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UNIVALENCE CRITERIA FOR A FAMILY OF INTEGRAL OPERATORS DEFINED BY GENERALIZED DIFFERENTIAL OPERATOR

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Abstract. In this paper we discuss some extensions of univalent conditions for a family of integral operators defined by generalized differential operators. Several other results are also considered.

Keywords: Univalent functions, Integral operators, Differential operators.

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1. Introduction

Let H be the class of functions analytic in the open unit disk $U = \{z : |z| < 1\}$ and $H[a, n]$ be the subclass of H consisting of functions of the form :

$$f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \dots . \tag{1}$$

Let A be the subclass of H consisting of functions of the form :

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{2}$$

and satisfy the following usual normalized condition $f(0) = f(0)' - 1 = 0$. Also let S denote the subclass of A consisting of functions $f(z)$ which are univalent in U.

If $f(z) = z + \sum\limits_{n=2}^{\infty} a_n z^n$ and $g(z) = z + \sum\limits_{n=2}^{\infty} b_n z^n$, then the Hadamard product or (convolution) of f and g is defined by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n, \ z \in U.$$

The authors in [13] have recently introduced a new generalized differential operator $D^k_{\alpha,\beta,\lambda,\delta}$, as the following:

Definition 1. *For $f \in A$ of the form (2) we define the following generalized differential operator*

$$D^0 f(z) = f(z)$$

$$D^1_{\alpha,\beta,\lambda,\delta} f(z) = [1 - (\lambda - \delta)(\beta - \alpha)] f(z) + (\lambda - \delta)(\beta - \alpha) z f'(z)$$

$$= z + \sum_{n=2}^{\infty} [(\lambda - \delta)(\beta - \alpha)(n-1) + 1] a_n z^n$$

$$\vdots$$

$$D^k_{\alpha,\beta,\lambda,\delta} f(z) = D^1_{\alpha,\beta,\lambda,\delta}\left(D^{k-1}_{\alpha,\beta,\lambda,\delta} f(z)\right)$$

$$D^k_{\alpha,\beta,\lambda,\delta} f(z) = z + \sum_{n=2}^{\infty} [(\lambda - \delta)(\beta - \alpha)(n-1) + 1]^k a_n z^n, \tag{3}$$

for $\alpha \geq 0,\ \beta \geq 0,\ \lambda > 0,\ \delta \geq 0,\ \lambda > \delta,\ \beta > \alpha$ and $k \in N_0 = N \cup \{0\}$.

Remark 1. (i) When $\alpha = 0,\ \delta = 0,\ \lambda = 1,\ \beta = 1$ we get Salagean differential operator (see[14]). (ii) When $\alpha = 0$ we get Darus and Ibrahim differential operator (see[6]). (iii) And when $\alpha = 0, \delta = 0, \lambda = 1$ we get Al- Oboudi differential operator(see [2]).

It is also interesting to see combination of operators given by Lupas [17].

Now, we begin by recalling each of the following theorems dealing with univalence criteria, which will be required in our present work.

Theorem 1.[10] *Let $f \in A$ and $\lambda \in C$. If $\Re(\lambda) > 0$ and*

$$\frac{1 - |z|^{2\Re(\lambda)}}{\Re(\lambda)} \left| \frac{z f''(z)}{f'(z)} \right| \leq 1 \quad (z \in U). \tag{4}$$

Then the function $F_\lambda(z)$ *given by*

$$F_\lambda(z) = \left(\alpha \int_0^z u^{\lambda-1} f(u)\, du\right)^{\frac{1}{\lambda}} \tag{5}$$

is in the univalent function class S *in* U.

Theorem 2.[9] *Let* $f \in A$ *satisfy the following condition*

$$\left|\frac{z^2 f'(z)}{[f(z)]^2} - 1\right| \leq 1 \quad (z \in U), \tag{6}$$

then the function $f(z)$ *is in the univalent function class* S *in* U.

Theorem 3.[12] *Let* $g \in A$ *satisfy the inequality in (6). Also let* $\lambda = a + ib\,(a, b \in R)$ *be a complex number with the components* a *and* b *constrained by*

$$a \in \left(0, \sqrt{3}\right] \quad and \quad a^4 + a^2b^2 - 9 \geq 0.$$

If

$$|g(z)| \leq 1 \quad (z \in U),$$

then the function $\mu_\lambda(z)$ *given by*

$$\mu_\lambda(z) = \left[(a+bi) \int_0^z u^{a+bi-1} \left(\frac{g(u)}{u}\right)^{\frac{1}{a+bi}} du\right]^{\frac{1}{a+bi}} \quad (\lambda = a + bi) \tag{7}$$

is in the univalent function class S *in* U.

Theorem 4.[12] *Let* $g \in A$ *satisfy the inequality in (6). Also let* $\lambda = a + ib\,(a, b \in R)$ *be a complex number with the components* a *and* b *constrained by*

$$a \in \left[\frac{3}{4}, \frac{3}{2}\right], \quad b \in \left[0, \frac{1}{2\sqrt{2}}\right]$$

and

$$8a^2 + 9b^2 - 18a + 9 \leq 0.$$

If

$$|g(z)| \leq 1 \quad (z \in U),$$

then the function $\psi_\lambda(z)$ *given by*

$$\psi_\lambda(z) = \left[(a+bi)\int_0^z (g(u))^{a+bi-1}du\right]^{\frac{1}{a+bi}} \quad (\lambda = a+bi) \tag{8}$$

is in the univalent function class S *in* U.

By using generalized differential operator given by (3), we introduce the following integral operator:

Definition 2. *Let* λ *a complex number,* $0 \leq \mu < 1$ *,* $j = \{1, 2, ..., n\}$*, we introduce the integral operator as follows:*

$$F^k_{n,\lambda,\mu}(z) = \left[\mu\left[n(\lambda-1)+1\right]\int_0^z \prod_{j=1}^n \left(\frac{D^k_{\alpha,\beta,\lambda,\delta}f_j(u)}{u}\right)^{\frac{1}{\lambda}} u^{n(\lambda-1)}du\right.$$

$$\left. + (1-\mu)\left[n(\lambda-1)+1\right]\int_0^z \prod_{j=1}^n \left(D^k_{\alpha,\beta,\lambda,\delta}f_j(u)\right)^{\lambda-1}du\right]^{\frac{1}{[n(\lambda-1)+1]}},$$

or

$$F^k_{n,\lambda,\mu}(z) = \left[\left[n(\lambda-1)+1\right]\left\{\mu\int_0^z \prod_{j=1}^n \left(\frac{D^k_{\alpha,\beta,\lambda,\delta}f_j(u)}{u}\right)^{\frac{1}{\lambda}} u^{n(\lambda-1)}du+\right.\right.$$

$$\left.\left.(1-\mu)\int_0^z \prod_{j=1}^n \left(D^k_{\alpha,\beta,\lambda,\delta}f_j(u)\right)^{\lambda-1}du\right\}\right]^{\frac{1}{[n(\lambda-1)+1]}}, \tag{9}$$

for $f_j \in A$*,* $j = \{1, 2, ..., n\}$ *and* $D^k_{\alpha,\beta,\lambda,\delta}$ *is defined by (3), where* $\alpha \geq 0$*,* $\beta \geq 0$*,* $\lambda > 0$*,* $\delta \geq 0$*,* $\lambda > \delta$*,* $\beta > \alpha$*,* $k \in N_0 = N \cup \{0\}$ *.*

Remark 2. When $n = 1$, $\mu = 1$, $k = 0$ we have the integral operator $F^k_{n,\lambda,\mu}$ reduces to the operator $F^0_{1,\lambda,1} \equiv F_{1,\lambda}$ which is related closely to some known integral operators investigated earlier in Univalent Function Theory (see for details [15]). The operator $F_{1,\lambda}$ was studied by Pescar [11]. Upon setting $n = \lambda = \mu = 1$, $k = 0$ in (9) we can obtain the integral operator $F^0_{1,1,1} \equiv F_{1,1}$ which was studied by Alexander [1]. When $\mu = 1$, $k = 0$, we have the integral operators, studied by Breaz [4]. When $\mu = k = 0$, $n = 1$ the integral operator $F^0_{1,\lambda,0} \equiv G_{1,\lambda}$ was studied by Moldoveanu [7]. When $\mu = k = 0$ we have the integral operators was introduced by Breaz and Breaz [5]. Furthermore, in their special case when $\mu = n = 1$, $k = 0$, $\lambda = a + ib$, $(a, b \in R)$ the integral operator $F^k_{n,\lambda,\mu}$ would obviously reduce to the integral operator (7) and when $\mu = k = 0$, $n = 1, \lambda = a + ib$, $(a, b \in R)$ the integral operator $F^k_{n,\lambda,\mu}$ would obviously reduce to the integral operator (8).

Now we need the following lemma to prove our main results:

Lemma 1._(General Schwarz Lemma [8]). Let the function $f(z)$ be regular in the disk_

$$U_R = \{z : z \in C \text{ and } |z| < R\},$$

with

$$|f(z)| < M, (z \in U_R)$$

for a fixed $M > 0$. If $f(z)$ has one zero with multiplicity order bigger than m for $z = 0$, then

$$|f(z)| \leq \frac{M}{R^m} |z|^m \quad (z \in U_R). \tag{10}$$

The equality in (10) can hold true only if

$$f(z) = e^{i\theta} \left(\frac{M}{R^m} \right) z^m,$$

where θ is a constant.

In the present paper we study further on univalence conditions involving the general integral operators given by (9).

2. Univalence condition associated with generalized integral operator $F^k_{n,\lambda,\mu}$ when $\mu = 1$

Theorem 5. _Let $M \geq 1$ and suppose that each of the functions $f_j \in A$, $j = \{1, 2, ..., n\}$ satisfies the inequality (6). Also let $\lambda = a + ib$, $(a, b \in R)$ be a complex number with the components a and b constrained by_

$$a \in \left(0, \sqrt{(2M+1)n}\right] \tag{11}$$

and

$$a^4 + a^2b^2 - [(2M+1)n]^2 \geq 0. \tag{12}$$

If

$$\left|D^k_{\alpha,\beta,\lambda,\delta} f_j\right| \le M \quad (z \in U,\ j = \{1, 2, ..., n\}).$$

Then the function $F^k_{n,\lambda,\mu}$ *when* $\mu = 1$ *is in the univalent function class* S *in* U.

Proof : We begin by setting

$$f(z) = \int_0^z \prod_{j=1}^n \left(\frac{D^k_{\alpha,\beta,\lambda,\delta} f_j(u)}{u} \right)^{\frac{1}{\lambda}} du$$

so that, obviously

$$f'(z) = \prod_{j=1}^n \left(\frac{D^k_{\alpha,\beta,\lambda,\delta} f_j(z)}{z} \right)^{\frac{1}{\lambda}} \tag{13}$$

and

$$f''(z) = \frac{1}{\lambda} \sum_{j=1}^n \left(\frac{D^k_{\alpha,\beta,\lambda,\delta} f_j(z)}{z} \right)^{\frac{(1-\lambda)}{\lambda}} \left(\frac{z \left(D^k_{\alpha,\beta,\lambda,\delta} f_j(z) \right)' - D^k_{\alpha,\beta,\lambda,\delta} f_j(z)}{z^2} \right)$$

$$\prod_{\substack{m=1 \\ (m \ne j)}}^n \left(\frac{D^k_{\alpha,\beta,\lambda,\delta} f_m(z)}{z} \right)^{\frac{1}{\lambda}}. \tag{14}$$

Thus from (13) and (14) we obtain

$$\frac{z f''(z)}{f'(z)} = \frac{1}{\lambda} \sum_{j=1}^n \left(\frac{z \left(D^k_{\alpha,\beta,\lambda,\delta} f_j(z) \right)'}{D^k_{\alpha,\beta,\lambda,\delta} f_j(z)} - 1 \right)$$

which readily shows that

$$\frac{1 - |z|^{2a}}{a} \left| \frac{z f''(z)}{f'(z)} \right| = \frac{1 - |z|^{2a}}{a} \frac{1}{\sqrt{a^2 + b^2}} \left| \sum_{j=1}^n \left(\frac{z \left(D^k_{\alpha,\beta,\lambda,\delta} f_j(z) \right)'}{D^k_{\alpha,\beta,\lambda,\delta} f_j(z)} - 1 \right) \right|$$

$$\le \frac{1 - |z|^{2a}}{a} \frac{1}{\sqrt{a^2 + b^2}} \sum_{j=1}^n \left| \frac{z \left(D^k_{\alpha,\beta,\lambda,\delta} f_j(z) \right)'}{D^k_{\alpha,\beta,\lambda,\delta} f_j(z)} - 1 \right|$$

$$\leq \frac{1-|z|^{2a}}{a}\frac{1}{\sqrt{a^2+b^2}}\sum_{j=1}^{n}\left(\left|\frac{z^2\left(D^k_{\alpha,\beta,\lambda,\delta}f_j(z)\right)'}{\left(D^k_{\alpha,\beta,\lambda,\delta}f_j(z)\right)^2}\right|\frac{\left|D^k_{\alpha,\beta,\lambda,\delta}f_j(z)\right|}{|z|}+1\right),\ \ (z\in U)\ .$$

Now, from the hypotheses of Theorem 5, we obtain

$$\left|D^k_{\alpha,\beta,\lambda,\delta}f_j(z)\right|\leq M\quad (z\in U,\ j=\{1,2,...,n\})$$

due to the General Schwarz Lemma, yields:

$$\left|D^k_{\alpha,\beta,\lambda,\delta}f_j(z)\right|\leq M|z|\quad (z\in U,\ j=\{1,2,...,n\}). \tag{15}$$

Therefore, by using the inequalities (6) and (15), we obtain the following inequality:

$$\frac{1-|z|^{2a}}{a}\left|\frac{zf''(z)}{f'(z)}\right|\leq\frac{1-|z|^{2a}}{a}\frac{(2M+1)n}{\sqrt{a^2+b^2}}\leq\frac{(2M+1)n}{a\sqrt{a^2+b^2}},\quad (z\in U).$$

Next, from (11) and (12), we have

$$\frac{1-|z|^{2a}}{a}\left|\frac{zf''(z)}{f'(z)}\right|\leq 1\quad (z\in U).$$

Finally, by applying Theorem 1, we conclude that when $\mu=1$, the function $F^k_{n,\lambda,\mu}$ given by (9) is in the univalent function class S in U. This evidently completes the proof of Theorem 5.

Taking $M=1$ in Theorem 5, we get the following:

Corollary 1. *Let each of the functions $f_j\in A$; $j=\{1,2,...,n\}$ and $D^k_{\alpha,\beta,\lambda,\delta}f_j(z)$ satisfies the inequality (6). Also let $\lambda=a+ib$, $(a,b\in R)$ be a complex number with the components a and b constrained by*

$$a\in\left(0,\sqrt{3n}\right]$$

and

$$a^4+a^2b^2-(3n)^2\geq 0.$$

If

$$\left|D^{k}_{\alpha,\beta,\lambda,\delta} f_j(z)\right| \le 1 \quad (z \in U,\ j = \{1,2,...,n\}),$$

then when $\mu = 1$, *the function* $F^{k}_{n,\lambda,\mu}(z)$ *defined by (9) is in the univalent functions class* S *in* U.

If we set $n = 1$ in Theorem 5, we can obtain the following:

Corollary 2. *Let* $M \ge 1$ *and suppose that* $f \in A$ *and* $D^{k}_{\alpha,\beta,\lambda,\delta} f(z)$ *satisfies the inequality (6). Also let* $\lambda = a + ib$, $(a, b \in R)$ *be a complex number with the components* a *and* b *constrained by*

$$a \in \left(0, \sqrt{2M+1}\right]$$

and

$$a^4 + a^2 b^2 - (2M+1)^2 \ge 0.$$

If

$$\left|D^{k}_{\alpha,\beta,\lambda,\delta} f(z)\right| \le M \quad (z \in U),$$

then the integral operator

$$F^{k}_{1,\lambda,1}(z) = \left[(a+ib)\int_0^z \left(\frac{D^{k}_{\alpha,\beta,\lambda,\delta} f(u)}{u}\right)^{\frac{1}{a+ib}} du\right]^{\frac{1}{a+ib}}$$

is in the univalent function class S *in* U.

Remark 2. When $k = 0$ in Corollary 2 provides an extension of Theorem 3 due to Pescar and Breaz [12].

Remark 3. If, in Theorem 5, we set $M = n = 1$, $k = 0$ again we obtain Theorem 3 due to Pescar and Breaz [12].

Univalence condition associated with generalized integral operator $F^{k}_{n,\lambda,\mu}$ when $\mu = 0$

Theorem 6. *Let* $M \ge 1$ *and suppose that each of the functions* $f_j \in A$, $j = \{1,2,...,n\}$ *satisfies the inequality (6). Also let* $\lambda = a + ib$, $(a, b \in R)$ *be a complex number with the components* a *and* b *constrained by*

$$a \in \left[\frac{(2M+1)n}{(2M+1)n+1}, \frac{(2M+1)n}{(2M+1)n-1}\right], \quad b \in \left[0, \frac{1}{\sqrt{[(2M+1)n]^2 - 1}}\right] \tag{16}$$

and

$$\left[(a-1)^2+b^2\right]\left[(2M+1)\,n\right]^2-a^2\leq 0. \tag{17}$$

If

$$\left|D^k_{\alpha,\beta,\lambda,\delta}f_j\left(z\right)\right|\leq\ M\quad(z\in\ U,\ j=\{1,2,...,n\}),$$

then when $\mu=0$, the function $F^k_{n,\lambda,\mu}$ defined by (9) is in the univalent function class S in U.

Proof: First of all when $\mu=0$ we recall from (9) that

$$F^k_{n,\lambda,\mu}\left(z\right)=\left(\left[n\left(\lambda-1\right)+1\right]\int\limits_0^z\prod_{j=1}^n\left(\frac{D^k_{\alpha,\beta,\lambda,\delta}f_j\left(u\right)}{u}\right)^{\lambda-1}u^{n(\lambda-1)}du\right)^{\frac{1}{[n(\lambda-1)+1]}},$$

for $f_j\in\ A;\ j=\{1,2,...,n\}$.

Let us now define the function $h\left(z\right)$ by

$$h\left(z\right)=\int\limits_0^z\prod_{j=1}^n\left(\frac{D^k_{\alpha,\beta,\lambda,\delta}f_j\left(u\right)}{u}\right)^{\lambda-1}du\quad\left(f_j\in\ A;\ j=\{1,2,...,n\}\right).$$

Then, since

$$h'\left(z\right)=\prod_{j=1}^n\left(\frac{D^k_{\alpha,\beta,\lambda,\delta}f_j\left(z\right)}{z}\right)^{\lambda-1}\quad\left(z\in\ U\right), \tag{18}$$

we see that $h\left(0\right)=0$ and $h'\left(0\right)=1$. Moreover, by noting that

$$h''\left(z\right)=\left(\lambda-1\right)\sum_{j=1}^n\left(\frac{D^k_{\alpha,\beta,\lambda,\delta}f_j\left(z\right)}{z}\right)^{\lambda-2}\left(\frac{z\left(D^k_{\alpha,\beta,\lambda,\delta}f_j\left(z\right)\right)'-D^k_{\alpha,\beta,\lambda,\delta}f_j\left(z\right)}{z^2}\right)$$

$$\prod_{\substack{m=1\\(m\neq j)}}^n\left(\frac{D^k_{\alpha,\beta,\lambda,\delta}f_m\left(z\right)}{z}\right)^{\lambda-1}, \tag{19}$$

we thus find from (18) and (19) that

$$\frac{zh''\left(z\right)}{h'\left(z\right)}=\left(\lambda-1\right)\sum_{j=1}^n\left(\frac{z\left(D^k_{\alpha,\beta,\lambda,\delta}f_j\left(z\right)\right)'}{D^k_{\alpha,\beta,\lambda,\delta}f_j\left(z\right)}-1\right)\quad\left(f_j\in\ A;\ j=\{1,2,...,n\}\right),$$

which readily shows that

$$\frac{1-|z|^{2a}}{a}\left|\frac{zh''(z)}{h'(z)}\right| = \frac{1-|z|^{2a}}{a}|\lambda-1|\left|\sum_{j=1}^{n}\left(\frac{z\left(D^k_{\alpha,\beta,\lambda,\delta}f_j(z)\right)'}{D^k_{\alpha,\beta,\lambda,\delta}f_j(z)}-1\right)\right|$$

$$\leq \frac{1-|z|^{2a}}{a}\sqrt{(a-1)^2+b^2}\sum_{j=1}^{n}\left|\frac{z\left(D^k_{\alpha,\beta,\lambda,\delta}f_j(z)\right)'}{D^k_{\alpha,\beta,\lambda,\delta}f_j(z)}-1\right|$$

$$\leq \frac{1-|z|^{2a}}{a}\sqrt{(a-1)^2+b^2}\sum_{j=1}^{n}\left(\left|\frac{z^2\left(D^k_{\alpha,\beta,\lambda,\delta}f_j(z)\right)'}{\left(D^k_{\alpha,\beta,\lambda,\delta}f_j(z)\right)^2}\right|\frac{\left|D^k_{\alpha,\beta,\lambda,\delta}f_j(z)\right|}{|z|}+1\right)$$

Therefore, by using the inequalities (6) and (15), we obtain

$$\frac{1-|z|^{2a}}{a}\left|\frac{zh''(z)}{h'(z)}\right| \leq \frac{1-|z|^{2a}}{a}(2M+1)\,n\sqrt{(a-1)^2+b^2}$$

$$\leq \frac{(2M+1)\,n\sqrt{(a-1)^2+b^2}}{a}$$

Now it follows from (16) and (17) that

$$\frac{1-|z|^{2a}}{a}\left|\frac{zh''(z)}{h'(z)}\right| \leq 1 \quad (z\in U).$$

Finally, by applying Theorem 1 for the function $h(z)$, we conclude that the function $F^k_{n,\lambda,\mu}$ defined by (9) is in the univalent function class S in U for the case $\mu=0$.

Next, taking $M=1$ in Theorem 6, we get the following:

Corollary 3. *Let each of the functions $f_j \in A$; $j=\{1,2,...,n\}$ and $D^k_{\alpha,\beta,\lambda,\delta}f_j(z)$ satisfies the inequality (6). Also let $\lambda = a+ib$, $(a,b\in R)$ be a complex number with the components a and b constrained by*

$$a\in\left[\frac{3n}{3n+1},\frac{3n}{3n-1}\right],\quad b\in\left[0,\frac{1}{\sqrt{9n^2-1}}\right]$$

and

$$9\left[(a-1)^2+b^2\right]n^2-a^2\leq 0.$$

If

$$\left|D^k_{\alpha,\beta,\lambda,\delta}f_j(z)\right|\leq 1 \quad (z\in U,\ j=\{1,2,...,n\}),$$

then the function $F^k_{n,\lambda,\mu}(z)$ *defined by (9) is in the univalent function class* S *in* U *for* $\mu=0$.

If we take $n=1$ in Theorem 6, we can have the following:

Corollary 4. *Let* $M\geq 1$ *and suppose that* $f\in A$ *and* $D^k_{\alpha,\beta,\lambda,\delta}f(z)$ *satisfies the inequality (6). Also let* $\lambda=a+ib$, $(a,b\in R)$ *be a complex number with the components* a *and* b *constrained by*

$$a\in\left[\frac{2M+1}{2M+2},\frac{2M+1}{2M}\right],\quad b\in\left[0,\frac{1}{2\sqrt{M(M+1)}}\right]$$

and

$$\left[(a-1)^2+b^2\right](2M+1)^2-a^2\leq 0.$$

If

$$\left|D^k_{\alpha,\beta,\lambda,\delta}f(z)\right|\leq M \quad (z\in U),$$

then the integral operator

$$F^k_{1,\lambda,0}(z)=\left[(a+ib)\int_0^z\left(D^k_{\alpha,\beta,\lambda,\delta}f(u)\right)^{a+ib-1}\right]^{\frac{1}{a+ib}}$$

is in the univalent function class S *in* U.

Remark 4. When $k=0$ in Corollary 3, reduce to Theorem 4 due to Pescar and Breaz [12].

Remark 5. If, in Theorem 6, we set $M=n=1$, $k=0$ again we obtain Theorem 4 due to Pescar and Breaz [12].

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