Acta Universitatis Apulensis No. 32/2012
ISSN: 1582-5329 pp. 243-250

ON A NEW STRONG DIFFERENTIAL SUBORDINATION

Georgia Irina Oros

Abstract. In this paper we define some new classes of analytic functions on $U \times \overline{U}$, which have as coefficients holomorphic functions in $\overline{U}$. Using those new classes, we give a new approach to the notion of strong subordination and we study certain strong differential subordinations.

2000 *Mathematics Subject Classification*: 30C80, 30C45, 30A20.
Keywords: analytic function, differential subordination, strong differential subordination, univalent function, dominant, best dominant.

Introductionand preliminaries

The concept of differential subordination was introduced in [2], [3] and developed in [4], by S.S. Miller and P.T. Mocanu. The concept of differential superordination was introduced in [5] as a dual problem of the differential subordination, by S.S. Miller and P.T. Mocanu. The concept of strong differential subordination was introduced in [1] by J.A. Antonino and S. Romaguera, and developed in [6], [7].

Denote by $\mathcal{H}(U \times \overline{U})$ the class of analytic functions in $U \times \overline{U}$,

$$U = \{z \in \mathbb{C} : \ |z| < 1\}, \quad \overline{U} = \{z \in \mathbb{C} : \ |z| \leq 1\}, \quad \partial U = \{z \in \mathbb{C} : \ |z| = 1\}.$$

For $a \in \mathbb{C}$ and n a positive integer, we denote by

$$\mathcal{H}\xi[a, n] = \{f(z, \xi) \in (U \times \overline{U}) : f(z, \xi) = a + a_n(\xi)z^n + a_{n+1}(\xi)z^{n+1} + \ldots\},$$

with $z \in U$, $\xi \in \overline{U}$, $a_k(\xi)$ holomorphic functions in $\overline{U}$, $k \geq n$.

Let

$$A\xi_n = \{f(z, \xi) \in \mathcal{H}(U \times \overline{U}) : f(z, \xi) = z + a_{n+1}(\xi)z^{n+1} + \ldots\},$$

with $z \in U$, $\xi \in \overline{U}$, $a_k(\xi)$ holomorphic functions in $\overline{U}$, $k \geq n+1$, and $A\xi_1 = A\xi$,

$$\mathcal{H}\xi_u(U) = \{f(z, \xi) \in \mathcal{H}\xi[a, n] : \ f(z, \xi) \text{ is univalent in } U \text{ for all } \xi \in \overline{U}\},$$

$$S\xi = \{f(z, \xi) \in A\xi_n : \ f(z, \xi) \text{ univalent in } U \text{ for all } \xi \in \overline{U}\}$$

denote the class of univalent functions in $\mathcal{H}(U \times \overline{U})$,

$$S^*\xi = \{f(z,\xi) \in A\xi : \ \mathrm{Re}\ \frac{z\frac{\partial f}{\partial z}(z,\xi)}{f(z,\xi)} > 0, \quad z \in U \text{ for all } \xi \in \overline{U}\}$$

denote the class of normalized starlike functions in $\mathcal{H}(U \times \overline{U})$,

$$K\xi = \{f(z,\xi) \in A\xi : \ \mathrm{Re}\ \left(\frac{z\frac{\partial^2 f}{\partial z^2}(z,\xi)}{\frac{\partial f}{\partial z}(z,\xi)} + 1 \right) \geq 0, \quad z \in U \text{ for all } \xi \in \overline{U}\}$$

denote the class of normalized convex functions in $\mathcal{H}(U \times \overline{U})$.
Let $A(p)\xi$ denote the subclass of the functions $f(z,\xi) \in \mathcal{H}(U \times \overline{U})$ of the form

$$f(z,\xi) = z^p + \sum_{k=p+1}^{\infty} a_k(\xi) z^k, \quad p \in N, \quad z \in U \text{ for all } \xi \in \overline{U}$$

and set $A(1)\xi = A\xi$.

In order to prove our main results we use the following new definitions, according to [1] and lemma according to [4].

Definition No. 1. Let $H(z,\xi)$, $f(z,\xi)$ be analytic in $U \times \overline{U}$. The function $f(z,\xi)$ is said to be strongly subordinate to $H(z,\xi)$, or $H(z,\xi)$ is said to be strongly superordinate to $f(z,\xi)$, if there exists a function w analytic in U, with $w(0) = 0$ and $|w(z)| < 1$ such that $f(z,\xi) = H(w(z),\xi)$, for all $\xi \in \overline{U}$. In such a case we write $f(z,\xi) \prec\prec H(z,\xi)$, $z \in U$, $\xi \in \overline{U}$.

Remark No. 1. (i) If $H(z,\xi)$ is analytic in $U \times \overline{U}$ and univalent in U, for all $\xi \in \overline{U}$, Definition 1 is equivalent to

$$H(0,\xi) = f(0,\xi) \text{ for all } \xi \in \overline{U} \text{ and } f(U \times \overline{U}) \subset \mathcal{H}(U \times \overline{U}).$$

(ii) If $H(z,\xi) \equiv H(z)$ and $f(z,\xi) \equiv f(z)$ then the strong subordination becomes the usual notion of subordination.

Definition No. 2. Let $\Psi : \mathbb{C}^3 \times U \times \overline{U} \to \mathbb{C}$ and let $h(z,\xi)$ be univalent in U for all $\xi \in \overline{U}$. If $p(z,\xi)$ is analytic in $U \times \overline{U}$ and satisfies the (second-order) strong differential subordination

$$\Psi\left(p(z,\xi), z\frac{\partial p(z,\xi)}{\partial z}, z^2\frac{\partial^2 p(z,\xi)}{\partial z^2}; z, \xi\right) \prec\prec h(z,\xi), \quad z \in U, \quad \xi \in \overline{U}, \qquad (1)$$

then $p(z,\xi)$ is called a solution of the strong differential subordination. The univalent function $q(z,\xi)$ is called a dominant of the solutions of the strong differential subordination, or simply a dominant, if $p(z,\xi) \prec\prec q(z,\xi)$ for all $p(z,\xi)$ satisfying

(1). A dominant $\widetilde{q}(z,\xi)$ that satisfies $\widetilde{q}(z,\xi) \prec\prec q(z,\xi)$, for all dominants $q(z,\xi)$ of (1) is said to be the best dominant of (1).

Note that the best dominant is unique up to a rotation of $U \times \overline{U}$.

Definition No. 3. We denote by Q_ξ the set of functions $q(\cdot,\xi)$ that are analytic and injective, as function of z on $\overline{U} \setminus E(q(z,\xi))$ where

$$E(q(z,\xi)) = \{\zeta \in \partial U : \lim_{z\to\zeta} q(z,\xi) = \infty\}$$

and are such that $q'(\zeta,\xi) \neq 0$ for $\zeta \in \partial U \setminus E(q(z,\xi))$, $\xi \in \overline{U}$.

The subclass of Q for which $q(0,\xi) = a$ is denoted by $Q(a)$.

We mention that all derivatives of first order or second-order that appear are derived in relation to the variable z.

Lemma No. 1. (S.S. Miller, P.T. Mocanu, [2], [4], [5, Lemma 9.2.3]) *Let $q(\cdot,\xi) \in Q_\xi$, with $q(0,\xi) = a$, and*

$$p(z,\xi) = a + a_n(\xi)z^n + a_{n+1}(\xi)z^{n+1} + \dots$$

be analytic in $U \times \overline{U}$ with $p(z,\xi) \not\equiv a$ and $n \geq 1$. If $p(\cdot,\xi)$ is not subordinated to $q(\cdot,\xi)$, then there exist points $z_0 = r_0 e^{i\theta_0} \in U$ and $\zeta_0 \in \partial U \setminus E(q(z,\xi))$ and $m \geq n \geq 1$ for which $p(U_{r_0} \times \overline{U}_{r_0}) \subset q(U \times \overline{U})$.

(i) $p(z_0,\xi) = q(z_0,\xi)$

(ii) $z_0 p'(z_0,\xi) = m\zeta_0 q'(\zeta_0,\xi)$ *and*

(iii) $\mathrm{Re}\, \dfrac{z_0 p''(z_0,\xi)}{p'(z_0,\xi)} + 1 \geq m \left[\mathrm{Re}\, \dfrac{\zeta_0 q''(\zeta_0,\xi)}{q'(\zeta_0,\xi)} + 1\right]$.

Remark. The proof is similar to the proof in [4].

Definition No. 4. [6] Let Ω_ξ be a set in $\mathbb{C}$, $q(\cdot,\xi) \in Q_\xi$ and n be a positive integer. The class of admissible functions $\Psi_n[\Omega_\xi, q(\cdot,\xi)]$ consists of those functions $\psi : \mathbb{C}^3 \times U \times \overline{U} \to \mathbb{C}$ that satisfy the admissibility condition:

$$(A) \qquad \psi(r,s,t;z,\xi) \notin \Omega_\xi$$

whenever

$$r = q(\zeta,\xi), \quad s = m\zeta q'(\zeta,\xi),$$

$$\mathrm{Re}\, \frac{t}{s} + 1 \geq m\mathrm{Re}\, \left[\frac{\zeta q''(\zeta,\xi)}{q'(\zeta,\xi)} + 1\right],$$

$$z \in U,\ \zeta \in \partial U \setminus E(q),\ \xi \in \overline{U} \text{ and } m \geq n.$$

We write $\Psi_1[\Omega_\xi, q(\cdot,\xi)]$ as $\Psi[\Omega_\xi, q(\cdot,\xi)]$.

In the special case when Ω_ξ is a simply connected domain, $\Omega_\xi \neq \mathbb{C}$, and $h(\cdot,\xi)$ is a conformal mapping of $U \times \overline{U}$ onto Ω_ξ we denote this class by $\Psi_n[h(\cdot,\xi), q(\cdot,\xi)]$.

If $\psi : \mathbb{C}^2 \times U \times \overline{U} \to \mathbb{C}$, then the admissibility condition (A) reduces to

$$(A') \qquad \psi(r, s; z.\xi) \notin \Omega_\xi,$$

whenever

$$r = q(\zeta, \xi),\ s = \zeta q'(\zeta, \xi), \quad z \in U,\ \zeta \in \partial U \setminus E(q(z, \xi)),\ \xi \in \overline{U},\ \text{and } m \geq n.$$

If $\psi : \mathbb{C} \times U \times \overline{U} \to \mathbb{C}$, then the admissibility condition (A) reduces to

$$(A'') \qquad \psi(r; z, \xi) \notin \Omega_\xi$$

whenever

$$r = q(\zeta, \xi), \quad z \in U, \quad \xi \in \overline{U}, \quad \zeta \in \partial U \setminus E(q(z, \xi)).$$

2.Main results

Theorem No. 1. *Let* $\psi \in \Psi_n[\Omega_\xi, q(\cdot, \xi)]$ *with* $q(0, \xi) = a$. *If* $p(\cdot, \xi) \in \xi[a, n]$ *satisfies*

$$(1) \qquad \psi(p(z, \xi), zp'(z, \xi), z^2 p''(z, \xi); z, \xi) \in \Omega_\xi, \quad z \in U,\ \xi \in \overline{U}$$

then

$$p(z, \xi) \prec\prec q(z, \xi), \quad z \in U,\ \xi \in \overline{U}.$$

Proof. Assume $p(z, \xi) \not\prec\prec q(z, \xi)$. By Lemma 1 there exist points $z_0 = r_0 e^{i\theta_0} \in U$ and $\zeta_0 \in \partial U \setminus E(q(z, \xi))$, and $m \geq n \geq 1$ that satisfy (i)-(iii) of Lemma 1.

Using these conditions with $r = p(z_0, \xi)$, $s = z_0 p'(z_0, \xi)$, $t = z_0^2 p''(z_0, \xi)$ and $z = z_0$ in Definition 3 we obtain

$$\psi(p(z_0, \xi), z_0 p'(z_0, \xi), z_0^2 p''(z_0, \xi); z_0, \xi) \notin \Omega_\xi.$$

Since this contradicts (1) we must have $p(z, \xi) \prec\prec q(z, \xi)$, $z \in U$, $\xi \in \overline{U}$. □

Remark No. 1. Upon examining the proof of Theorem 1 it is easy to see that the theorem also holds if condition (1) is replaced by

$$(1') \qquad \psi(p(w(z), \xi), w(z)p'(w(z), \xi), w^2(z)p''(w(z), \xi); w(z), \xi) \in \Omega_\xi,$$

$z \in U$, $\xi \in \overline{U}$, where $w(z)$ is any function mapping U into U.

On checking the definitions of Q_ξ and $\Psi_n[\Omega_\xi, q(\cdot, \xi)]$ we see that the hypothesis of Theorem 1 requires that $q(\cdot, \xi)$ behave very nicely on the boundary of U. If this is not true or if the behavior of $q(\cdot, \xi)$ is not known, it may still be possible to prove that $p(z, \xi) \prec\prec q(z, \xi)$, $z \in U$, $\xi \in \overline{U}$ by the following limiting procedure.

Theorem No. 2. *Let $\Omega_\xi \subset \mathbb{C}$ and let $q(\cdot,\xi) \in S\xi$, for all $\xi \in \overline{U}$, with $q(0,\xi) = a$. Let $\psi \in \Psi_n[\Omega_\xi, q_\rho(\cdot,\xi)]$ for some $\rho \in (0,1)$, where $q_\rho(z,\xi) = q(\rho z,\xi)$. If $p(\cdot,\xi) \in \xi[a,n]$ and*

$$\psi(p(z,\xi), zp'(z,\xi), z^2p''(z,\xi); z,\xi) \in \Omega_\xi$$

then

$$p(z,\xi) \prec\prec q(z,\xi), \quad z \in U,\ \xi \in \overline{U}.$$

Proof. The function $q_\rho(\cdot,\xi)$ is univalent in U for all $\xi \in \overline{U}$ and therefore $E(q_\rho(z,\xi))$ is empty and $q_\rho(\cdot,\xi) \in Q_\xi$. The class $\Psi_n[\Omega_\xi, q_\rho(\cdot,\xi)]$ is an admissible class and from Theorem 1 we obtain $p(z,\xi) \prec\prec q_\rho(z,\xi)$. Since $q_\rho(z,\xi) \prec\prec q(z,\xi)$ we deduce

$$p(z,\xi) \prec\prec q(z,\xi), \quad z \in U,\ \xi \in \overline{U}. \quad \square$$

We next consider the special situation when $\Omega_\xi \neq \mathbb{C}$ is a simply connected domain. In this case $\Omega_\xi = h(U \times \overline{U})$ where $h(\cdot,\xi)$ is a conformal mapping of $U \times \overline{U}$ onto Ω_ξ and the class $\Psi_n[h(U \times \overline{U}), q(\cdot,\xi)]$ is written as $\Psi_n[h(\cdot,\xi), q(\cdot,\xi)]$.

The following result is an immediate consequence of Theorem 1.

Corollary No. 1. *Let $\psi \in \Psi_n[h(\cdot,\xi), q(\cdot,\xi)]$ with $q(0,\xi) = a$.*

If $p(.,\xi) \in \xi[a,n]$, $\psi(p(z,\xi), zp'(z,\xi), z^2p''(z,\xi); z,\xi)$ is analytic in $U \times \overline{U}$ and

$$\psi(p(z,\xi), zp'(z,\xi), z^2p''(z,\xi); z,\xi) \prec\prec h(z,\xi),$$

then

$$p(z,\xi) \prec\prec q(z,\xi), \quad z \in U,\ \xi \in \overline{U}.$$

This result can be extended to those cases in which the behavior of $q(\cdot,\xi)$ on the boundary of U is unknown by the following theorem.

Theorem No. 3. *Let $h(\cdot,\xi) \in S\xi$, for all $\xi \in \overline{U}$ and $q(\cdot,\xi) \in S\xi$, for all $\xi \in \overline{U}$, with $q(0,\xi) = a$ and set $q_\rho(z,\xi) = q(\rho z,\xi)$ and $h_\rho(z,\xi) = h(\rho z,\xi)$. Let $\psi \in \mathbb{C}^3 \times U \times \overline{U} \to \mathbb{C}$ satisfy one of the following conditions:*

(i) $\psi \in \Psi_n[h(\cdot,\xi), q_\rho(\cdot,\xi)]$, for some $\rho \in (0,1)$, or

(ii) there exists $\rho_0 \in (0,1)$ such that $\psi \in \Psi_n[h_\rho(\cdot,\xi), q_\rho(\cdot,\xi)]$ for all $\rho \in (\rho_0,1)$.

If $p(.,\xi) \in \xi[a,n]$, $\psi(p(z,\xi), zp'(z,\xi), z^2p''(z,\xi); z,\xi)$ is analytic in $U \times \overline{U}$ and

$$\psi(p(z,\xi), zp'(z,\xi), z^2p''(z,\xi); z,\xi) \prec\prec h(z,\xi) \tag{3}$$

then

$$p(z,\xi) \prec\prec q(z,\xi), \quad z \in U,\ \xi \in \overline{U}.$$

Proof. Case (i). By applying Theorem 2 we obtain $p(z,\xi) \prec\prec q_\rho(z,\xi)$. Since $q_\rho(z,\xi) \prec\prec q(z,\xi)$ we deduce

$$p(z,\xi) \prec\prec q(z,\xi), \quad z \in U,\ \xi \in \overline{U}.$$

Case (ii). If we let $p_\rho(z,\xi) = p(\rho z,\xi)$, then

$$\psi(p_\rho(z,\xi), zp'_\rho(z,\xi), z^2 p''_\rho(z,\xi); z,\xi)$$

$$= \psi(p(\rho z,\xi), \rho z p'(\rho z,\xi), \rho^2 z^2 p''(\rho z,\xi); \rho z,\xi) \in h_\rho(U \times \overline{U}).$$

By using Theorem 2 and the comment associated with (1'), with $w(z) = \rho z$, we obtain

$$p_\rho(z,\xi) \prec\prec q_\rho(z,\xi)$$

for $\rho \in (\rho_0, 1)$. By letting $\rho \to 1$ we obtain

$$p(z,\xi) \prec\prec q(z,\xi), \quad z \in U,\ \xi \in \overline{U}. \quad \square$$

The next two theorems yield best dominants of the strong differential subordination (3).

Theorem No. 4. *Let $h(\cdot,\xi) \in S\xi$, for all $\xi \in \overline{U}$ and let $\psi : \mathbb{C}^3 \times U \times \overline{U} \to \mathbb{C}$. Suppose that the differential equation*

$$\psi(q(z,\xi), zq'(z,\xi), z^2 q''(z,\xi); z,\xi) = h(z,\xi), \quad z \in U,\ \xi \in \overline{U} \tag{4}$$

has a solution $q(\cdot,\xi)$, with $q(0,\xi) = a$, and one of the following conditions is satisfied:

(i) $q(\cdot,\xi) \in Q_\xi$ and $\psi \in \Psi[h(\cdot,\xi), q(\cdot,\xi)]$;

(ii) $q(\cdot,\xi) \in S\xi$, for all $\xi \in \overline{U}$ and $\psi \in \Psi[h(\cdot,\xi), q_\rho(\cdot,\xi)]$, for some $\rho \in (0,1)$ or

(iii) $q(\cdot,\xi) \in S\xi$, for all $\xi \in \overline{U}$ and there exists $\rho_0 \in (0,1)$ such that $\psi \in \Psi[h_\rho(\cdot,\xi), q_\rho(\cdot,\xi)]$ for all $\rho \in (\rho_0, 1)$.

If $p(\cdot,\xi) \in \xi[a,1]$ and $\psi(p(z,\xi), zp'(z,\xi), z^2 p''(z,\xi); z,\xi)$ is analytic in $U \times \overline{U}$ and if $p(\cdot,\xi)$ satisfies

$$\psi(p(z,\xi), zp'(z,\xi), z^2 p''(z,\xi); z,\xi) \prec\prec h(z,\xi), \quad z \in U,\ \xi \in \overline{U} \tag{5}$$

then $p(z,\xi) \prec\prec q(z,\xi)$ and $q(\cdot,\xi)$ is the best dominant.

Proof. By applying Theorem 2 and Theorem 3 we deduce that $q(\cdot,\xi)$ is a dominant of (5). Since $q(\cdot,\xi)$ satisfies (4), it is a solution of (5) and therefore q will be dominated by all dominants of (5). Hence $q(\cdot,\xi)$ will be the best dominant of (5). $\square$

Theorem No. 5. *Let $h(\cdot,\xi) \in S\xi$, for all $\xi \in \overline{U}$ and $\psi : \mathbb{C}^3 \times U \times \overline{U} \to \mathbb{C}$. Suppose that the differential equation*

$$\psi(q(z,\xi), nzq'(z,\xi), n(n-1)zq'(z,\xi) + n^2 z^{2n} q''(z,\xi); z,\xi) = h(z,\xi), \tag{6}$$

$z \in U$, $\xi \in \overline{U}$ has a solution $q(\cdot,\xi)$, with $q(0,\xi) = a$, and one of the following conditions is satisfied:

(i) $q(\cdot,\xi) \in Q_\xi$ and $\psi \in \Psi_n[h(\cdot,\xi), q(\cdot,\xi)]$;

(ii) $q(\cdot,\xi) \in S\xi$, for all $\xi \in \overline{U}$ and $\psi \in \Psi_n[h(\cdot,\xi), q_\rho(\cdot,\xi)]$, for some $\rho \in (0,1)$, or

(iii) $q(\cdot,\xi) \in S\xi$, for all $\xi \in \overline{U}$ and there exists $\rho_0 \in (0,1)$ such that $\psi \in \Psi_n[h_\rho(\cdot,\xi), q_\rho(\cdot,\xi)]$ for all $\rho \in (\rho_0, 1)$.

If $p(\cdot,\xi) \in \xi[a,n]$, $\psi(p(z,\xi), zp'(z,\xi), z^2p''(z,\xi); z,\xi)$ is analytic in $U \times \overline{U}$ and $p(\cdot,\xi)$ satisfies

$$\psi(p(z,\xi), zp'(z,\xi), z^2p''(z,\xi); z,\xi) \prec\prec h(z,\xi), \tag{7}$$

$z \in U$, $\xi \in \overline{U}$, then $p(z,\xi) \prec\prec q(z,\xi)$, $z \in U$, $\xi \in \overline{U}$, and $q(\cdot,\xi)$ is the best dominant.

Proof. By applying Theorem 2 and Theorem 3 we deduce that $q(\cdot,\xi)$ is a dominant of (7). If we let $p(z,\xi) = q(z^n,\xi)$, then

$$zp'(z,\xi) = nz^n q'(z^n,\xi)$$

and

$$z^2p''(z,\xi) = n(n-1)z^n q'(z^n) + n^2 z^{2n} q''(z^n,\xi).$$

Therefore from (6) we obtain

$$\psi(p(z,\xi), zp'(z,\xi), z^2p''(z,\xi); z,\xi) = h(z^n,\xi) \prec\prec h(z,\xi), \quad z \in U,\ \xi \in \overline{U}.$$

Since $p(U \times \overline{U}) = q(U \times \overline{U})$, we conclude that $q(\cdot,\xi)$ is best dominant. $\square$

Example No. 1. Let $q(z,\xi) = 1 + \dfrac{\xi}{2}z$,

$$h(z,\xi) = q(z,\xi) + zq'(z,\xi) + z^2q''(z,\xi) = 1 + \xi z.$$

If $\psi(p(z,\xi), zp'(z,\xi), z^2p''(z,\xi); z,\xi) = p(z,\xi) + zp'(z,\xi) + z^2p''(z,\xi)$ is analytic in $U \times \overline{U}$ and satisfies

$$p(z,\xi) + zp'(z,\xi) + z^2p''(z,\xi) \prec\prec h(z,\xi) = 1 + \xi z$$

then

$$p(z,\xi) \prec\prec q(z,\xi) = 1 + \xi z, z \in U,\ \xi \in \overline{U}$$

and $q(\cdot, \xi)$ is the best dominant.

References

[1] J.A. Antonino and S. Romaguera, *Strong differential subordination to Briot-Bouquet differential equations*, Journal of Differential Equations, **114**(1994), 101-105.

[2] S.S. Miller, P.T., Mocanu, *Second order differential inequalities in the complex plane*, J. Math. Anal. Appl., **65**(1978), 298-305.

[3] S.S. Miller and P.T. Mocanu, *Differential subordinations and univalent functions*, Michig. Math. J., **28**(1981), 157-171.

[4] S.S. Miller and P.T. Mocanu, *Differential subordinations. Theory and applications*, Pure and Applied Mathematics, Marcel Dekker, Inc., New York, 2000.

[5] S.S. Miller and P.T. Mocanu, *Subordinants of differential superordinations*, Complex variables, **48**(10), 2003, 815-826.

[6] Georgia Irina Oros and Gheorghe Oros, *Strong differential subordination*, Turkish Journal of Mathematics, **33**(2009), 249-257.

[7] G. I. Oros, *Briot-Bouquet strong differential subordination*, Journal Computational Analysis and Applications, vol. 14, **4**(2012), 733-737.

Georgia Irina Oros
Department of Mathematics
University of Oradea
Str. Universităţii, No.1, 410087 Oradea, Romania
email:*georgia_oros_ro@yahoo.co.uk*