_i^{j_1} - x_i^{j_2}\right)\left(\overline{x}^{j_1} - \overline{x}^{k}\right)\int\limits_0^{\frac{\pi}{2}} I_1(\theta)\, d\theta +$$
$$+\frac{S_j}{4\pi\nu}\left[q_{ijk}^{21}\right]\int\limits_0^{\frac{\pi}{2}} \cos\theta I_2(\theta)\, d\theta + \frac{S_j}{4\pi\nu}\left[q_{ijk}^{31}\right]\int\limits_0^{\frac{\pi}{2}} \sin\theta I_2(\theta)\, d\theta +$$
$$+\frac{S_j}{4\pi\nu}\left[q_{ijk}^{23}\right]\int\limits_0^{\frac{\pi}{2}} \sin\theta\cos\theta I_3(\theta)\, d\theta +$$
$$+\frac{S_j}{4\pi\nu}\left[\left(x_i^{j_2} - x_i^{j_1}\right)\left(\overline{x}^{j_3} - \overline{x}^{j_1}\right)\right]\int\limits_0^{\frac{\pi}{2}} \cos^2\theta I_3(\theta)\, d\theta +$$
$$+\frac{S_j}{4\pi\nu}\left[\left(x_i^{j_3} - x_i^{j_1}\right)\left(\overline{x}^{j_3} - \overline{x}^{j_1}\right)\right]\int\limits_0^{\frac{\pi}{2}} \sin^2\theta I_3(\theta)\, d\theta$$

$$\overline{E}_k^j = \frac{S_j}{4\pi\nu}\left(x_i^{j_1} - x_i^{j_2}\right)\left(\overline{x}^{j_1} - \overline{x}^{k}\right)\int\limits_0^{\frac{\pi}{2}} \cos\theta I_2(\theta)\, d\theta +$$
$$+\frac{S_j}{4\pi\nu}\left[q_{ijk}^{21}\right]\int\limits_0^{\frac{\pi}{2}} \cos^2\theta I_3(\theta)\, d\theta + \frac{S_j}{4\pi\nu}\left[q_{ijk}^{31}\right]\int\limits_0^{\frac{\pi}{2}} \sin\theta\cos\theta I_3(\theta)\, d\theta +$$
$$+\frac{S_j}{4\pi\nu}\left[q_{ijk}^{23}\right]\int\limits_0^{\frac{\pi}{2}} \sin\theta\cos^2\theta I_4(\theta)\, d\theta +$$
$$+\frac{S_j}{4\pi\nu}\left[\left(x_i^{j_2} - x_i^{j_1}\right)\left(\overline{x}^{j_3} - \overline{x}^{j_1}\right)\right]\int\limits_0^{\frac{\pi}{2}} \cos^3\theta I_4(\theta)\, d\theta +$$
$$+\frac{S_j}{4\pi\nu}\left[\left(x_i^{j_3} - x_i^{j_1}\right)\left(\overline{x}^{j_3} - \overline{x}^{j_1}\right)\right]\int\limits_0^{\frac{\pi}{2}} \sin^2\theta\cos\theta I_4(\theta)\, d\theta$$

$$\overline{F}_k^j = \frac{S_j}{4\pi\nu}\left(x_i^{j_1} - x_i^{j_2}\right)\left(\overline{x}^{j_1} - \overline{x}^k\right)\int_0^{\frac{\pi}{2}} \sin\theta I_2(\theta)\,d\theta +$$

$$+\frac{S_j}{4\pi\nu}\left[q_{ijk}^{21}\right]\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta I_3(\theta)\,d\theta + \frac{S_j}{4\pi\nu}\left[q_{ijk}^{31}\right]\int_0^{\frac{\pi}{2}} \sin^2\theta I_3(\theta)\,d\theta +$$

$$+\frac{S_j}{4\pi\nu}\left[q_{ijk}^{23}\right]\int_0^{\frac{\pi}{2}} \sin^2\theta\cos\theta I_4(\theta)\,d\theta +$$

$$+\frac{S_j}{4\pi\nu}\left[\left(x_i^{j_2} - x_i^{j_1}\right)\left(\overline{x}^{j_3} - \overline{x}^{j_1}\right)\right]\int_0^{\frac{\pi}{2}} \cos^2\theta\sin\theta I_4(\theta)\,d\theta +$$

$$+\frac{S_j}{4\pi\nu}\left[\left(x_i^{j_3} - x_i^{j_1}\right)\left(\overline{x}^{j_3} - \overline{x}^{j_1}\right)\right]\int_0^{\frac{\pi}{2}} \sin^3\theta I_4(\theta)\,d\theta$$

$$q_{ijk}^{21} = \left(x_i^{j_1} - x_i^k\right)\left(\overline{x}^{j_2} - \overline{x}^{j_1}\right) + \left(x_i^{j_2} - x_i^{j_1}\right)\left(\overline{x}^{j_1} - \overline{x}^k\right),$$
$$q_{ijk}^{31} = \left(x_i^{j_1} - x_i^k\right)\left(\overline{x}^{j_3} - \overline{x}^{j_1}\right) + \left(x_i^{j_3} - x_i^{j_1}\right)\left(\overline{x}^{j_1} - \overline{x}^k\right),$$
$$q_{ijk}^{23} = \left(x_i^{j_2} - x_i^{j_1}\right)\left(\overline{x}^{j_3} - \overline{x}^{j_1}\right) + \left(x_i^{j_2} - x_i^{j_1}\right)\left(\overline{x}^{j_2} - \overline{x}^{j_1}\right)$$

The coefficients noted $J_1, J_2, J_3, I_1, I_2, I_3, I_4$can be computed analytically and they have the expressions:

$\mathrm{J}_1(\theta) = \frac{1}{a}\sqrt{a\rho^2 + 2b\rho + c} - \frac{\sqrt{c}}{a} - \frac{b}{a}J.$

$\mathrm{J}_2(\theta) = \frac{a\rho - 3b}{2a^2}\sqrt{a\rho^2 + 2b\rho + c} + \frac{3b}{2a^2}\sqrt{c} + \frac{3b^2 - ac}{2a^2}J.$

$$J_3(\theta) = \frac{2a^2\rho^2 - 5ab\rho + 15b^2 - 4ac}{6a^3}\sqrt{a\rho^2 + 2b\rho + c} - \frac{15b^2 - 4ac}{6a^3}\sqrt{c} - \frac{5b^3 - 3bc}{2a^3}J.$$

where

$$J = \frac{1}{\sqrt{a}}\arcsin\frac{a\rho + b}{\sqrt{ac - b^2}} - \frac{1}{\sqrt{a}}\arcsin\frac{b}{\sqrt{ac - b^2}}$$

$$I_1(\theta) = \frac{1}{a}\sqrt{a\rho^2+2b\rho+c} - \frac{\sqrt{c}}{a} - \frac{b}{a\sqrt{a}}\ln\frac{b+a\rho+\sqrt{a\rho^2+2b\rho+c}}{b+\sqrt{ac}}.$$

$$I_2(\theta) = \frac{\sqrt{c}}{\delta} - \frac{b\rho+c}{\sqrt{a\rho^2+2b\rho+c}}\frac{1}{\delta},\ I_3(\theta) = -\frac{\rho^2}{a\sqrt{a\rho^2+2b\rho+c}} + \frac{2}{a}I_2 - \frac{b}{a}I_4$$

$$I_4(\theta) = \frac{(b^2-\delta)\rho+bc}{a\delta\sqrt{a\rho^2+2b\rho+c}} - \frac{b\sqrt{c}}{a\delta} + \frac{1}{a\sqrt{a}}\ln\frac{\sqrt{a(a\rho^2+2b\rho+c)}+a\rho+b}{b+\sqrt{ac}} \qquad (14)$$

We evaluate the singular integrals. Considering now that the triangle, noted T_k , has a corner point in $\overline{x}^k$, which we take as node number one after a renotation, we calculate the singular integrals occurring in (10) using the following relations: $\overline{\xi} = \overline{x}^k + \left(\overline{x}^{j_1} - \overline{x}^k\right) r\cos\theta + \left(\overline{x}^{j_2} - \overline{x}^k\right) r\sin\theta$, $\overline{f} = \overline{f}^k + \left(\overline{f}^{j_2} - \overline{f}^k\right) r\cos\theta + \left(\overline{f}^{j_3} - \overline{f}^k\right) r\sin\theta$. Denoting by $\overline{e}_{jk}[\theta] = \left(\overline{x}^{j_2} - \overline{x}^k\right)\cos\theta + \left(\overline{x}^{j_3} - \overline{x}^k\right)\sin\theta$, we get $r^k = \left\|\xi - \overline{x}^k\right\| = \|r\overline{e}_{jk}(\theta)\| = r\|\overline{e}_{jk}(\theta)\|$, and the coefficients have in this case the following expressions:

$\mathrm{A}^j_{ki} = \frac{S_j}{4\pi\nu}\int\limits_0^{\frac{\pi}{2}} \frac{1}{\left\|\overline{e}_{jk}(\theta)\right\|} d\theta,$

$\mathrm{B}^j_{ki} = \frac{S_j}{4\pi\nu}\int\int\limits_{T_j} \frac{\cos\theta}{\left\|\overline{e}_{jk}(\theta)\right\|}\frac{\rho^2(\theta)}{2} d\theta,$

$\mathrm{C}^j_{ki} = \frac{S_j}{4\pi\nu}\int\int\limits_{T_j} \frac{\sin\theta}{\left\|\overline{e}_{jk}(\theta)\right\|}\frac{\rho^2(\theta)}{2} d\theta$

$\mathrm{q}_{ijk} = \left(x_i^{j_2} - x_i^k\right)\left(\overline{x}^{j_3} - \overline{x}^k\right) + \left(x_i^{j_2} - x_i^k\right)\left(\overline{x}^{j_2} - \overline{x}^k\right).$

$\overline{D}^j_k = \frac{S_j}{4\pi\nu}[q_{ijk}]\int\limits_0^{\frac{\pi}{2}} \frac{\sin\theta\cos\theta}{\left\|\overline{e}_{jk}(\theta)\right\|^3} d\theta + \frac{S_j}{4\pi\nu}\left[\left(x_i^{j_2} - x_i^k\right)\left(\overline{x}^{j_3} - \overline{x}^k\right)\right]\int\limits_0^{\frac{\pi}{2}} \frac{\cos^2\theta}{\left\|\overline{e}_{jk}(\theta)\right\|^3} d\theta +$

$+ \frac{S_j}{4\pi\nu}\left[\left(x_i^{j_3} - x_i^k\right)\left(\overline{x}^{j_3} - \overline{x}^k\right)\right]\int\limits_0^{\frac{\pi}{2}} \frac{\sin^2\theta}{\left\|\overline{e}_{jk}(\theta)\right\|^3} d\theta$

$\overline{E}^j_k = \frac{S_j}{4\pi\nu}[q_{ijk}]\int\limits_0^{\frac{\pi}{2}} \frac{\sin\theta\cos^2\theta}{\left\|\overline{e}_{jk}(\theta)\right\|^3}\frac{\rho^2(\theta)}{2} d\theta +$

$$+\frac{S_j}{4\pi\nu}\left[\left(x_i^{j_2}-x_i^k\right)\left(\overline{x}^{j_3}-\overline{x}^k\right)\right]\int_0^{\frac{\pi}{2}}\frac{\cos^3\theta}{\left\|\overline{e}_{jk}(\theta)\right\|^3}\frac{\rho^2(\theta)}{2}d\theta+$$

$$+\frac{S_j}{4\pi\nu}\left[\left(x_i^{j_3}-x_i^k\right)\left(\overline{x}^{j_3}-\overline{x}^k\right)\right]\int_0^{\frac{\pi}{2}}\frac{\sin^2\theta\cos\theta}{\left\|\overline{e}_{jk}(\theta)\right\|^3}\frac{\rho^2(\theta)}{2}d\theta$$

$$\overline{F}_k^j=\frac{S_j}{4\pi\nu}\left[q_{ijk}\right]\int_0^{\frac{\pi}{2}}\frac{\sin^2\theta\cos\theta}{\left\|\overline{e}_{jk}(\theta)\right\|^3}\frac{\rho^2(\theta)}{2}d\theta+$$

$$+\frac{S_j}{4\pi\nu}\left[\left(x_i^{j_2}-x_i^k\right)\left(\overline{x}^{j_3}-\overline{x}^k\right)\right]\int_0^{\frac{\pi}{2}}\frac{\cos^2\theta\sin\theta}{\left\|\overline{e}_{jk}(\theta)\right\|^3}\frac{\rho^2(\theta)}{2}d\theta+ \tag{15}$$

$$+\frac{S_j}{4\pi\nu}\left[\left(x_i^{j_3}-x_i^k\right)\left(\overline{x}^{j_3}-\overline{x}^k\right)\right]\int_0^{\frac{\pi}{2}}\frac{\sin^3\theta}{\left\|\overline{e}_{jk}(\theta)\right\|^3}\frac{\rho^2(\theta)}{2}d\theta \tag{16}$$

As shown in the previous paragraphs all the system' s coefficients depend only on the coordinates of the nodes chosen for the boundary discretization, and on some simple integrals that are one-dimensional and can be evaluated with a math soft or with the usual rules of the numerical integration. After some manipulation we obtain a linear system of $3M$ equations with 3 M unknowns, the scalar components of the M nodal values of the unknown function$\overline{f}$, $\overline{f}^j\left(f_1^j,f_2^j,f_3^j\right)$, $j=\overline{1,M}$, having the following form:

$$\sum_{j=1}^{M}A_i^jf_i^j+\sum_{j=1}^{M}\overline{B}_i^j\cdot\overline{f}^j=-\nu_{i0},i=\overline{1,3}, \tag{17}$$

where A_i^j is the coefficient of f_i^j , $i=\overline{1,3}$, $j=\overline{1,M}$ obtained as a sum of all the contributions that exist.After solving the system we may compute the velocity for the M nodes chosen for the discretization of the boundary, and everywhere on it, and also in the fluid region, and further the pressure on the boundary and in the fluid region.

References

[1] Brebbia, C.A., Telles, J.C.F., Wobel, L.C.: *Boundary Element Theory and Application in Engineering*, Springer-Verlag, Berlin,1984

[2] Grecu, L.: *A new boundary element approach for the Stokes flow*, Proceedings of "microCAD 2005" International Scientific Conference, Section G, 63-68

[3] Grecu, L.: *A boundary element approach for evaluating the fluid action over the obstacle for the Stokes flow*, Proceedings of "microCAD 2005" International Scientific Conference, Section G, 69-72

[4] Dragos, L.: *Principiile mecanicii mediilor continue*, Ed.Tehnica, Bucuresti,1983

[5] Dragos, L.: *Metode Matematice in Aerodinamica (Mathematical Methods in Aerodinamics)*, Editura Academiei Romane, Bucuresti, 2000

[6] Lifanov, I.K.: *Singular integral equations and discrete vortices*, VSP, Utrecht, TheNetherlands, 1996

[7] Petrila,T.,Kohr,M.: *Some boundary element techniques for Stokes flows (PartI)*, Rev. D'Anallyse Numerique et de Theorie de L'Approximation 21(1992), 167-182.

Grecu Luminiţa
Department of Applied Sciences
University of Craiova, Faculty of Engineering and Management of the Technological Systems, Drobeta Turnu Severin Address: Călugăreni no.1 Dr. Tr. Severin, cod 220037
email: luminita.grecu@imst.ro; lumigrecu@hotmail.com