Acta Universitatis Apulensis No. 33/2013
ISSN: 1582-5329 pp. 175-191

REPEATED LOW-DENSITY BURST ERROR LOCATING CODES

B.K. Dass and Surbhi Madan

Abstract. This paper deals with derivation of bounds for linear codes which are able to detect and locate 2-repeated low-density burst errors occurring within a sub-block. An example of such a code has also been provided.

2000 *Mathematics Subject Classification*: 94B20, 94B65, 94B25.

1. Introduction

In some communication systems errors occur predominantly in the form of bursts. However, in some cases transient disturbances, such as lightening, might induce bursts in which most of the characters are received correctly. This kind of error has been called by A. D. Wyner [13] as *low-density burst*, defined as follows:

Definition 1. *A low-density burst of length b with weight w is an n-tuple whose only non-zero components are confined to b consecutive positions, the first and the last of which is non-zero, with w $(w \leq b)$ non-zero components within such b consecutive digits.*

A study of low-density burst error detecting and correcting linear codes has been made in Sharma and Dass [11] and Dass [2].

The nature of burst errors varies from channel to channel depending upon the type of channels or the kind of errors occurring during the process of transmission. In very busy communication channels errors repeat themselves. Berardi, Dass and Verma [1] introduced repeated burst errors. For systems where certain disturbances cause occurrence of burst errors in such a way that over a given length some digits are received correctly while others are corrupted that is, not all digits inside a burst are in error, development of repeated low-density burst error detecting and correcting codes was desired. A study of such codes was initiated by Dass and Verma [6]. Am m-repeated low-density burst of length b with weight $w(w \leq b)$ is defined as follows:

Definition 2. *An m-repeated low-density burst of length b or less with weight w or less is a vector of length n which is sum of m disjoint low-density bursts of length at most b.*

For example, (010202001001200) is a 2-repeated low-density burst of length 5 with weight 3 over $GF(3)$.

The concept of error location, lying midway between error detection and error correction, was introduced by Wolf and Elspas [12]. In an error locating code, each block of received digits considered as subdivided into mutually exclusive sub-blocks. An EL code permits the detection of errors occurring within a single sub-block, the sub-block containing errors being identified. This error location technique is an attractive alternative to the conventional error detection in decision feedback communications. The main advantage is, clearly, that, rather than requiring retransmission of the whole word as in a traditional two way error detecting communication channel, it is possible to resend just the affected blocks where errors have been detected. This leads to a faster and more efficient data transmission, allowing the word length to be longer and the sub-block length to be as small as desired. Error location has also been studied by E. Fujiwara and M. Kitakami [8] for fault isolation and reconfiguration in dependable computer systems. In byte organized semiconductor memory cards, upon location of faulty position on the card, it is possible to switch that byte alone instead of replacing the memory card.

Wolf and Elspas [12] obtained results in the form of bounds over the number of parity-check digits required for binary codes capable of detecting and locating a single sub-block containing random errors. A study of codes locating burst errors and low-density burst errors has been made by Dass [3], [4]. In our earlier paper [5], we have obtained bounds for codes locating 2-repeated and m-repeated burst errors occurring within a *single* sub-block. This paper presents a study of codes dealing with the location of 2-repeated low-density burst errors occurring within a *single* sub-block.

The development of codes locating repeated low-density burst errors economizes in the number of parity-check digits in comparison to the usual low-density burst errors locating codes. In this paper, lower and upper bounds on the number of parity check digits required for the existence of such codes are obtained. An example of such a code is also included. Throughout the paper, we consider a block of n digits, consisting of r check digits, and $k = n - r$ information digits, subdivided into s mutually exclusive sub-blocks, each sub-block containing $t = n/s$ digits.

2. 2-Repeated Low-Density Burst Error Locating Codes

Under syndrome decoding, an (n, k) linear error locating code (EL code) over $GF(q)$ capable of detecting and locating a single sub-block containing 2-repeated low-density burst of length b or less with weight w or less must satisfy the following conditions:

(i) The syndrome resulting from the occurrence of an error which is a 2-repeated low-density burst of length b or less with weight w or less within any one sub-block must be different from the all zeros syndrome.
(ii) The syndrome resulting from the occurrence of any 2-repeated low-density burst error of length b or less with weight w or less within a single sub-block must be distinct from the syndrome resulting likewise from any other 2-repeated low-density burst error of length b or less with weight w or less within *any other* sub-block.

In the following text we obtain two results. The first result gives a lower bound on the number of parity check digits required for the existence of a linear code over $GF(q)$ capable of detecting and locating a single sub-block containing errors that are 2-repeated low-density bursts of length b or less with weight w or less. In the second result, we derive an upper bound on the number of check digits, which assures the existence of such a code.

Theorem 1. *The number of check digits r required for an (n, k) linear EL code over $GF(q)$ that detects and locates 2-repeated low-density bursts of length b or less with weight w or less within a single corrupted sub-block is bounded from below by*

$$r \geq \ log_q(1 + s(q^{2w} - 1)). \tag{1}$$

Proof. Let V be an (n, k) linear code over $GF(q)$ that locates 2-repeated low-density burst of length b or less with weight w or less within a single sub-block. The maximum number of distinct syndromes available using r check digits is q^r. The proof proceeds by first counting the number of syndromes that are required to be distinct by conditions (i) and (ii) and then imposing this number to be less than or equal to q^r.

Let X consist of all those vectors whose non zero components are confined within any two *fixed* distinct sets of b consecutive components of any one sub-block, say the i^{th}, such that from each set of b consecutive components the non-zero components are confined to some *fixed* $w (w \leq b)$ components.

We claim that the syndromes of all elements of X should be distinct; else any x_1, x_2 in X having the same syndrome would imply that the syndrome of $x_1 - x_2$ which is also an element of X and hence a 2-repeated low-density burst of length b or less with weight w or less within the same sub-block, becomes zero, in violation of condition (i).

Also, since the code locates a single sub-block containing 2-repeated low-density burst of length b or less with weight w or less, the syndromes produced by the similar vectors in different sub-blocks must be distinct by condition (ii).

Thus, the syndromes of vectors which are 2-repeated low-density bursts in fixed positions, whether in the same sub-block or in different sub-blocks, must be distinct.

(It may be noted that different fixed components may be chosen in different sub-blocks)
As there are $q^{2w} - 1$ distinct non-zero syndromes corresponding to vectors in any single sub-block and there are s sub-blocks in all, we must have at least

$$1 + s(q^{2w} - 1)$$

distinct syndromes, including the all zeros syndrome.
Therefore, we must have

$$q^r \geq \ 1 + s(q^{2w} - 1)$$

that is,

$$r \geq \ log_q(1 + s(q^{2w} - 1)).$$

□

Remark 1. *It is worth noticing that the result obtained in Theorem* 1 *is independent of* t*, the length of the sub-block. Thus the bound obtained in equation* (1) *remains valid for all* t*, so long as* $w \leq b \leq t$ *and* $n = st$*.*

Remark 2. *For* $w = b$*, the weight consideration over the burst becomes redundant and the bound obtained in Theorem* 1 *coincides with the lower bound on check digits for the location of* 2*-repeated bursts of length* b *or less occurring within a sub-block [5].*

An upper bound on the number r of check digits required for the existence of such a code is given in Theorem 2. The proof involves relative modifications of the procedure used to establish Varshamov-Gilbert-Sacks bound by constructing a parity check matrix for such a code (see Sacks [10], also Theorem 4.7, Peterson and Weldon [9]). This technique not only ensures the existence of such a code but also gives a method for its actual construction.

Theorem 2. *An (n, k) linear EL code over* $GF(q)$ *capable of detecting* 2*-repeated low-density burst of length* b *or less with weight* w *or less*($w \leq b$) *occurring within a single sub-block and of locating that sub-block can always be constructed using* r

check digits where r is the smallest integer satisfying the inequality

$$
\begin{aligned}
q^r >\Bigg(& [1+(q-1)]^{(b-1,w-1)}\Big\{q^{w-1}((q-1)(t-b-w+1)+1) \\
& + (q-1)^2 \sum_{i=w+1}^{b}(t-b-i+1)[1+(q-1)]^{(i-2,w-2)}\Big\} + \sum_{i=w}^{2w-1}\binom{b-1}{i}(q-1)^i \\
& +\sum_{k=1}^{b-1}\sum_{r_1,r_2,r_3}\binom{b-k-1}{r_1}\binom{k}{r_2}\binom{b-k-1}{r_3}(q-1)^{r_1+r_2+r_3+1}\Bigg) \\
& \times\Bigg(1+(s-1)\Bigg((q-1)[1+(q-1)]^{(b-1,w-1)}\Bigg(\binom{t-2b+2}{2} \\
& \quad \times (q-1)[1+(q-1)]^{(b-1,w-1)} + \binom{t-2b+1}{1}[1+(q-1)]^{(b-1,\min(w,b-1))} \\
& \quad + \sum_{i=t-2b+2}^{t-b-w+1}[1+(q-1)]^{(t-i-b+1,w)} + \sum_{i=t-b-w+2}^{t-b+1} q^{t-i-b+1}\Bigg) \\
& + \Bigg(\binom{t-2b+2}{1}\sum_{k_1=0}^{b-2}\sum_{r_4,r_5,r_6} + \sum_{i=t-2b+3}^{t-b}\sum_{k_1=0}^{t-i-b}\sum_{r_4,r_5,r_6}\Bigg) \\
& \qquad \times \binom{k_1}{r_4}\binom{b-k_1-1}{r_5}\binom{k_1}{r_6}(q-1)^{r_4+r_5+r_6+2} \\
& \qquad + \binom{t-b+1}{1}(q-1)[1+(q-1)]^{(b-1;w,\min(2w-1,b-1))} \\
& \qquad + [1+(q-1)]^{(b-1,\min(2w,b-1))} - 1\Bigg)\Bigg), \qquad (2)
\end{aligned}
$$

where $0 \le r_1 \le w-2$, $1 \le r_2 \le 2w-2$, $0 \le r_3 \le w-1$, $r_2+r_3 \ge w$, $r_1+r_2+r_3 \le 2w-2$;
$0 \le r_4 \le w-1$, $1 \le r_5 \le 2w-2$, $0 \le r_6 \le w-2$, $r_4+r_5 \ge w$, $r_4+r_5+r_6 \le 2w-2$;
$[1+(q-1)]^{(m,r)}$ *denotes the incomplete binomial expansion of* $(1+x)^m$ *upto the term* x^r *in ascending powers of* x *and* $[1+(q-1)]^{(m;r_1,r_2)}$ *denotes the incomplete binomial expansion of* $(1+x)^m$ *from the term* x^{r_1} *upto the term* x^{r_2} *in ascending powers of* x.

Proof. We shall prove the result by constructing an appropriate $(n-k)\times n$ parity check matrix H for the desired code. Suppose that the columns of the first $s-1$ sub-blocks of H and the first $j-1$ columns $h_1, h_2, \cdots, h_{j-1}$ of the s^{th} sub-block have been appropriately added. We now lay down conditions to add the j^{th} column h_j as follows:

For the detection of 2-repeated low-density bursts of length b or less with weight w or less in the s^{th} sub-block, h_j should not be a linear combination of any $w-1$ or less columns from amongst the immediately preceding $b-1$ columns together with any linear combination of w or less columns from any set of b consecutive columns from the earlier chosen $j-1$ columns of the s^{th} sub-block. In other words,

$$h_j \neq (\alpha_1 h_{i_1} + \cdots + \alpha_{w-1} h_{i_w-1}) + (\beta_1 h_{p_1} + \cdots + \beta_w h_{p_w}) \tag{3}$$

where $\alpha_i, \beta_i \in GF(q)$ and h_i are any $w-1$ columns amongst immediately preceding $b-1$ columns $h_{j-b+1}, h_{j-b+2}, \cdots, h_{j-1}$ and h_p are any w or less columns from a set of b consecutive columns among all the $j-1$ columns.

The number of ways in which the coefficients α_i and β_p can be chosen is (see [2], [6])

$$\Bigg([1+(q-1)]^{(b-1,w-1)} \Big\{ q^{w-1}((q-1)(j-b-w+1)+1) + (q-1)^2 \sum_{i=w+1}^{b} (j-b-i+1)[1+(q-1)]^{(i-2,w-2)} \Big\} + \sum_{i=w}^{2w-1} \binom{b-1}{i} (q-1)^i + \sum_{k=1}^{b-1} \sum_{r_1,r_2,r_3} \binom{b-k-1}{r_1} \binom{k}{r_2} \binom{b-k-1}{r_3} (q-1)^{r_1+r_2+r_3+1} \Bigg), \tag{4}$$

where $0 \leq r_1 \leq w-2$, $1 \leq r_2 \leq 2w-2$, $0 \leq r_3 \leq w-1$, $r_2 + r_3 \geq w$, $r_1 + r_2 + r_3 \leq 2w-2$.

Further, according to condition (ii), for location of 2-repeated low-density bursts of length b or less having weight w or less, h_j should not be a linear combination of any $w-1$ or less columns from amongst the immediately preceding $b-1$ columns together with linear combination of any w or less columns from any set of b consecutive columns from the earlier chosen $j-1$ columns of the s^{th} sub-block together with linear combination of two sets of w or less columns out of b or less consecutive columns each from *any other* sub-block that is,

$$\begin{aligned} h_j \neq\ & (\alpha_1 h_{i_1} + \alpha_2 h_{i_2} + \cdots + \alpha_{w-1} h_{i_w-1}) + (\beta_1 h_{l_1} + \beta_2 h_{l_2} + \cdots + \beta_w h_{l_w}) \\ & + (\gamma_1 h_{p_1} + \gamma_2 h_{p_2} + \cdots + \gamma_w h_{p_w}) + (\delta_1 h_{q_1} + \delta_2 h_{q_2} + \cdots + \delta_w h_{q_w}), \end{aligned} \tag{5}$$

where $\alpha_i, \beta_i, \gamma_i, \delta_i \in GF(q)$, not all γ_i, δ_i zero and h_i's are any $w-1$ columns amongst $h_{j-b+1}, h_{j-b+2}, \cdots, h_{j-1}$ and h_l's are any w columns from a set of b consecutive columns from the previously chosen $j-1$ columns of s^{th} sub-block and both h_p's and h_q's are sets of w columns from any b consecutive columns each from *any other* sub-block.

The number of ways in which the coefficients α_i and β_i can be chosen is the same as enumerated in (4). Also, the number of linear combinations corresponding to the last two terms on the R.H.S. of (5), is the same as the number of 2-repeated low-density bursts of length b or less with weight w or less within a sub-block of length t, excluding the vector of all zeros; this number in a sub-block of length t is given in [7] and amounts to

$$\begin{aligned}
&(q-1)[1+(q-1)]^{(b-1,w-1)}\left(\binom{t-2b+2}{2}(q-1)[1+(q-1)]^{(b-1,w-1)}\right.\\
&+\binom{t-2b+1}{1}[1+(q-1)]^{(b-1,\min(w,b-1))}+\sum_{i=t-2b+2}^{t-b-w+1}[1+(q-1)]^{(t-i-b+1,w)}\\
&\left.+\sum_{i=t-b-w+2}^{t-b+1}q^{t-i-b+1}\right)+\left(\binom{t-2b+2}{1}\sum_{k_1=0}^{b-2}\sum_{r_4,r_5,r_6}+\sum_{i=t-2b+3}^{t-b}\sum_{k_1=0}^{t-i-b}\sum_{r_4,r_5,r_6}\right)\\
&\quad\times\binom{k_1}{r_4}\binom{b-k_1-1}{r_5}\binom{k_1}{r_6}(q-1)^{r_4+r_5+r_6+2}\\
&+\binom{t-b+1}{1}(q-1)[1+(q-1)]^{(b-1;w,\min(2w-1,b-1))}\\
&+[1+(q-1)]^{(b-1,\min(2w,b-1))}-1,
\end{aligned}\tag{6}$$

where $0\le r_4\le w-1$, $1\le r_5\le 2w-2$, $0\le r_6\le w-2$, $r_4+r_5\ge w$, $r_4+r_5+r_6\le 2w-2$.

Since there are $s-1$ previously chosen sub-blocks, the number of such linear combinations becomes

$$(s-1)\cdot expr(6).\tag{7}$$

Thus, for location of 2-repeated low-density burst within a sub-block, the number of linear combinations to which h_j can not be equal to is the product computed in expr(4) and expr(7) that is,

$$expr(4)\cdot expr(7).\tag{8}$$

Hence, for detection and location of 2-repeated low-density burst within a sub-block, the total number of linear combinations that h_j can not be equal to is the sum of linear combinations in (4) and (8).
At worst, all these combinations might yield a distinct sum. Therefore, h_j can be

added to the s^{th} sub- block of H provided that

$$q^r > expr(4) + expr(8)$$

that is,

$$q^r > expr(4)\Big(1 + expr(7)\Big).$$

For completing the s^{th} sub-block of length t, replacing j by t gives the result as stated in (2). □

Remark 3. *For $w = b$, the weight consideration over the burst becomes redundant and the inequality in* (2) *reduces to*

$$q^r > q^{2(b-1)}[(t-2b+1)(q-1)+1]\Bigg\{1 + (s-1)\Bigg\{q^{2b-2}\Bigg\{q + (q-1) \times \Bigg[(q-1)\binom{t-2b+2}{2}) + \binom{t-2b+1}{1}\Bigg]\Bigg\} - 1\Bigg\}\Bigg\},$$

which coincides with the sufficient condition for location of 2*-repeated burst errors occurring within a sub-block [5].*

We conclude this section with an example.

Example 1 Consider a $(27, 14)$ binary code with the 13×27 parity-check matrix H given by

$$H = \begin{bmatrix}
1\,0\,0\,0\,0\,0\,0\,0\,0 & 0\,0\,0\,0\,1\,0\,0\,0\,0 & 0\,0\,0\,0\,0\,0\,0\,0\,0 \\
0\,1\,0\,0\,0\,0\,0\,0\,0 & 0\,0\,0\,0\,1\,0\,0\,0\,0 & 0\,0\,0\,0\,0\,0\,0\,0\,1 \\
0\,0\,1\,0\,0\,0\,0\,0\,0 & 0\,0\,0\,0\,1\,0\,0\,0\,0 & 0\,0\,0\,0\,0\,0\,0\,1\,1 \\
0\,0\,0\,1\,0\,0\,0\,0\,0 & 0\,0\,0\,0\,0\,0\,0\,0\,0 & 0\,0\,0\,0\,0\,1\,1\,0\,1 \\
0\,0\,0\,0\,1\,0\,0\,0\,0 & 0\,0\,0\,0\,0\,0\,0\,1\,1 & 0\,0\,0\,0\,0\,0\,0\,0\,0 \\
0\,0\,0\,0\,0\,1\,0\,0\,0 & 0\,0\,0\,0\,0\,0\,0\,1\,1 & 0\,0\,0\,1\,1\,0\,1\,0\,0 \\
0\,0\,0\,0\,0\,0\,1\,0\,0 & 0\,0\,0\,0\,1\,0\,0\,1\,1 & 0\,0\,1\,0\,1\,0\,0\,0\,0 \\
0\,0\,0\,0\,0\,0\,0\,1\,0 & 0\,0\,0\,0\,1\,0\,0\,1\,1 & 0\,1\,0\,0\,1\,0\,0\,0\,1 \\
0\,0\,0\,0\,0\,0\,0\,0\,1 & 0\,0\,0\,0\,0\,0\,0\,1\,1 & 1\,0\,0\,0\,1\,0\,1\,1\,1 \\
0\,0\,0\,0\,0\,0\,0\,0\,0 & 1\,0\,0\,0\,0\,1\,0\,0\,1 & 0\,0\,0\,1\,1\,0\,1\,1\,0 \\
0\,0\,0\,0\,0\,0\,0\,0\,0 & 0\,1\,0\,0\,0\,1\,1\,0\,0 & 0\,0\,1\,0\,1\,1\,1\,1\,0 \\
0\,0\,0\,0\,0\,0\,0\,0\,0 & 0\,0\,1\,0\,0\,1\,1\,0\,1 & 0\,1\,0\,0\,1\,1\,1\,0\,0 \\
0\,0\,0\,0\,0\,0\,0\,0\,0 & 0\,0\,0\,1\,0\,0\,1\,0\,1 & 1\,0\,0\,0\,1\,1\,0\,1\,0
\end{bmatrix}$$

This matrix has been constructed by the synthesis procedure outlined in the proof of Theorem 2 by taking $b = 3$, $w = 2$, $t = 9$ over $GF(2)$. It can be seen from

Table 1 that the syndromes of all distinct 2-repeated low-density bursts of length 3 or less with weight 2 or less in any sub-block are non-zero, showing thereby that the code *detects* all low-density bursts of length 3 or less with weight 2 or less occurring within a sub-block.
Further, it has been verified through MS Excel program that the syndromes resulting from the occurrence of 2-repeated low-density bursts of length 3 or less with weight 2 or less within a single sub-block are distinct from the syndromes resulting likewise from any such burst within *any other* sub-block, thereby ensuring that the code *locates* any 2-repeated low-density bursts of length 3 or less with weight 2 or less occurring within single sub-block.

Table 1. Error Patterns - Syndrome vectors

Sub-block 1

ERROR VECTORS	SYNDROMES	ERROR VECTORS	SYNDROMES
101101000 000000000 000000000	1011010000000	000101000 000000000 000000000	0001010000000
101010100 000000000 000000000	1010101000000	000010111 000000000 000000000	0000101110000
101001010 000000000 000000000	1010010100000	000010110 000000000 000000000	0000101100000
101000101 000000000 000000000	1010001010000	000010101 000000000 000000000	0000101010000
101110000 000000000 000000000	1011100000000	000010100 000000000 000000000	0000101000000
101011000 000000000 000000000	1010110000000	000001011 000000000 000000000	0000010110000
101001100 000000000 000000000	1010011000000	000001010 000000000 000000000	0000010100000
101000110 000000000 000000000	1010001100000	000000101 000000000 000000000	0000001010000
101000011 000000000 000000000	1010000110000	110101000 000000000 000000000	1101010000000
101100000 000000000 000000000	1011000000000	110010100 000000000 000000000	1100101000000
101010000 000000000 000000000	1010100000000	110001010 000000000 000000000	1100010100000
101001000 000000000 000000000	1010010000000	110000101 000000000 000000000	1100001010000
101000100 000000000 000000000	1010001000000	110110000 000000000 000000000	1101100000000
101000010 000000000 000000000	1010000100000	110011000 000000000 000000000	1100110000000
101000001 000000000 000000000	1010000010000	110001100 000000000 000000000	1100011000000
101000000 000000000 000000000	1010000000000	110000110 000000000 000000000	1100001100000
010110100 000000000 000000000	0101101000000	110000011 000000000 000000000	1100000110000
010101010 000000000 000000000	0101010100000	110100000 000000000 000000000	1101000000000
010100101 000000000 000000000	0101001010000	110010000 000000000 000000000	1100100000000
010111000 000000000 000000000	0101110000000	110001000 000000000 000000000	1100010000000
010101100 000000000 000000000	0101011000000	110000100 000000000 000000000	1100001000000
010100110 000000000 000000000	0101001100000	110000010 000000000 000000000	1100000100000
010100011 000000000 000000000	0101000110000	110000001 000000000 000000000	1100000010000
010110000 000000000 000000000	0101100000000	110000000 000000000 000000000	1100000000000
010101000 000000000 000000000	0101010000000	011010100 000000000 000000000	0110101000000
010100100 000000000 000000000	0101001000000	011001010 000000000 000000000	0110010100000
010100010 000000000 000000000	0101000100000	011000101 000000000 000000000	0110001010000
010100001 000000000 000000000	0101000010000	011011000 000000000 000000000	0110110000000
010100000 000000000 000000000	0101000000000	011001100 000000000 000000000	0110011000000
001011010 000000000 000000000	0010110100000	011000110 000000000 000000000	0110001100000
001010101 000000000 000000000	0010101010000	011000011 000000000 000000000	0110000110000
001011100 000000000 000000000	0010111000000	011010000 000000000 000000000	0110100000000
001010110 000000000 000000000	0010101100000	011001000 000000000 000000000	0110010000000
001010011 000000000 000000000	0010100110000	011000100 000000000 000000000	0110001000000
001011000 000000000 000000000	0010110000000	011000010 000000000 000000000	0110000100000
001010100 000000000 000000000	0010101000000	011000001 000000000 000000000	0110000010000
001010010 000000000 000000000	0010100100000	011000000 000000000 000000000	0110000000000
001010001 000000000 000000000	0010100010000	001101010 000000000 000000000	0011010100000
001010000 000000000 000000000	0010100000000	001100101 000000000 000000000	0011001010000
000101101 000000000 000000000	0001011010000	001101100 000000000 000000000	0011011000000
000101110 000000000 000000000	0001011100000	001100110 000000000 000000000	0011001100000
000101011 000000000 000000000	0001010110000	001100011 000000000 000000000	0011000110000
000101100 000000000 000000000	0001011000000	001101000 000000000 000000000	0011010000000
000101010 000000000 000000000	0001010100000	001100100 000000000 000000000	0011001000000
000101001 000000000 000000000	0001010010000	001100010 000000000 000000000	0011000100000

Sub-block 1

ERROR VECTORS	SYNDROMES	ERROR VECTORS	SYNDROMES
001100001 000000000 000000000	0011000010000	001001010 000000000 000000000	0010010100000
001100000 000000000 000000000	0011000000000	001000101 000000000 000000000	0010001010000
000110101 000000000 000000000	0001101010000	001001100 000000000 000000000	0010011000000
000110110 000000000 000000000	0001101100000	001000110 000000000 000000000	0010001100000
000110011 000000000 000000000	0001100110000	001000011 000000000 000000000	0010000110000
000110100 000000000 000000000	0001101000000	001001000 000000000 000000000	0010010000000
000110010 000000000 000000000	0001100100000	001000100 000000000 000000000	0010001000000
000110001 000000000 000000000	0001100010000	001000010 000000000 000000000	0010000100000
000110000 000000000 000000000	0001100000000	001000001 000000000 000000000	0010000010000
000011011 000000000 000000000	0000110110000	001000000 000000000 000000000	0010000000000
000011010 000000000 000000000	0000110100000	000100101 000000000 000000000	0001001010000
000011001 000000000 000000000	0000110010000	000100110 000000000 000000000	0001001100000
000011000 000000000 000000000	0000110000000	000100011 000000000 000000000	0001000110000
000001101 000000000 000000000	0000011010000	000100100 000000000 000000000	0001001000000
000001100 000000000 000000000	0000011000000	000100010 000000000 000000000	0001000100000
000000110 000000000 000000000	0000001100000	000100001 000000000 000000000	0001000010000
100101000 000000000 000000000	1001010000000	000100000 000000000 000000000	0001000000000
100010100 000000000 000000000	1000101000000	000010011 000000000 000000000	0000100110000
100001010 000000000 000000000	1000010100000	000010010 000000000 000000000	0000100100000
100000101 000000000 000000000	1000001010000	000010001 000000000 000000000	0000100010000
100110000 000000000 000000000	1001100000000	000010000 000000000 000000000	0000100000000
100011000 000000000 000000000	1000110000000	000001001 000000000 000000000	0000010010000
100001100 000000000 000000000	1000011000000	000001000 000000000 000000000	0000010000000
100000110 000000000 000000000	1000001100000	000000100 000000000 000000000	0000001000000
100000011 000000000 000000000	1000000110000	111010000 000000000 000000000	1110100000000
100100000 000000000 000000000	1001000000000	111100000 000000000 000000000	1111000000000
100010000 000000000 000000000	1000100000000	011101000 000000000 000000000	0111010000000
100001000 000000000 000000000	1000010000000	011110000 000000000 000000000	0111100000000
100000100 000000000 000000000	1000001000000	001110100 000000000 000000000	0011101000000
100000010 000000000 000000000	1000000100000	001111000 000000000 000000000	0011110000000
100000001 000000000 000000000	1000000010000	000111010 000000000 000000000	0001110100000
100000000 000000000 000000000	1000000000000	000111100 000000000 000000000	0001111000000
010010100 000000000 000000000	0100101000000	000011101 000000000 000000000	0000111010000
010001010 000000000 000000000	0100010100000	000011110 000000000 000000000	0000111100000
010000101 000000000 000000000	0100001010000	000001111 000000000 000000000	0000011110000
010011000 000000000 000000000	0100110000000	111000000 000000000 000000000	1110000000000
010001100 000000000 000000000	0100011000000	011100000 000000000 000000000	0111000000000
010000110 000000000 000000000	0100001100000	001110000 000000000 000000000	0011100000000
010000011 000000000 000000000	0100000110000	000111000 000000000 000000000	0001110000000
010010000 000000000 000000000	0100100000000	000011100 000000000 000000000	0000111000000
010001000 000000000 000000000	0100010000000	000001110 000000000 000000000	0000011100000
010000100 000000000 000000000	0100001000000	000000111 000000000 000000000	0000001110000
010000010 000000000 000000000	0100000100000	000000011 000000000 000000000	0000000110000
010000001 000000000 000000000	0100000010000	000000010 000000000 000000000	0000000100000
010000000 000000000 000000000	0100000000000	000000001 000000000 000000000	0000000010000

Sub-block 2

ERROR VECTORS	SYNDROMES	ERROR VECTORS	SYNDROMES
000000000 101101000 000000000	0000000000101	000000000 000101000 000000000	0000000001111
000000000 101010100 000000000	1110001101101	000000000 000010111 000000000	1110001101100
000000000 101001010 000000000	0000111110100	000000000 000010110 000000000	1110110010111
000000000 101000101 000000000	0000111110110	000000000 000010101 000000000	1110110011100
000000000 101110000 000000000	1110001101011	000000000 000010100 000000000	1110001100111
000000000 101011000 000000000	1110001100100	000000000 000001011 000000000	0000000000101
000000000 101001100 000000000	0000000000011	000000000 000001010 000000000	0000111111110
000000000 101000110 000000000	0000111111101	000000000 000000101 000000000	0000111111100
000000000 101000011 000000000	0000000000001	000000000 110101000 000000000	0000000000011
000000000 101100000 000000000	0000000001011	000000000 110010100 000000000	1110001101011
000000000 101010000 000000000	1110001101010	000000000 110001010 000000000	0000111110010
000000000 101001000 000000000	0000000000100	000000000 110000101 000000000	0000111110000
000000000 101000100 000000000	0000000001101	000000000 110110000 000000000	1110001101101
000000000 101000010 000000000	0000111111010	000000000 110011000 000000000	1110001100010
000000000 101000001 000000000	0000111110001	000000000 110001100 000000000	0000000000101
000000000 101000000 000000000	0000000001010	000000000 110000110 000000000	0000111111011
000000000 010110100 000000000	1110001100010	000000000 110000011 000000000	0000000000111
000000000 010101010 000000000	0000111111011	000000000 110100000 000000000	0000000001101
000000000 010100101 000000000	0000111111001	000000000 110010000 000000000	1110001101100
000000000 010111000 000000000	1110001101011	000000000 110001000 000000000	0000000000010
000000000 010101100 000000000	0000000001100	000000000 110000100 000000000	0000000001011
000000000 010100110 000000000	0000111110010	000000000 110000010 000000000	0000111111100
000000000 010100011 000000000	0000000001110	000000000 110000001 000000000	0000111110111
000000000 010110000 000000000	1110001100101	000000000 110000000 000000000	0000000001100
000000000 010101000 000000000	0000000001011	000000000 011010100 000000000	1110001100001
000000000 010100100 000000000	0000000000010	000000000 011001010 000000000	0000111111000
000000000 010100010 000000000	0000111110101	000000000 011000101 000000000	0000111111010
000000000 010100001 000000000	0000111111110	000000000 011011000 000000000	1110001101000
000000000 010100000 000000000	0000000000101	000000000 011001100 000000000	0000000001111
000000000 001011010 000000000	1110110011100	000000000 011000110 000000000	0000111110001
000000000 001010101 000000000	1110110011110	000000000 011000011 000000000	0000000001101
000000000 001011100 000000000	1110001101011	000000000 011010000 000000000	1110001100110
000000000 001010110 000000000	1110110010101	000000000 011001000 000000000	0000000001000
000000000 001010011 000000000	1110001101001	000000000 011000100 000000000	0000000000001
000000000 001011000 000000000	1110001101100	000000000 011000010 000000000	0000111110110
000000000 001010100 000000000	1110001100101	000000000 011000001 000000000	0000111111101
000000000 001010010 000000000	1110110010010	000000000 011000000 000000000	0000000000110
000000000 001010001 000000000	1110110011001	000000000 001101010 000000000	0000111111101
000000000 001010000 000000000	1110001100010	000000000 001100101 000000000	0000111111111
000000000 000101101 000000000	0000111110011	000000000 001101100 000000000	0000000001010
000000000 000101110 000000000	0000111111000	000000000 001100110 000000000	0000111110100
000000000 000101011 000000000	0000000000100	000000000 001100011 000000000	0000000001000
000000000 000101100 000000000	0000000001000	000000000 001101000 000000000	0000000001101
000000000 000101010 000000000	0000111111111	000000000 001100100 000000000	0000000000100
000000000 000101001 000000000	0000111110100	000000000 001100010 000000000	0000111110011

Sub-block 2

ERROR VECTORS	SYNDROMES	ERROR VECTORS	SYNDROMES
000000000 001100001 000000000	0000111111000	000000000 001001010 000000000	0000111111100
000000000 001100000 000000000	0000000000011	000000000 001000101 000000000	0000111111110
000000000 000110101 000000000	1110110011101	000000000 001001100 000000000	0000000001011
000000000 000110110 000000000	1110110010110	000000000 001000110 000000000	0000111110101
000000000 000110011 000000000	1110001101010	000000000 001000011 000000000	0000000001001
000000000 000110100 000000000	1110001100110	000000000 001001000 000000000	0000000001100
000000000 000110010 000000000	1110110010001	000000000 001000100 000000000	0000000000101
000000000 000110001 000000000	1110110011010	000000000 001000010 000000000	0000111110010
000000000 000110000 000000000	1110001100001	000000000 001000001 000000000	0000111111001
000000000 000011011 000000000	1110001100101	000000000 001000000 000000000	0000000000010
000000000 000011010 000000000	1110110011110	000000000 000100101 000000000	0000111111101
000000000 000011001 000000000	1110110010101	000000000 000100110 000000000	0000111110110
000000000 000011000 000000000	1110001101110	000000000 000100011 000000000	0000000001010
000000000 000001101 000000000	0000111110010	000000000 000100100 000000000	0000000000110
000000000 000001100 000000000	0000000001001	000000000 000100010 000000000	0000111110001
000000000 000000110 000000000	0000111110111	000000000 000100001 000000000	0000111111010
000000000 100101000 000000000	0000000000111	000000000 000100000 000000000	0000000000001
000000000 100010100 000000000	1110001101111	000000000 000010011 000000000	1110001101011
000000000 100001010 000000000	0000111110110	000000000 000010010 000000000	1110110010000
000000000 100000101 000000000	0000111110100	000000000 000010001 000000000	1110110011011
000000000 100110000 000000000	1110001101001	000000000 000010000 000000000	1110001100000
000000000 100011000 000000000	1110001100110	000000000 000001001 000000000	0000111110101
000000000 100001100 000000000	0000000000001	000000000 000001000 000000000	0000000001110
000000000 100000110 000000000	0000111111111	000000000 000000100 000000000	0000000000111
000000000 100000011 000000000	0000000000011	000000000 111010000 000000000	1110001101110
000000000 100100000 000000000	0000000001001	000000000 111100000 000000000	0000000001111
000000000 100010000 000000000	1110001101000	000000000 011101000 000000000	0000000001001
000000000 100001000 000000000	0000000000110	000000000 011110000 000000000	1110001100111
000000000 100000100 000000000	0000000001111	000000000 001110100 000000000	1110001100100
000000000 100000010 000000000	0000111111000	000000000 001111000 000000000	1110001101101
000000000 100000001 000000000	0000111110011	000000000 000111010 000000000	1110110011111
000000000 100000000 000000000	0000000001000	000000000 000111100 000000000	1110001101000
000000000 010010100 000000000	1110001100011	000000000 000011101 000000000	1110110010010
000000000 010001010 000000000	0000111111010	000000000 000011110 000000000	1110110011001
000000000 010000101 000000000	0000111111000	000000000 000001111 000000000	0000000000010
000000000 010011000 000000000	1110001101010	000000000 111000000 000000000	0000000001110
000000000 010001100 000000000	0000000001101	000000000 011100000 000000000	0000000000111
000000000 010000110 000000000	0000111110011	000000000 001110000 000000000	1110001100011
000000000 010000011 000000000	0000000001111	000000000 000111000 000000000	1110001101111
000000000 010010000 000000000	1110001100100	000000000 000011100 000000000	1110001101001
000000000 010001000 000000000	0000000001010	000000000 000001110 000000000	0000111111001
000000000 010000100 000000000	0000000000011	000000000 000000111 000000000	0000000001100
000000000 010000010 000000000	0000111110100	000000000 000000011 000000000	0000000001011
000000000 010000001 000000000	0000111111111	000000000 000000010 000000000	0000111110000
000000000 010000000 000000000	0000000000100	000000000 000000001 000000000	0000111111011

Sub-block 3

ERROR VECTORS	SYNDROMES	ERROR VECTORS	SYNDROMES
000000000 000000000 101101000	0001011011010	000000000 000000000 000101000	0001010001111
000000000 000000000 101010100	0001000110100	000000000 000000000 000010111	0100001001100
000000000 000000000 101001010	0011001001111	000000000 000000000 000010110	0011001111100
000000000 000000000 101000101	0110011111011	000000000 000000000 000010101	0110001010001
000000000 000000000 101110000	0000000100010	000000000 000000000 000010100	0001001100001
000000000 000000000 101011000	0001010101101	000000000 000000000 000001011	0100000101010
000000000 000000000 101001100	0000011001100	000000000 000000000 000001010	0011000011010
000000000 000000000 101000110	0011011010110	000000000 000000000 000000101	0110010101110
000000000 000000000 101000011	0101001111000	000000000 000000000 110101000	0001010111100
000000000 000000000 101100000	0000011011101	000000000 000000000 110010100	0001001010010
000000000 000000000 101010000	0000010101010	000000000 000000000 110001010	0011000101001
000000000 000000000 101001000	0001001010010	000000000 000000000 110000101	0110010011101
000000000 000000000 101000100	0001011001011	000000000 000000000 110110000	0000001000100
000000000 000000000 101000010	0010001001000	000000000 000000000 110011000	0001011001011
000000000 000000000 101000001	0111001100101	000000000 000000000 110001100	0000010101010
000000000 000000000 101000000	0000001010101	000000000 000000000 110000110	0011010110000
000000000 000000000 010110100	0001011001011	000000000 000000000 110000011	0101000011110
000000000 000000000 010101010	0011010110000	000000000 000000000 110100000	0000010111011
000000000 000000000 010100101	0110000000100	000000000 000000000 110010000	0000011001100
000000000 000000000 010111000	0001001010010	000000000 000000000 110001000	0001000110100
000000000 000000000 010101100	0000000110011	000000000 000000000 110000100	0001010101101
000000000 000000000 010100110	0011000101001	000000000 000000000 110000010	0010000101110
000000000 000000000 010100011	0101010000111	000000000 000000000 110000001	0111000000011
000000000 000000000 010110000	0000001010101	000000000 000000000 110000000	0000000110011
000000000 000000000 010101000	0001010101101	000000000 000000000 011010100	0001000000111
000000000 000000000 010100100	0001000110100	000000000 000000000 011001010	0011001111100
000000000 000000000 010100010	0010010110111	000000000 000000000 011000101	0110011001000
000000000 000000000 010100001	0111010011010	000000000 000000000 011011000	0001010011110
000000000 000000000 010100000	0000010101010	000000000 000000000 011001100	0000011111111
000000000 000000000 001011010	0011010100001	000000000 000000000 011000110	0011011100101
000000000 000000000 001010101	0110000010101	000000000 000000000 011000011	0101001001011
000000000 000000000 001011100	0000000100010	000000000 000000000 011010000	0000010011001
000000000 000000000 001010110	0011000111000	000000000 000000000 011001000	0001001100001
000000000 000000000 001010011	0101010010110	000000000 000000000 011000100	0001011111000
000000000 000000000 001011000	0001010111100	000000000 000000000 011000010	0010001111011
000000000 000000000 001010100	0001000100101	000000000 000000000 011000001	0111001010110
000000000 000000000 001010010	0010010100110	000000000 000000000 011000000	0000001100110
000000000 000000000 001010001	0111010001011	000000000 000000000 001101010	0011011010110
000000000 000000000 001010000	0000010111011	000000000 000000000 001100101	0110001100010
000000000 000000000 000101101	0111000100001	000000000 000000000 001101100	0000001010101
000000000 000000000 000101110	0010000001100	000000000 000000000 001100110	0011001001111
000000000 000000000 000101011	0100010100010	000000000 000000000 001100011	0101011100001
000000000 000000000 000101100	0000000010001	000000000 000000000 001101000	0001011001011
000000000 000000000 000101010	0011010010010	000000000 000000000 001100100	0001001010010
000000000 000000000 000101001	0110010111111	000000000 000000000 001100010	0010011010001

Sub-block 3

ERROR VECTORS	SYNDROMES	ERROR VECTORS	SYNDROMES
000000000 000000000 001100001	0111011111100	000000000 000000000 001001010	0011001011110
000000000 000000000 001100000	0000011001100	000000000 000000000 001000101	0110011101010
000000000 000000000 000110101	0110011011001	000000000 000000000 001001100	0000011011101
000000000 000000000 000110110	0011011110100	000000000 000000000 001000110	0011011000111
000000000 000000000 000110011	0101001011010	000000000 000000000 001000011	0101001101001
000000000 000000000 000110100	0001011101001	000000000 000000000 001001000	0001001000011
000000000 000000000 000110010	0010001101010	000000000 000000000 001000100	0001011011010
000000000 000000000 000110001	0111001000111	000000000 000000000 001000010	0010001011001
000000000 000000000 000110000	0000001110111	000000000 000000000 001000001	0111001110100
000000000 000000000 000011011	0100011010101	000000000 000000000 001000000	0000001000100
000000000 000000000 000011010	0011011100101	000000000 000000000 000100101	0110000100110
000000000 000000000 000011001	0110011001000	000000000 000000000 000100110	0011000001011
000000000 000000000 000011000	0001011111000	000000000 000000000 000100011	0101010100101
000000000 000000000 000001101	0111010101001	000000000 000000000 000100100	0001000010110
000000000 000000000 000001100	0000010011001	000000000 000000000 000100010	0010010010101
000000000 000000000 000000110	0011010000011	000000000 000000000 000100001	0111010111000
000000000 000000000 100101000	0001010011110	000000000 000000000 000100000	0000010001000
000000000 000000000 100010100	0001001110000	000000000 000000000 000010011	0101011010010
000000000 000000000 100001010	0011000001011	000000000 000000000 000010010	0010011100010
000000000 000000000 100000101	0110010111111	000000000 000000000 000010001	0111011001111
000000000 000000000 100110000	0000001100110	000000000 000000000 000010000	0000011111111
000000000 000000000 100011000	0001011101001	000000000 000000000 000001001	0110000110111
000000000 000000000 100001100	0000010001000	000000000 000000000 000001000	0001000000111
000000000 000000000 100000110	0011010010010	000000000 000000000 000000100	0001010011110
000000000 000000000 100000011	0101000111100	000000000 000000000 111010000	0000010001000
000000000 000000000 100100000	0000010011001	000000000 000000000 111100000	0000011111111
000000000 000000000 100010000	0000011101110	000000000 000000000 011101000	0001011101001
000000000 000000000 100001000	0001000010110	000000000 000000000 011110000	0000000010001
000000000 000000000 100000100	0001010001111	000000000 000000000 001110100	0001010101101
000000000 000000000 100000010	0010000001100	000000000 000000000 001111000	0001000110100
000000000 000000000 100000001	0111000100001	000000000 000000000 000111010	0011001101101
000000000 000000000 100000000	0000000010001	000000000 000000000 000111100	0000011101110
000000000 000000000 010010100	0001001000011	000000000 000000000 000011101	0111001010110
000000000 000000000 010001010	0011000111000	000000000 000000000 000011110	0010001111011
000000000 000000000 010000101	0110010001100	000000000 000000000 000001111	0101010110100
000000000 000000000 010011000	0001011011010	000000000 000000000 111000000	0000001110111
000000000 000000000 010001100	0000010111011	000000000 000000000 011100000	0000011101110
000000000 000000000 010000110	0011010100001	000000000 000000000 001110000	0000000110011
000000000 000000000 010000011	0101000001111	000000000 000000000 000111000	0001001110000
000000000 000000000 010010000	0000011011101	000000000 000000000 000011100	0000001100110
000000000 000000000 010001000	0001000100101	000000000 000000000 000001110	0010010000100
000000000 000000000 010000100	0001010111100	000000000 000000000 000000111	0100010110011
000000000 000000000 010000010	0010000111111	000000000 000000000 000000011	0101000101101
000000000 000000000 010000001	0111000010010	000000000 000000000 000000010	0010000011101
000000000 000000000 010000000	0000000100010	000000000 000000000 000000001	0111000110000

Remark 4. *Since it is not desired to distinguish between (detectable) error combinations occurring within the same sub-block, it is not necessary that their corresponding syndromes be distinct. In fact, for better coding efficiency such syndromes should be identical whenever possible.*
(We may observe that the error patterns (000000000 000000000 000011100), (000000000 000000000 100110000) and (000000000 000000000 011000000) all have syndrome (0000001100110); the errors (000000000 000000000 000111000) and (000000000 000000000 100010100) both have syndrome (0001001110000).)

References

[1] L. Berardi, B.K. Dass and Rashmi Verma, *On 2-repeated burst error detecting codes*, Journal of Statistical Theory and Practice, 3(2), (2009), 381–391.
[2] B.K. Dass, *A sufficient bound for codes correcting bursts with weight constraints*, Journal of the Association for Computing Machinery, 22(4), (1975), 501–503.
[3] B.K. Dass, *Burst error locating codes*, J. Inf. and Optimization Sciences, 3(1), (1982), 77–80.
[4] B.K. Dass, *Low-Density burst error locating linear codes*, IEE Proc., 129(E)(4), (1984), 145–146.
[5] B.K. Dass and Surbhi Madan, *Repeated burst error locating linear codes*, Discrete Mathematics, Algorithms and Applications, 2 (2010), 181–188.
[6] B.K. Dass and Rashmi Verma, *Repeated low-density burst error detecting codes*, Journal of the Korean Mathematical Society, 48(3), (2011), 475–486.
[7] B.K. Dass and Rashmi Verma, *Bounds for 2-repeated low-density burst error correcting linear codes*, Learning Manual of Workshop on Optimization and Information Theory with their Applications, JUET, Guna, 24-27 March 2011, 89-123.
[8] E. Fujiwara and M. Kitakami, *A class of Error-locating codes for byte organized memory systems*, IEEE Trans. Inform. Theory, 40(6), (1994), 1858–1865.
[9] W.W. Peterson and E.J. Weldon Jr., *Error-correcting codes*, Second Edition, The MIT Press, Mass, 1972.
[10] G.E. Sacks, *Multiple error correction by means of parity-checks*, IRE Trans. Inform. Theory IT, 4, (1958), 145–147.
[11] B.D. Sharma and B.K. Dass, *Extended Varsharmov-Gilbert and sphere-packing bounds for burst correcting codes*, IEEE Trans. Inform. Theory, **IT 20**, (1974), 291–292.
[12] J. Wolf and B. Elspas, *Error-locating codes-A New concept in error control*, IEEE Transactions on Information Theory, 9(2), (1963), 113–117.
[13] A.D. Wyner, *Low-density-burst-correcting codes*, IEEE Trans. Information Theory, **IT-9** (1963), 124.

Prof. (Dr.) B.K. Dass
Department of Mathematics
University of Delhi
Delhi - 110 007, India
email:*dassbk@rediffmail.com*
Dr. Surbhi Madan
Department of Mathematics
Shivaji College (University of Delhi)
New Delhi - 110 027, India
email:*surbhimadan@gmail.com*