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S_1-PARACOMPACT SPACES

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Abstract. In this paper we introduce a new class of spaces which will be called the class of S_1-paracompact spaces.We characterize S_1-paracompact spaces and study their basic properties.The relationships between S_1-paracompact spaces and other well-known spaces are investigated.

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1. Introduction and preliminaries

In 1963, Levine [8] introduced and studied the concept of semi-open sets in topological spaces. In [1] , Al-Zoubi used semi-open sets to defined the class of s-expandable spaces. A space (X, T) is said to be s-expandable space if for every s-locally finite collection $\mathcal{F} = \{F_\alpha : \alpha \in I\}$ of subsets of X there exists a locally finite collection $\mathcal{G} = \{G_\alpha : \alpha \in I\}$ of open subsets of X such that $F_\alpha \subseteq G_\alpha$ for each $\alpha \in I$. In [1], Theorem 3.4, a space (X, T) is s-expandable if every semi-open cover of X has a locally finite open refinement.

In section 2 of this work we introduce and study a new class of spaces, namely S_1-paracompact spaces, and we provide several characterizations of S_1-paracompact spaces and investigate the relaionship between S_1-paracompact spaces and other well-known spaces such as paracompact spaces, s-expandable spaces, nearly paracompact spaces and semi-compac spaces. Finally, in section 3, we deal with some basic properties of S_1-paracompact spaces , i.e. subspaces, sum, inverse image and product. Throughout this work a space will always mean a topological space on which no separation axioms are assumed unless explicitly stated. Let (X, T) be a space and A be a subset of X. The closure of A, the interior of A and the relative topology on A will be denoted by $cl(A)$, $int(A)$ and T_A respectively. A is called semi-open subset of (X, T) ([8]) if there exists an open set U of X such that $U \subseteq A \subseteq cl(U)$. The complement of a semi-open set is called a semi-closed set ([2]). The semiclosure of A ([2]), denoted by $scl(A)$, is the smallest semi-closed set that contains A.

A is called regular open if $A = int(cl(A))$.The family of all semi-open (resp. regular open) subsets of (X,T) is denoted by $SO(X,T)$ (resp. $RO(X,T)$).

Definition 1.1. *A collection $\mathcal{F} = \{F_\alpha : \alpha \in I\}$ of subsets of a space (X,T) is said to be locally finite(resp. s-locally finite[1]), if for each $x \in X$, there exists $U \in T$ (resp. $U \in SO(X,T)$) containing x and U intersects at most finitely many members of $\mathcal{F}$.*

Definition 1.2. *A space (X,T) is said to be:*

(a) *semi-compact* [3] *if every semi-open cover of X has a finite subcover.*

(b) *semi-regular* [4] *if for each semi-closed set F and each point $x \notin F$, there exist disjoint semi-open sets U and V such that $x \in U$ and $F \subseteq V$. This is equivalent to, for each $U \in SO(X,T)$ and for each $x \in U$, there exists $V \in SO(X,T)$ such that $x \in V \subseteq scl(V) \subseteq U$.*

(c) *extremally disconnected (briefly e.d.) if the closure of every open set in (X,T) is open.*

Lemma 1.3 [10]. *If (X,T) is e.d., then $scl(U) = cl(U)$ for each $U \in SO(X,T)$.*

Proposition 1.4. *Let (X,T) be an e.d. semi-regular space. Then:*

(a) $SO(X,T) = T$.

(b) (X,T) *is regular.*

Proof. (a) Let $U \in SO(X,T)$ and $x \in U$. Since (X,T) is semi-regular, there exists $V \in SO(X,T)$ such that $x \in V \subseteq scl(V) \subseteq U$. Now, choose $W \in T$ such that $W \subseteq V \subseteq cl(W)$. But (X,T) is e.d., therefore, by Lemma 1.3, $cl(W) = cl(V) = scl(V)$ is an open set containing x such that $cl(W) \subseteq U$.Thus $U \in T$.

(c) Follows from part (a) and Lemma 1.3.

Lemma 1.5.([2]). *If A is an open set in (X,T) and $B \in SO(X,T)$ then $A \cap B \in SO(X,T)$.*

2. S_1-Paracompact Spaces

Definition 2.1. *A space (X,T) is said to be S_1-paracompact space if every semi-open cover of X has a locally finite open refinement.*

Every S_1-paracompact space is s-expandable (Theorem 3.4 of [1])but the converse is not true as may be seen from the following example.

Example 2.2. Let $X = \{1,2,3\}$ and $T = \{\emptyset, X, \{1\}, \{2\}, \{1,2\}\}$. Then (X,T) is s-expandable (every finite space is s-expandable) but not S_1-paracompact since $\{\{1,2\},\{2,3\}\}$ is a semi-open cover of X which admits no locally finite open refinement.

For any space (X,T) ,we have $RO(X,T) \subseteq T \subseteq SO(X,T)$.Therefore the following implications are obvious.

S_1-paracompact $\Rightarrow$ paracompact $\Rightarrow$ nearly paracompact.

Where a space (X,T) is said to be nearly paracompact if every reguler open cover

of X has a locally finite open refinement. Moreover we have the following theorem.
Theorem 2.3. *Let (X,T) be an e.d. semi-regular space. Then the following are equivalent:*

(a) (X,T) *is nearly paracompact.*

(b) (X,T) *is paracompact.*

(c) (X,T) *is S_1-paracompact.*

Proof. $(a) \to (b)$: Follows from Proposition 1.4 and the fact that if a space (X,T) is regular then $T \subseteq RO(X,T)$.

$(b) \to (c)$: Follows from Proposition 1.4 (a).

The following examples show that the condeitions "e.d. and semi-regular" on the space (X,T) in the above Theorem are essential.

Example 2.4. (a) Let $X = R$ be the set of the real numbers with the topology $T = \{\emptyset, X, \{1\}\}$. Then (X,T) is an e.d. paracompact space but not S_1-paracompact since $\{\{1,x\} : x \in X\}$ is a semi-open cover of X which admits no locally finite open refinement.

(b) Consider the space (X,T) where $X = R$ and

$$T = \{U \subseteq R : 0 \notin U\} \bigcup \{U \subseteq R : 0 \in U \, and \, R - U \, is \, finite\}.$$

Then (X,T) is semi-regular and paracompact but not S_1-paracompact since it is not s-expandable (Example 3.3,[1]).

(c) Let $X = R$ with the topology $T = \{U \subseteq R : 0 \in U\} \bigcup \{\emptyset\}$. Then (X,T) is an e.d. nearly paracompact space ($RO(X,T) = \{X, \emptyset\}$) but not paracompact since $\{\{0,x\} : x \in X\}$ is an open cover of X which admits no locally finite open refinement.

Theorem 2.5. *If (X,T) is an S_1 $-$paracompact T_1-space, then $T = SO(X,T) = T^\alpha$.*

Proof. Let U be a semi-open in (X,T). For each $y \notin U$, we choose an open set V_y containing y and $x \notin V_y$. Therefore the collection $\mathcal{V} = \{V_y : y \notin U\} \cup \{U\}$ is a semi-open cover of (X,T) and so it has a locally finite open refinement $\mathcal{W}$. Put $V = \cup\{W \in \mathcal{W} : x \in W\}$. Then V is an open set containing x and $V \subseteq U$. Thus U is open in (X,T).

Corollary 2.6. *Let (X,T) be an S_1 $-$paracompact space.*

(a) *If (X,T) is T_1 then it is e.d.*

(b) *If (X,T) is T_2 then it is semi-reguler.*

Proof. (a) Let $U \in T$. Then $cl(U) \in SO(X,T)$ and so $cl(U) \in T$ by Theorem 2.5.
(b) (X,T) is a paracompact T_2-space. Therefore by Lemma 5.1.4 of ([6]), (X,T) is reguler. Now let $U \in SO(X,T)$and $x \in U$. By Theorem 2.5, $U \in T$ and so there exists an open set $V \in T$ such that $x \in V \subset cl(V) \subset U$. Thus $x \in V \subset scl(V) \subset cl(V) \subset U$. It follows that (X,T) is semi-reguler.

Note that R with the cofinite topology is S_1 −paracompact but not semi-reguler. Recall that a space (X,T) is called semi-compact ([3]), if every semi-open cover of X has a finite subcover. We note that S_1 −paracompactness and semi-compactness are independent, since in Example 2.2**,** $(X,T)$is semi-compact (X is finite) but it is not S_1 −paracompact. On the other hand, the space (X,T_{dis}) where X is an infinite set is S_1 −paracompact but it is not semi-compact.

Theorem 2.7. *Let (X,T) be a T_2-space. Then the following are equivalent:*
(a) (X,T) *is semi-compact.*
(b) (X,T) *is S_1 −paracompact and compact.*

Proof. (a)→(b): Suppose that (X,T) is semi-compact. Then by Theorem 2.4 of ([3]), X is finite and so T is the discrete topology. Therefore (X,T) is S_1 −paracompact.
(b) →(a) : As (X,T) is compact T_2-space, every locally finite family of open sets is finite.

For a space (X,T), we denote by T_Ψ([3]) the topology on X which has $SO(X,T)$ as a subbase.
It is clear that if a space (X,T) is e.d., then $T_\Psi = SO(X,T)$.
Note that in Example 2.4 part (a), the space (X,T) is paracompact and e.d while (X,T_Ψ) is not paracompact since $T_\Psi = \{U : 1 \in U\} \cup \{\phi\}$.

Proposition 2.8. *Let (X,T) be an e.d. space.*
(a) *If (X,T) is S_1 −paracompact then (X,T_Ψ) is S_1 −paracompact.*
(b) *(X,T_Ψ) is S_1 −paracompact if and only if $(X,T_\Psi)$is paracompact.*

Proof. (a) Follows from the fact that for any space (X,T), $T \subseteq T_\Psi$ and $SO(X,T_\Psi) \subseteq SO(X,T)$.
(b) Since (X,T) is e.d. then by Lemma 3.7 of ([1]), $SO(X,T_\Psi) = SO(X,T)$ and thus $T_\Psi = SO(X,T_\Psi)$,

Example 2.9. The converse of part (a) of Proposition 2.8 is not true in general. To see that, consider $X = \{1,2,3\}$ with $T = \{\phi, X,\{1\}\}$, then it is easy to see that (X,T) is e.d. and $(X,T_\Psi)$is S_1 −paracompact but (X,T) is not.

Corollary 2.10. *Let (X,T) be an e.d. semi-reguler space. Then (X,T) is S_1−paracompact if and only if (X,T_Ψ) is paracompact.*

Proof. Necessity follows from Proposition 2.8. For sufficiently, since(X,T) e.d. then $SO(X,T) = SO(X,T_\Psi)$and so by Proposition 1.4 part (a), $T_\Psi = SO(X,T) = SO(X,T_\Psi) = T$.

Recall that a subset A of a space (X,T) is said to be α−set if $A \subset int(cl(int(A)))$.The family of all α−sets of a space (X,T),denote by T^α, forms a topology on X, finer than T.

Lemma 2.11. (a) *For any space (X,T), $SO(X,T^\alpha) = SO(X,T)$*([9]).
(b) *For a space (X,T), if (X,T^α) is normal, then $T = T^\alpha$*([5]).

Theorem 2.12. *Let (X,T) be a T_2−space. Then (X,T) is S_1 −paracompact space* *if and only if* (X, T^α) *is* S_1 *−paracompact space.*

Proof. Necessity, follows from Lemma 2.11 and the fact that $T \subset T^\alpha$.For sufficiency, suppose that (X, T^α) is S_1 −paracompact. Then (X, T^α) is a paracompact T_2−space and so it is normal ([6]). Therefore, by Lemma 2.11, $T = T^\alpha$. On the other hand $SO(X, T^\alpha) = SO(X, T)$(Lemma 2.11) and so (X, T) is S_1 −paracompact.

Note that, in Example **2.9**, the space (X, T) is not S_1 −paracompact. However, (X, T^α) is S_1 −paracompact.Therefore the condition (X, T) is T_2 in Theorem 2.12, can not be dropped.

Theorem 2.13. *If each semi-open cover of a space* (X, T) *has an open* σ*-locally finite refinement, then each semi-open cover of* X *has a locally finite refinement.*

Proof. Let $\mathcal{U}$ be a semi-open cover of X. Let $\mathcal{V} = \cup_{n\in N}\mathcal{V}_n$ be an open σ-locally finite refinement of $\mathcal{U}$, where $\mathcal{V}_n$ is locally finite. For each $n \in N$ and each $V \in \mathcal{V}_n$, let $V_n^{/} = V - \cup_{k<n}\mathcal{V}_k^*$ where $\mathcal{V}_k^* = \cup\{V : V \in \mathcal{V}_k\}$ and put $\mathcal{V}_n^{/} = \{V_n^{/} : V \in \mathcal{V}_n\}$. Now, put $\mathcal{W} = \{V_n^{/} : n \in N, V \in \mathcal{V}_n\} = \cup\{\mathcal{V}_n^{/} : n \in N\}$. We show $\mathcal{W}$ is a locally finite refinement of $\mathcal{U}$. Let $x \in X$ and let n be the first positive integer such that $x \in \mathcal{V}_n^*$. Therefore $x \in V^{/}$for some $V^{/} \in \mathcal{V}_n^{/}$. Thus $\mathcal{W}$ is a cover of X. To show $\mathcal{W}$ is locally finite, let $x \in X$ and n be the first positive integer such that $x \in \mathcal{V}_n^*$. Then $x \in V$ for some $V \in \mathcal{V}_n$. Now, $V \cap V^{/}{=}\phi$ for each $V^{/} \in \mathcal{V}_k$ and for each $k > n$.Therfore, V can intersect at most the elements of $\mathcal{V}_k^{/}$ for $k \leq n$. Since $\mathcal{V}_k^{/}$ is locally finite for each $k \leq n$, so we choose an open set $O_{x(k)}$ containing x such that $O_{x(k)}$ meets at most finitely many members of $\mathcal{V}_k^{/}$. Finally, put $O_x = V \cap (\cap_{k=1}^n O_{x(k)})$.Then O_x is an open set containing x such that O_x meets at most finitely many members of $\mathcal{W}$.

Theorem 2.14. Let(X, T)be a semi-reguler space. If each semi-open cover of a space X has a locally finite refinement, then each semi-open cover of X has a locally finite semi-closed refinement.

Proof. Let $\mathcal{U}$ be a semi-open cover of X. For each $x \in X$, pick $U_x \in U$ such that $x \in U_x$. Since (X, T) is semi-reguler, then there exists $V_x \in SO(X, T)$such that $x \in V_x \subset scl(V_x) \subset U_x$. The family $\mathcal{V} = \{V_x : x \in X\}$is a semi-open cover of X and so, by asumption, has a locally finite refinement $\mathcal{W} = \{W_\alpha : \alpha \in I\}$. The collection $scl(\mathcal{W}) = \{scl(W_\alpha) : \alpha \in I\}$ is locally finite such that for each $\alpha \in I$, if $W_\alpha \subset V_x$, then $scl(W_\alpha) \subset U$ for some $U \in \mathcal{U}$. Thus $scl(\mathcal{W})$ is a semi-closed locally finite refinement of $\mathcal{U}$.

For the next theorem we will use the statements:

(a) (X, T) S_1 −paracompact.

(b) Each semi-open cover of X has a σ-locally finite open refinement.

(c) Each semi-open cover of X has a locally finite refinement.

(d) Each semi-open cover of$X$ has a locally finite semi-closed refinement.

Theorem 2.15. *If* (X, T)*is a semi-reguler space then* $(a) \to (b) \to (c) \to (d)$.

Proof. $(a) \rightarrow (b)$ is obvious.
$(b) \rightarrow (c)$ Follows from Theorem 2.13.
$(c) \rightarrow (d)$ Follows from Theorem 2.14.
In Example 2.2, (X, T) is semi- regular in which $\{1\}$, $\{2\}$ and $\{3\}$ are semi-closed sets. Therefore, (X, T) satisfies statement (d) of Theorem 2.15, but it is not S_1 $-$paracompact.

Note that if (X, T) is also e.d. in Theorem 2.15, then $(d) \rightarrow (a)$.

3-Properties of S_1 -paracompact Spaces

In this section we study some basic properties of S_1-paracompact spaces such as subspaces, sums, products, images and inverse images under some types of functions.

Theorem 3.1. *Every reguler open subspace of an S_1-paracompact space is S_1-paracompact.*

Proof. Let (X, T)be an S_1 $-$paracompact space and A be a regular open subspace of (X, T). Let $\mathcal{U}$=$\{U_\alpha : \alpha \in I\}$be a semi-open cover of A such that $U_\alpha \in SO(A, T_A)$ for each $\alpha \in I$. Since A is an open subset of X then $U_\alpha \in SO(X, T)$ for each $\alpha \in I$. Therefore the family $\mathcal{V} = \{U_\alpha : \alpha \in I\} \cup \{X - A\}$is a semi-open cover of X. Let $\mathcal{W} = \{W_\beta : \beta \in B\}$be a locally finite open refinement of $\mathcal{U}$ in (X, T). Then the family $\{W_\beta \cap A : \beta \in B\ \}$ is a locally finite open refinement of A in (A, T_A), Thus(A, T_A) is S_1-paracompact.

Corollary 3.2. *Every clopen subspace of an S_1-paracompact space is S_1-paracompact.*

Definition 3.3.[6]*Let $\{(X_\alpha, T_\alpha) : \alpha \in I\}$ be a collection of topological spaces such that $X_\alpha \bigcap X_\beta = \emptyset$ for each $\alpha \neq \beta$. Let $X = \cup_{\alpha\in I} X_\alpha$ be topologized by $T = \{G \subseteq X : G \cap X_\alpha \in T_\alpha\ ,\ \alpha \in I\}$. Then (X, T) is called the sum of the spaces $\{(X_\alpha, T_\alpha) : \alpha \in I\}$ and we write $X = \oplus_{\alpha\in I} X_\alpha$.*

Theorem 3.4. *The topological sum $\oplus_{\alpha\in I} X_\alpha$ is S_1-paracompact if and only if the space (X_α, T_α) is S_1-paracompact for each $\alpha \in I$.*

Proof. Necessity follows from Corollary 3.2, since (X_α, T_α) is a clopen subspace of the space $\oplus_{\alpha\in I} X_\alpha$, for each $\alpha \in I$. To prove sufficiency, let $\mathcal{U}$ be a semi- open cover of $\oplus_{\alpha\in I} X_\alpha$. For each $\alpha \in I$ the family $\mathcal{U}_\alpha = \{U \cap X_\alpha : U \in \mathcal{U}\}$ is a semi- open cover of the S_1-paracompact space (X_α, T_α) . Therefore $\mathcal{U}_\alpha$ has a locally finite open refinement $\mathcal{V}_\alpha$ in (X_α, T_α). Put $\mathcal{V} = \cup_{\alpha\in I} \mathcal{V}_\alpha$. It is clear that $\mathcal{V}$ is a locally finite open refinement of $\mathcal{U}$ .Thus $\oplus_{\alpha\in I} X_\alpha$is S_1-paracompact.

Recall that a function $f : (X, T) \rightarrow (Y, M)$ is said to be irresolute ([2]) , if $f^{-1}(U) \in SO(X, T)$ for every $U \in SO(Y, M)$. It is well known that every continuous open surjective function is irresolute (Theorem 1.8 of [2]).

Theorem 3.5. *Let $f : (X, T) \rightarrow (Y, M)$be a continuous, open, and closed surjective function such $f^{-1}(y)$ is compact for each $y \in Y$. If (X, T) is S_1 $-$paracompact, then* *so is* (Y, M).

Proof. Let $\mathcal{V} = \{V_\alpha : \alpha \in I\}$be a semi-open cover of (Y, M). Since f is irresolute, the collection $\mathcal{U} = f^{-1}(\mathcal{V}) = \{f^{-1}(V_\alpha) : \alpha \in I\}$is a semi-open cover of the S_1 −paracompact (X, T) space and so it has a locally finite open refinement, say $\mathcal{W}$. The collection $f(\mathcal{W})$ is a locally finite open refinement of $\mathcal{V}$ in (Y, M).

Definition 3.6. *A function* $f : (X, T) \to (Y, M)$ *is said to be semi-closed* ([10]) *if* $f(A) \in SC(Y, M)$ *for every closed subset* A *of* X.

Proposition 3.7. [1] A *function* $f : (X, T) \to (Y, M)$ *is semi-closed if and only if for every* $y \in Y$ *and every open set* U *in* (X, T) *which contains* $f^{-1}(y)$*, there exists* $V \in SO(Y, M)$ *such that* $y \in V$ *and* $f^{-1}(V) \subseteq U$.

Theorem 3.8. *Let* $f : (X, T) \to (Y, M)$ *be a continuous semi-closed surjection and*$f^{-1}(y)$ *is compact for each* $y \in Y$. *If* (Y, M) *is* S_1 *−paracompact space then* (X, T) *is paracompact.*

Proof. Let $\mathcal{U} = \{U_\alpha, \alpha \in I\}$be an open cover of X. For each $y \in Y$ and for each $x \in f^{-1}(y)$ choose $\alpha(x) \in I$ such that $x \in U_{\alpha(x)}$. Therefore the collection $\{U_{\alpha(x)} : x \in f^{-1}(y)\}$ is an open cover of $f^{-1}(y)$ and so there exists a finite subset $I(y)$ of I such that $f^{-1}(y) \subset \cup_{\alpha(x)\in I(y)} U_{\alpha(x)} = U_y$. But f is semi-closed so there exists a semi-open setV_y containing y and $f^{-1}(V_y) \subset U_y$. Thus $\mathcal{V} = \{V_y : y \in Y\}$is a semi-open cover of Y and so it has a locally finite open refinement say $\mathcal{W} = \{W_\beta$ $:\beta \in B$ $\}$. Since f is continuous, then the family $\{f^{-1}(W_\beta) : \beta \in B$ $\}$ is an open locally finite cover of X such that for each $\beta \in B$, $f^{-1}(W_\beta) \subset U_y$ for some $y \in Y$. Now, the family $\{f^{-1}(W_\beta)\Lambda U_y : \beta \in B, y \in \mathcal{V}$ $\}$is a locally finite open refinement of $\mathcal{U}$, where $f^{-1}(W_\beta)\Lambda U_y = \{$ $f^{-1}(W_\beta) \cap U_y : \beta \in B, \alpha(x) \in I(y)\}$. Therefore (X, T) is paracompact.

We finally study products of S_1 −paracompact spaces. Note that the space (X, T) where$X$= {1,2} and $T = \{\phi, X, \{1\}\}$ is an e.d. S_1 −paracompact (compact) space while $(X, T) \times (X, T)$ is not S_1 −paracompact since {{(1,1),(2,2)},{(1,1), (1,2),(2,1)}}is a semi-open cover of $X \times X$ which admits no locally finite open refinement.

Corollary 3.9. If (X, T) is compact, (Y, M) is S_1 −paracompact and
$(X, T) \times (Y, M)$ is e.d. semi-reguler, then $(X, T) \times (Y, M)$ is S_1 −paracompact.

Proof. Since (Y, M) is paracompact then $(X, T) \times (Y, M)$ is paracompact (see[6]). Therefore $(X, T) \times (Y, M)$ is S_1− paracompact by Theorem 2.3.

In the above paragraph $(X, T) \times (X, T)$ is e.d. but not semi-reguler. Therefore the condition " semi-reguler " on $(X, T) \times (Y, M)$ in the above Corollary can not be dropped. On the other hand $(R, T) \times (R, T_{dis})$ (where (R, T)as in Example 2.4, part (b)) is semi-reguler but not S_1 −paracompact since it is not s-expandable (Example 3.14,[1]).

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