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UNSTEADY FLOW OF A DUSTY FLUID THROUGH AN INCLINED OPEN RECTANGULAR CHANNEL

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Abstract. An analytical study is made on unsteady flow of a dusty fluid through an inclined open rectangular channel. The flow is due to the influence of time dependent pressure gradients i.e., impulsive, transition and motion for a finite time is considered along with the effect of the movement of the plates and the effect of uniform magnetic field. Flow analysis is carried out in Frenet frame field system and exact solutions of the problem are obtained by solving the partial differential equations using Variable Separable and Laplace transform methods. Further graphs drawn for different values of inclined angle and on basis of these the conclusions are given. Finally, the expressions for skin-friction at the boundaries are obtained.

2000 *Mathematics Subject Classification*: 76T10, 76T15.

1. Introduction

The study of fluid flow and dust particles through porous media and through permeable bed has been made by many mathematicians. Using the equations given by P.G.Saffman [17], several authors have developed special problems under various assumptions. This study has proved to be useful in the movement of dust, laden air and the use of dust in the cooling of dust particles on viscous flows, Besides, it has a great importance in petroleum industry and in the purification of crude oil. Other important application of such flows is the dust entrainment in a cloud during nuclear explosion.

Some researchers like Liu [14], Michael and Miller [15], Ghosh [19], Chamkha [8], Amos [1], Datta [9], Agrawal and Varshney [2], Saxena and Sharma have studied various problems under different initial and boundary conditions. Shri Ram, B.K.Gupta and N.P.Singh [18] have studied unsteady flow of a dusty viscous stratified fluid through an inclined open rectangular channel.

To investigate the kinematical properties of fluid flows in the field of fluid mechanics some researchers like Kanwal [13], Truesdell [20], Indrasena [12], Purushotham [16], Bagewadi and Gireesha [3][4] have applied differential geometry techniques. Further, recently the authors [10][11] have studied the flow of unsteady dusty fluid under varying different pressure gradients like constant, periodic and exponential.

In the present paper, laminar flow of an unsteady, electrically conducting, incompressible fluid with embedded non-conducting identical spherical particles through a long open rectangular channel under the influence of magnetic field and a time varying pressure gradient. Further by considering the fluid and dust particles to be at rest initially, the exact solutions are obtained for velocities of fluid and dust particles and also the skin friction at the boundary is calculated. The effect of inclined angle on the velocities of fluid and dust are shown graphically.

2.Equations of Motion

The equations of motion of unsteady viscous incompressible fluid with uniform distribution of dust particles are given by [17]:

For fluid phase

$$\nabla \cdot \overrightarrow{u} = 0, \qquad \text{(Continuity)} \tag{1}$$

$$\begin{aligned} \frac{\partial \overrightarrow{u}}{\partial t} &+ (\overrightarrow{u} \cdot \nabla)\overrightarrow{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \overrightarrow{u} \\ &+ \frac{kN}{\rho}(\overrightarrow{v} - \overrightarrow{u}) + g\sin\gamma - \frac{\sigma B^2}{\rho}\overrightarrow{u} \qquad \text{(Linear Momentum)} \end{aligned} \tag{2}$$

For dust phase

$$\nabla \cdot \overrightarrow{v} = 0, \qquad \text{(Continuity)} \tag{3}$$

$$\frac{\partial \overrightarrow{v}}{\partial t} + (\overrightarrow{v} \cdot \nabla)\overrightarrow{v} = \frac{k}{m}(\overrightarrow{u} - \overrightarrow{v}) \qquad \text{(Linear Momentum)} \tag{4}$$

We have following nomenclature:

$\overrightarrow{u}$−velocity of the fluid phase, $\overrightarrow{v}$−velocity of dust phase, ρ−density of the gas, p−pressure of the fluid, N−number density of dust particles, ν−kinematic viscosity, $k = 6\pi a\mu$−Stoke's resistance (drag coefficient), a−spherical radius of dust particle, m−mass of the dust particle, μ−the co-efficient of viscosity of fluid particles, $t-$time, $g-$ the acceleration due to gravity, $\sigma-$ is the electrical conductivity, $B-$ is variable electromagnetic induction, $\gamma-$ is inclined angle.

Let $\overrightarrow{s}, \overrightarrow{n}, \overrightarrow{b}$ be triply orthogonal unit vectors tangent, principal normal, binormal respectively to the spatial curves of congruences formed by fluid phase velocity and dusty phase velocity lines respectively as shown in the figure-1.

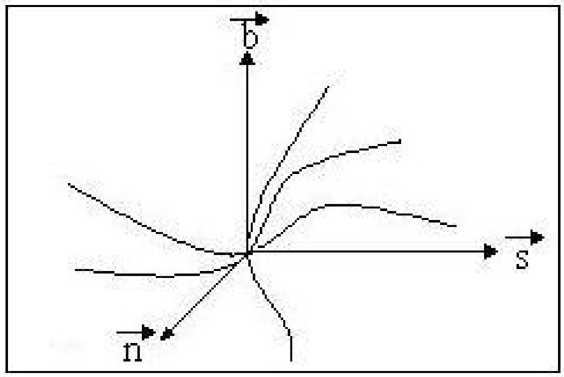

Figure-1: Frenet Frame Field System

Geometrical relations are given by Frenet formulae [6]

$$\begin{aligned}
&i) \quad \frac{\partial \overrightarrow{s}}{\partial s} = k_s \overrightarrow{n}, \ \frac{\partial \overrightarrow{n}}{\partial s} = \tau_s \overrightarrow{b} - k_s \overrightarrow{s}, \ \frac{\partial \overrightarrow{b}}{\partial s} = -\tau_s \overrightarrow{n} \\
&ii) \quad \frac{\partial \overrightarrow{n}}{\partial n} = k'_n \overrightarrow{s}, \ \frac{\partial \overrightarrow{b}}{\partial n} = -\sigma'_n \overrightarrow{s}, \ \frac{\partial \overrightarrow{s}}{\partial n} = \sigma'_n \overrightarrow{b} - k'_n \overrightarrow{n} \\
&iii) \quad \frac{\partial \overrightarrow{b}}{\partial b} = k''_b \overrightarrow{s}, \ \frac{\partial \overrightarrow{n}}{\partial b} = -\sigma''_b \overrightarrow{s}, \ \frac{\partial \overrightarrow{s}}{\partial b} = \sigma''_b \overrightarrow{n} - k''_b \overrightarrow{b} \\
&iv) \quad \nabla.\overrightarrow{s} = \theta_{ns} + \theta_{bs}; \ \nabla.\overrightarrow{n} = \theta_{bn} - k_s; \ \nabla.\overrightarrow{b} = \theta_{nb} \qquad (5)
\end{aligned}$$

where $\partial/\partial s$, $\partial/\partial n$ and $\partial/\partial b$ are the intrinsic differential operators along fluid phase velocity (or dust phase velocity) lines, principal normal and binormal. The functions (k_s, k'_n, k''_b) and $(\tau_s, \sigma'_n, \sigma''_b)$ are the curvatures and torsions of the above curves and θ_{ns} and θ_{bs} are normal deformations of these spatial curves along their principal normal and binormal respectively.

3.Formulation and Solution of the Problem

Let us consider an unsteady laminar flow of an incompressible, Newtonian, electrically conducting dusty fluid. The fluid is flowing down in an open inclined channel, the walls of the channel being normal to the surface of the bottom. The bottom is assumed to be inclined at an angle γ $(0 < \gamma < \pi/2)$ to the horizontal. It is assumed that the binormal direction $\overrightarrow{b}$ is along the central line in the direction of flow of fluid at the free surface, and $\overrightarrow{n}$ along the depth and $\overrightarrow{s}$ is along the width of the channel as shown in the figure-2.

The flow is due to the influence of pressure gradient varying with time. Both the fluid and the dust particle clouds are supposed to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size. The number density of the dust particles is taken as a constant throughout the flow. The velocity components of both fluid and dust particles are respectively given by:

$$\left\{ \begin{array}{ll} u_s = 0; & u_n = 0; \\ v_s = 0; & v_n = 0; \end{array} \right\} \tag{6}$$

where (u_s, u_n, u_b) and (v_s, v_n, v_b) are velocity components of fluid and dust particles respectively.

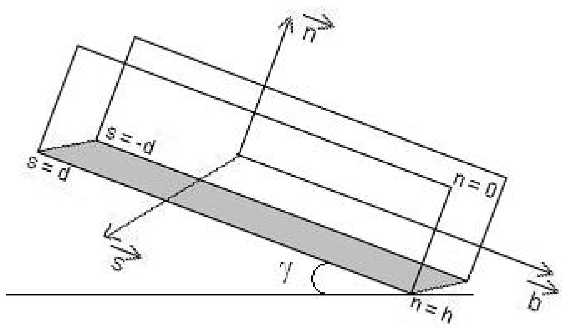

Figure 2: Schematic diagram of dusty fluid flow in a rectangular channel.

By virtue of system of equations (5) the intrinsic decomposition of equations (2) and (4) using equation (6) give the following forms:

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial s} + \nu\left(\tau_s k_s u_b - 2\sigma'_n \frac{\partial u_b}{\partial n}\right) \tag{7}$$

$$\begin{aligned} 0 &= -\frac{1}{\rho}\frac{\partial p}{\partial n} + \nu\left(\sigma'_n k'_n u_b + k''_b \sigma''_b u_b - 2\tau_s \frac{\partial u_b}{\partial s}\right) \\ \frac{\partial u_b}{\partial t} &= -\frac{1}{\rho}\frac{\partial p}{\partial b} + \nu\left[\frac{\partial^2 u_b}{\partial s^2} + \frac{\partial^2 u_b}{\partial n^2} - C_r u_b\right] + \frac{kN}{\rho}(v_b - u_b) \end{aligned} \tag{8}$$

$$
\begin{aligned}
&+ \quad g\sin\gamma - \frac{\sigma B^2}{\rho}u_b && (9)\\
\frac{\partial v_b}{\partial t} &= \frac{k}{m}(u_b - v_b) && (10)\\
v_b^2 k_b'' &= 0 && (11)
\end{aligned}
$$

where $C_r = (\tau_s^2 + \sigma_n'^2 + k''^2{}_b)$ is called curvature number [5].

From equation (11) we see that $v_b^2 k_b'' = 0$ which implies either $v_b = 0$ or $k_b'' = 0$. The choice $v_b = 0$ is impossible, since if it happens then $u_b = 0$, which shows that the flow doesn't exist. Hence $k_b'' = 0$, it suggests that the curvature of the streamline along binormal direction is zero. Thus no radial flow exists.

The flow in the porous media is governed by the Darcy's equation,

$$Q = \frac{K_0}{\mu}\left(-\frac{\partial p}{\partial b} + \rho\, g\sin\gamma\right) \tag{12}$$

where Q is the velocity in the porous media and K_0 is the variable permeability of the medium.

The condition at the interface of the free flow region and porous medium, following Beavers and Joseph [7] is given by

$$
\begin{aligned}
\left(\frac{\partial u_s}{\partial n}\right)_{n=h} &= \frac{-\alpha}{\sqrt{K}}(u_s - Q)\\
\left(\frac{\partial v_s}{\partial n}\right)_{n=h} &= \frac{-\alpha}{\sqrt{K}}(v_s - Q)
\end{aligned}
$$

From equations (9) and (12),we have

$$\frac{\partial u_b}{\partial t} = \frac{Q\mu}{\rho K_0} + \nu\left[\frac{\partial^2 u_b}{\partial s^2} + \frac{\partial^2 u_b}{\partial n^2} - C_r u_b\right] + \frac{kN}{\rho}(v_b - u_b) - \frac{\sigma B^2}{\rho}u_b \tag{13}$$

By defining the depth of the channel h as the characteristic length and the mean flow velocity U_0 as the characteristic velocity. We introduce the following non-dimensional quantities.

$$
\begin{aligned}
s^* &= \frac{s}{h},\quad n^* = \frac{n}{h},\quad b^* = \frac{b}{h},\quad u_b^* = \frac{\mu M^2 u_b}{U_0},\quad v_b^* = \frac{\mu M^2 v_b}{U_0},\quad Q^* = \frac{\mu M^2 Q}{U_0}\\
k_0^* &= \frac{k_0}{h^2},\quad t_0^* = \frac{\mu t}{\rho h^2},\quad u_1^* = \frac{\rho M^2 h^3 u_1}{U_0},\quad u_0^* = \frac{\rho M^2 h^2 u_0}{U_0},\quad T^* = \frac{\mu T}{\rho h^2}
\end{aligned}
$$

The equations (10) and (13)transformed to(after dropping the asterisks over them)

$$\frac{\partial u_b}{\partial t} = P + \nu\left[\frac{\partial^2 u_b}{\partial s^2} + \frac{\partial^2 u_b}{\partial n^2} - C_r u_b\right] + \frac{l}{\omega}(v_b - u_b) - M^2 u_b \qquad (14)$$

$$\frac{\partial v_b}{\partial t} = \frac{1}{\omega}(v_b - u_b) \qquad (15)$$

where $M^2 = \frac{\sigma B^2 h^2}{\mu}$, $P = \frac{M^2 h^2}{U_0}\left[-\frac{\partial p}{\partial b} + \rho g \sin\gamma\right]$, $l = \frac{mN}{\rho}$ and $\omega = \frac{m\mu}{kh^2\rho}$.

CASE-1. Impulsive Motion:

Consider the case of impulsive motion, in which the boundary conditions are

$$\begin{aligned} P(t) &= P_0 \ \delta(t) \\ u_b &= 0 \qquad at \quad s = \pm d \\ u_b &= u_0 \ \delta(t) \quad at \quad n = h \\ \frac{\partial u_b}{\partial n} &= u_1 \ \delta(t) \quad at \quad n = 0 \end{aligned}$$

where $\delta(t)$ is the Dirac delta function and p_0, u_0 and u_1 are constants. After non dimensionalizing,

$$\begin{aligned} P(t) &= P_0 \ \delta(t) \\ u_b &= 0 \qquad at \quad s = \pm r \\ u_b &= u_0 \ \delta(t) \quad at \quad n = 1 \\ \frac{\partial u_b}{\partial n} &= u_1 \ \delta(t) \quad at \quad n = 0 \quad where \quad r = \frac{d}{h} \end{aligned}$$

Let U_b and V_b are given by

$$U_b = \int_0^\infty e^{-xt} \ u_b \ dt \quad \text{and} \quad V_b = \int_0^\infty e^{-xt} \ v_b \ dt \qquad (16)$$

denote the Laplace transforms of u_b and v_b respectively.
Then (14) and (15) becomes,

$$xU_b = P(x) + \nu\left(\frac{\partial^2 U_b}{\partial s^2} + \frac{\partial^2 U_b}{\partial n^2} - C_r U_b\right) + \frac{l}{\omega}(V_b - U_b) - M^2 U_b \quad (17)$$

$$V_b = \frac{U_b}{(1 + x\omega)} \qquad (18)$$

and boundary conditions are,

$$\begin{aligned} P(x) &= P_0/x \\ U_b &= 0 \ \text{ at } \ s = \pm r \\ U_b &= u_0 \ \text{ at } \ n = 1 \\ \frac{\partial U_b}{\partial n} &= u_1 \ \text{ at } \ n = 0 \end{aligned} \tag{19}$$

From equations (17) and (18) we obtain, the following equation

$$\frac{\partial^2 U_b}{\partial s^2} + \frac{\partial^2 U_b}{\partial n^2} - q^2 U_b + R = 0 \tag{20}$$

where

$$q^2 = \left(C_r + M^2 + \frac{x}{\nu} + \frac{xl}{\nu(1 + x\omega)} \right), \quad R = \frac{I}{\nu x} \ \ \textit{and} \quad I = \frac{M^2 h^2}{u_0} (p_0 + \rho g \sin\gamma)$$

To solve equation (17) we assume the solution in the following form

$$U_b(s, n) = w_1(s, n) + w_2(s) \tag{21}$$

Substitution of $U_b(s, n)$ in equation (20) yields

$$\frac{\partial^2 w_1}{\partial s^2} + \frac{\partial^2 w_2}{\partial s^2} + \frac{\partial^2 w_1}{\partial n^2} - q^2(w_1 + w_2) + R = 0$$

so that if w_2 satisfies

$$\frac{\partial^2 w_2}{\partial s^2} - q^2 w_2 + R = 0$$

then

$$\frac{\partial^2 w_1}{\partial s^2} + \frac{\partial^2 w_1}{\partial n^2} - q^2 w_1 = 0 \tag{22}$$

In similar manner if $U_b(s, n)$ is inserted in no slip boundary conditions, one can obtain

$$\left\{ \begin{array}{l} U_b(r, n) = w_1(r, n) + w_2(r) = 0, \ U_b(-r, n) = w_1(-r, n) + w_2(-r) = 0, \\ U_b(s, 1) = w_1(s, 1) + w_2(s) = u_0, \ \ \frac{\partial U_b}{\partial n}(s, 0) = \frac{\partial w_1}{\partial n}(s, 0) = u_1 \end{array} \right\}$$

By solving the problem

$$\frac{\partial^2 w_2}{\partial s^2} - q^2 w_2 + R = 0, \quad w_2(r) = 0, \quad w_2(-r) = 0$$

we obtain the solution in the form

$$w_2(s) = \frac{R}{q^2}\left(\frac{\cosh(qr) - \cos(qs)}{\cos(qr)}\right) \tag{23}$$

Using variable separable method, the solution of the problem (22) with the conditions

$$w_1(r,n) = 0, \quad w_1(-r,n) = 0, \quad w_1(s,1) = u_0 - w_2(s), \quad \frac{\partial w_1}{\partial n}(s,0) = u_1$$

is obtained in the form

$$w_1(s,n) \;=\; \sum_{r_1=0}^{\infty} \sin\left(\frac{r_1\pi}{r}s\right)\left(c_{r_1}e^{An} + D_{r_1}e^{-An}\right) \tag{24}$$

where $A = \sqrt{\frac{q^2r^2+r_1^2\pi^2}{r^2}}$

Now by substituting (23) and (24) in (21) we have

$$\begin{aligned} U_b(s,n) \;=\;& \frac{R}{q^2}\left(\frac{\cosh(qr) - \cosh(qs)}{\cosh(qr)}\right) + \frac{2R}{q^2}\sum_{r_1=0}^{\infty} \sin\left(\frac{r_1\pi}{r}s\right) \\ \times\;& \left\{\frac{(-1)^{r_1}q^2}{A^2 r_1\pi} + \frac{r_1\pi}{A^2r^2\cosh(qr)} - \frac{1}{r_1\pi}\right\}\frac{\cosh(An)}{\cosh(A)} \\ +\;& \frac{2u_0}{\pi}\sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\frac{\cosh(An)}{\cosh(A)} \\ +\;& \frac{2u_1}{\pi}\sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\frac{\sinh[(n-1)A]}{A\cosh(A)} \end{aligned}$$

Using U_b in equation (20) one can see that

$$\begin{aligned} V_b(s,n) \;=\;& \frac{R}{q^2(1+x\omega)}\left(\frac{\cosh(qr) - \cosh(qs)}{\cosh(qr)}\right) + \frac{2R}{q^2(1+x\omega)}\sum_{r_1=0}^{\infty} \sin\left(\frac{r_1\pi}{r}s\right) \\ \times\;& \left\{\frac{(-1)^{r_1}q^2}{A^2 r_1\pi} + \frac{r_1\pi(1+x\omega)}{A^2r^2\cosh(qr)} - \frac{1}{r_1\pi}\right\}\frac{\cosh(An)}{\cosh(A)} \end{aligned}$$

$$
\begin{aligned}
&+ \quad \frac{2u_0}{\pi(1+x\omega)}\sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\frac{\cosh(An)}{\cosh(A)}\\
&+ \quad \frac{2u_1}{\pi(1+x\omega)}\sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\frac{\sinh[(n-1)A]}{A\cosh(A)}
\end{aligned}
$$

By taking inverse Laplace transformation to U_b and V_b, we obtain u_b and v_b as follows:

$$
\begin{aligned}
u_b(s,n,t) &= \frac{I}{\nu}\left\{\left[\frac{\cosh(Xr)-\cosh(Xs)}{X^2\cosh(Xr)}\right]+\frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\right.\\
&\times \left.\cos\left[\frac{(2r_2+1)\pi}{2r}s\right]\left[\frac{e^{x_3t}(1+x_3\omega)^2)}{x_3[l+(1+x_3\omega)^2]}+\frac{e^{x_4t}(1+x_4\omega)^2}{x_4[l+(1+x_4\omega)^2]}\right]\right\}\\
&+ \frac{2I}{\nu\pi}\sum_{r_1=0}^{\infty}\frac{(-1)^{r_1}}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\left\{\frac{\cosh(Yn)}{Y^2\cosh(Y)}-\frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\right.\\
&\times \left.\cos\left[\frac{(2r_2+1)\pi}{2}n\right]\left[\frac{e^{x_7t}(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8t}(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right]\right\}\\
&+ \frac{2\pi I}{\nu r^2}\sum_{r_1=0}^{\infty}r_1\sin\left(\frac{r_1\pi}{r}s\right)\left\{\frac{\cosh(Yn)}{X^2Y^2\cosh(Y)\cosh(Xr)}\right.\\
&- \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{\cosh(\beta n)}{\beta^2\cosh(\beta)}\left[\frac{e^{x_3t}(1+x_3\omega)^2}{x_3[l+(1+x_3\omega)^2]}\right.\\
&+ \left.\frac{e^{x_4t}(1+x_4\omega)^2}{x_4[l+(1+x_4\omega)^2]}\right]-\frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\cos\left[\frac{(2r_2+1)\pi}{2}n\right]\\
&\times \left.\alpha_1^2\cos(\alpha_1 r)\left[\frac{e^{x_7t}(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8t}(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right]\right\}\\
&- \frac{2I}{\nu\pi}\sum_{r_1=0}^{\infty}\frac{1}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\left\{\frac{\cosh(Yn)}{X^2\cosh(Y)}+\nu\pi\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{\alpha_1^2}(2r_2+1)\right.\\
&\times \left.\cos\left[\frac{(2r_2+1)\pi}{2}n\right]\left[\frac{e^{x_7t}(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8t}(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right]\right\}\\
&+ 2u_0\nu\sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\sum_{r_2=0}^{\infty}(-1)^{r_2}(2r_2+1)\\
&\times \cos\left[\frac{(2r_2+1)\pi}{2}n\right]\left[\frac{e^{x_7t}(1+x_7\omega)^2}{[l+(1+x_7\omega)^2]}+\frac{e^{x_8t}(1+x_8\omega)^2}{[l+(1+x_8\omega)^2]}\right]
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{4u_1\nu}{\pi}\sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\sum_{r_2=0}^{\infty}(-1)^{r_2} \\
&\times \quad \sin\left[\frac{(2r_2+1)\pi}{2}(n-1)\right]\left[\frac{e^{x_7t}(1+x_7\omega)^2}{[l+(1+x_7\omega)^2]}+\frac{e^{x_8t}(1+x_8\omega)^2}{[l+(1+x_8\omega)^2]}\right]
\end{aligned}
$$

$$
\begin{aligned}
v_b(s,n,t) \quad &= \quad \frac{I}{\nu}\left\{\left[\frac{\cosh(Xr)-\cosh(Xs)}{X^2\cosh(Xr)}\right]+\frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\right. \\
&\times \quad \left.\cos\left[\frac{(2r_2+1)\pi}{2r}s\right]\left[\frac{e^{x_3t}(1+x_3\omega))}{x_3[l+(1+x_3\omega)^2]}+\frac{e^{x_4t}(1+x_4\omega)}{x_4[l+(1+x_4\omega)^2]}\right]\right\} \\
&+ \quad \frac{2I}{\nu\pi}\sum_{r_1=0}^{\infty}\frac{(-1)^{r_1}}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\left\{\frac{\cosh(Yn)}{Y^2\cosh(Y)}-\frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\right. \\
&\times \quad \left.\cos\left[\frac{(2r_2+1)\pi}{2}n\right]\left[\frac{e^{x_7t}(1+x_7\omega)}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8t}(1+x_8\omega)}{x_8[l+(1+x_8\omega)^2]}\right]\right\} \\
&+ \quad \frac{2\pi I}{\nu r^2}\sum_{r_1=0}^{\infty}r_1\sin\left(\frac{r_1\pi}{r}s\right)\left\{\frac{\cosh(Yn)}{X^2Y^2\cosh(Y)\cosh(Xr)}\right. \\
&- \quad \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{\cosh(\beta n)}{\beta^2\cosh(\beta)}\left[\frac{e^{x_3t}(1+x_3\omega)}{x_3[l+(1+x_3\omega)^2]}\right. \\
&+ \quad \left.\frac{e^{x_4t}(1+x_4\omega)}{x_4[l+(1+x_4\omega)^2]}\right]-\frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\cos\left[\frac{(2r_2+1)\pi}{2}n\right] \\
&\times \quad \left.\alpha_1^2\cos(\alpha_1 r)\left[\frac{e^{x_7t}(1+x_7\omega)}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8t}(1+x_8\omega)}{x_8[l+(1+x_8\omega)^2]}\right]\right\} \\
&- \quad \frac{2I}{\nu\pi}\sum_{r_1=0}^{\infty}\frac{1}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\left\{\frac{\cosh(Yn)}{X^2\cosh(Y)}+\nu\pi\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{\alpha_1^2}(2r_2+1)\right. \\
&\times \quad \left.\cos\left[\frac{(2r_2+1)\pi}{2}n\right]\left[\frac{e^{x_7t}(1+x_7\omega)}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8t}(1+x_8\omega)}{x_8[l+(1+x_8\omega)^2]}\right]\right\} \\
&+ \quad 2u_0\nu\sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\sum_{r_2=0}^{\infty}(-1)^{r_2}(2r_2+1) \\
&\times \quad \cos\left[\frac{(2r_2+1)\pi}{2}n\right]\left[\frac{e^{x_7t}(1+x_7\omega)}{[l+(1+x_7\omega)^2]}+\frac{e^{x_8t}(1+x_8\omega)}{[l+(1+x_8\omega)^2]}\right] \\
&+ \quad \frac{4u_1\nu}{\pi}\sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\sum_{r_2=0}^{\infty}(-1)^{r_2}
\end{aligned}
$$

$$\times \quad \sin\left[\frac{(2r_2+1)\pi}{2}(n-1)\right]\left[\frac{e^{x_7 t}(1+x_7\omega)}{[l+(1+x_7\omega)^2]}+\frac{e^{x_8 t}(1+x_8\omega)}{[l+(1+x_8\omega)^2]}\right]$$

Shearing Stress (Skin Friction):
The Shear stress at the boundaries $s = r$, $s = -r$ and $n = 0$, $n = 1$ are given by

$$\begin{aligned}
D_{rn} &= \frac{I\mu}{\nu}\frac{\sin h(Xr)}{\cos h(Xr)} + \frac{2I\mu}{r}\sum_{r_2=0}^{\infty}\left[\frac{e^{x_3 t}(1+x_3\omega)^2}{x_3[l+(1+x_3\omega)^2]}+\frac{e^{x_4 t}(1+x_4\omega)^2}{x_4[l+(1+x_4\omega)^2]}\right] \\
&- \frac{2I\mu}{\nu}\left\{\frac{\sin h(Yn)}{Y\cos h(Y)} - \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\cos\left[\frac{(2r_2+1)\pi}{2}n\right]\right. \\
&\times \left.\left[\frac{e^{x_7 t}(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8 t}(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right]\right\} - \frac{2I\mu\pi^2}{r^3\nu}\sum_{r_1=0}^{\infty} r_1^2(-1)^{r_1} \\
&\times \left\{\frac{\cos h(Yn)}{X^2Y^2\cos h(Y)\cos h(Xr)} - \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{\cosh(\beta n)}{\beta^2\cosh(\beta)}\right. \\
&\times \left[\frac{e^{x_3 t}(1+x_3\omega)^2}{x_3[l+(1+x_3\omega)^2]}+\frac{e^{x_4 t}(1+x_4\omega)^2}{x_4[l+(1+x_4\omega)^2]}\right] - \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)} \\
&\times \left.\frac{\cos\left[\frac{(2r_2+1)\pi}{2}n\right]}{\alpha_1^2\cos(\alpha_1 r)}\left[\frac{e^{x_7 t}(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8 t}(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right]\right\} \\
&+ \frac{2I\mu}{r\nu}\sum_{r_1=0}^{\infty}(-1)^{r_1}\left\{\frac{\cos h(Yn)}{X^2\cos h(Y)} - \nu\pi\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{\alpha_1^2}(2r_2+1)\right. \\
&\times \left.\cos\left[\frac{(2r_2+1)\pi}{2}n\right]\left[\frac{e^{x_7 t}(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8 t}(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right]\right\} \\
&- \frac{2u_0\nu\mu\pi}{r}\sum_{r_1=0}^{\infty}[(-1)^{r_1}-1]\sum_{r_2=0}^{\infty}(-1)^{r_2}(2r_2+1)\cos\left[\frac{(2r_2+1)\pi}{2}n\right] \\
&\times \left[\frac{e^{x_7 t}(1+x_7\omega)^2}{[l+(1+x_7\omega)^2]}+\frac{e^{x_8 t}(1+x_8\omega)^2}{[l+(1+x_8\omega)^2]}\right] - \frac{4u_1\nu\mu}{r} \\
&\times \sum_{r_1=0}^{\infty}[(-1)^{r_1}-1]\sum_{r_2=0}^{\infty}(-1)^{r_2}\sin\left[\frac{(2r_2+1)\pi}{2}(n-1)\right] \\
&\times \left[\frac{e^{x_7 t}(1+x_7\omega)^2}{[l+(1+x_7\omega)^2]}+\frac{e^{x_8 t}(1+x_8\omega)^2}{[l+(1+x_8\omega)^2]}\right]
\end{aligned}$$

$$
\begin{aligned}
D_{-rn} &= -D_{rn} \\
D_{s0} &= \frac{I\mu}{\nu}\frac{\sin h(Xs)}{\cos h(Xr)} + \frac{2I\mu}{r}\sum_{r_2=0}^{\infty}(-1)^{r_2}\sin\left[\frac{(2r_2+1)\pi}{2r}s\right] \\
&\times \left[\frac{e^{x_3t}(1+x_3\omega)^2}{x_3[l+(1+x_3\omega)^2]} + \frac{e^{x_4t}(1+x_4\omega)^2}{x_4[l+(1+x_4\omega)^2]}\right] + \frac{8I\mu}{r\pi}\sum_{r_1=0}^{\infty}(-1)^{r_1} \\
&\times \cos\left[\frac{r_1\pi}{r}s\right]\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\left[\frac{e^{x_7t}(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]} + \frac{e^{x_8t}(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right] \\
&- \frac{2I\pi^2\mu}{r^3\nu}\sum_{r_1=0}^{\infty}r_1^2\cos\left[\frac{r_1\pi}{r}s\right]\left\{\frac{1}{X^2Y^2\cos h(Y)\cos h(Xr)} - \frac{4\nu}{\pi}\right. \\
&\times \sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{1}{\beta^2\cosh(\beta)}\left[\frac{e^{x_3t}(1+x_3\omega)^2}{x_3[l+(1+x_3\omega)^2]} + \frac{e^{x_4t}(1+x_4\omega)^2}{x_4[l+(1+x_4\omega)^2]}\right] \\
&- \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{1}{\alpha_1^2\cos(\alpha_1 r)}\left[\frac{e^{x_7t}(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}\right. \\
&+ \left.\left.\frac{e^{x_8t}(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right]\right\} + \frac{2I\mu}{r\nu}\sum_{r_1=0}^{\infty}\cos\left[\frac{r_1\pi}{r}s\right]\left\{\frac{1}{X^2\cos h(Y)}\right. \\
&- \left.\nu\pi\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{\alpha_1^2}(2r_2+1)\left[\frac{e^{x_7t}(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]} + \frac{e^{x_8t}(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right]\right\} \\
&- \frac{2u_0\pi\nu\mu}{r}\sum_{r_1=0}^{\infty}[1-(-1)^{r_1}]\cos\left[\frac{r_1\pi}{r}s\right]\sum_{r_2=0}^{\infty}(-1)^{r_2}(2r_2+1) \\
&\times \left[\frac{e^{x_7t}(1+x_7\omega)^2}{[l+(1+x_7\omega)^2]} + \frac{e^{x_8t}(1+x_8\omega)^2}{[l+(1+x_8\omega)^2]}\right] + \frac{4u_1\nu\mu}{r}\sum_{r_1=0}^{\infty}[1-(-1)^{r_1}] \\
&\times \cos\left[\frac{r_1\pi}{r}s\right]\left[\frac{e^{x_7t}(1+x_7\omega)^2}{[l+(1+x_7\omega)^2]} + \frac{e^{x_8t}(1+x_8\omega)^2}{[l+(1+x_8\omega)^2]}\right] \\
D_{s1} &= \frac{2I\mu}{\pi\nu}\sum_{r_1=0}^{\infty}\frac{(-1)^{r_1}}{r_1}\sin\left[\frac{r_1\pi}{r}s\right]\left\{\frac{\sin h(Y)}{Y\cos h(Y)} + 2\nu\left[\frac{e^{x_7t}(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}\right.\right. \\
&+ \left.\left.\frac{e^{x_8t}(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right]\right\} + \frac{2I\mu\pi}{r^2\nu}\sum_{r_1=0}^{\infty}r_1\sin\left(\frac{r_1\pi}{r}s\right) \\
&\times \left\{\frac{\sin h(Y)}{X^2Y\cos h(Y)\cos h(Xr)} - \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{\sinh(\beta)}{\beta\cosh(\beta)}\right.
\end{aligned}
$$

$$
\begin{aligned}
\times & \left[\frac{e^{x_3 t}(1+x_3\omega)^2}{x_3[l+(1+x_3\omega)^2]}+\frac{e^{x_4 t}(1+x_4\omega)^2}{x_4[l+(1+x_4\omega)^2]}\right]+2\nu\sum_{r_2=0}^{\infty}\frac{1}{\alpha_1^2\cos(\alpha_1 r)} \\
\times & \left[\frac{e^{x_7 t}(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8 t}(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right]\Bigg\}-\frac{2I\mu}{\nu\pi} \\
\times & \sum_{r_1=0}^{\infty}\frac{1}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\left\{\frac{Y\sin h(Y)}{X^2Y\cos h(Y)}+\frac{\nu\pi^2}{2}\sum_{r_2=0}^{\infty}(-1)^{r_2}\frac{(2r_2+1)^2}{\alpha_1^2}\right. \\
\times & \left.\sin\left[\frac{(2r_2+1)\pi}{2}\right]\left[\frac{e^{x_7 t}(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8 t}(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right]\right\} \\
- & u_0\mu\nu\pi\sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\sum_{r_2=0}^{\infty}(2r_2+1)^2\left[\frac{e^{x_7 t}(1+x_7\omega)^2}{[l+(1+x_7\omega)^2]}\right. \\
+ & \left.\frac{e^{x_8 t}(1+x_8\omega)^2}{[l+(1+x_8\omega)^2]}\right]+2u_1\mu\nu\sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right) \\
\times & \sum_{r_2=0}^{\infty}(-1)^{r_2}(2r_2+1)\left[\frac{e^{x_7 t}(1+x_7\omega)^2}{[l+(1+x_7\omega)^2]}+\frac{e^{x_8 t}(1+x_8\omega)^2}{[l+(1+x_8\omega)^2]}\right] \\
+ & \frac{I\mu}{\nu X}\frac{\sin h(Xs)}{\cos h(Xr)}+\frac{2I\mu}{r}\sum_{r_2=0}^{\infty}(-1)^{r_2}\sin\left[\frac{(2r_2+1)\pi}{2r}s\right] \\
\times & \left[\frac{e^{x_3 t}(1+x_3\omega)^2}{x_3[l+(1+x_3\omega)^2]}+\frac{e^{x_4 t}(1+x_4\omega)^2}{x_4[l+(1+x_4\omega)^2]}\right]-\frac{2\mu I}{\nu r}\sum_{r_1=0}^{\infty}(-1)^{r_1} \\
\times & \cos\left[\frac{(r_1)\pi}{r}s\right]\frac{\sin h(Y)}{Y\cos h(Y)}-\frac{2I\pi^2\mu}{r^3\nu}\sum_{r_1=0}^{\infty}r_1^2\cos\left(\frac{r_1\pi}{r}s\right) \\
\times & \left\{\frac{\cos h(Y)}{X^2Y^2\cos h(Y)\cos h(Xr)}-\frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{1}{\beta^2}\right. \\
\times & \left.\left[\frac{e^{x_3 t}(1+x_3\omega)^2}{x_3[l+(1+x_3\omega)^2]}+\frac{e^{x_4 t}(1+x_4\omega)^2}{x_4[l+(1+x_4\omega)^2]}\right]\right\}+\frac{2\mu I}{\nu r}\sum_{r_1=0}^{\infty}\cos(\frac{r_1\pi}{r}s)\frac{1}{X^2}
\end{aligned}
$$

CASE-2. Transition Motion:

Consider the case of transition motion, in which

$$
\begin{aligned}
P(t) &= P_0\ H(t)\ e^{-\lambda t} \\
u_b &= 0 \qquad at \quad s=\pm d \\
u_b &= u_0\ H(t)\ e^{-\lambda t} \quad at \quad n=h \\
\frac{\partial u_b}{\partial n} &= u_1\ H(t)\ e^{-\lambda t} \quad at \quad n=0
\end{aligned}
$$

where $H(t)$ is the Heaviside unit step function and p_0, u_0 and u_1 are constants.

By applying the same procedure as in case-1, we obtain the expressions for u_b and v_b as

$$
\begin{aligned}
u_b(s,n,t) &= \frac{J}{\nu}\left\{\rho g \sin\gamma\left[\frac{\cosh(Xr)-\cosh(Xs)}{X^2\cosh(Xr)}\right] + p_0 e^{-\phi t}\right.\\
&\times \left[\frac{\cosh(\alpha_2 r)-\cosh(\alpha_2 s)}{\alpha_2^2\cosh(\alpha_2 r)}\right] + \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\cos\left[\frac{(2r_2+1)\pi}{2r}s\right]\\
&\times \left[\frac{e^{x_3 t}(1+x_3\omega)^2\left[p_0 x_3+\rho g\sin\gamma(x_3+\phi)\right]}{x_3(x_3+\phi)[l+(1+x_3\omega)^2]}\right.\\
&+ \left.\left.\frac{e^{x_4 t}(1+x_4\omega)^2\left[p_0 x_4+\rho g\sin\gamma(x_4+\phi)\right]}{x_4(x_4+\phi)[l+(1+x_4\omega)^2]}\right]\right\} + \frac{2J}{\pi\nu}\sum_{r_1=0}^{\infty}\frac{(-1)^{r_1}}{r_1}\\
&\times \sin\left(\frac{r_1\pi}{r}s\right)\left\{\rho g\sin\gamma\frac{\cosh(Yn)}{Y^2\cosh(Y)} + p_0 e^{-\phi t}\frac{\cosh(\alpha_3 n}{\alpha_3^2\cosh(\alpha_3)}\right.\\
&- \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\cos\left[\frac{(2r_2+1)\pi}{2}n\right]\\
&\times \left[\frac{e^{x_7 t}(1+x_7\omega)^2\left[p_0 x_7+\rho g\sin\gamma(x_7+\phi)\right]}{x_7(x_7+\phi)[l+(1+x_7\omega)^2]}\right.\\
&+ \left.\left.\frac{e^{x_8 t}(1+x_8\omega)^2\left[p_0 x_8+\rho g\sin\gamma(x_8+\phi)\right]}{x_8(x_8+\phi)[l+(1+x_8\omega)^2]}\right]\right\} + \frac{2\pi J}{\nu r^2}\\
&\times \sum_{r_1=0}^{\infty} r_1\sin\left(\frac{r_1\pi}{r}s\right)\left\{\rho g\sin\gamma\left[\frac{\cosh(Yn)}{X^2Y^2\cosh(Y)\cosh(Xr)}\right]\right.\\
&+ p_0 e^{-\phi t}\left[\frac{\cosh(\alpha_3 n}{\alpha_2^2\alpha_3^2\cosh(\alpha_3)\cosh(\alpha_2 r)}\right] + \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\\
&\times \frac{\cos\left[\frac{(2r_2+1)\pi}{2}n\right]}{\alpha_1^2\cos(\alpha_1 r)}\left[\frac{e^{x_7 t}(1+x_7\omega)^2\left[p_0 x_7+\rho g\sin\gamma(x_7+\phi)\right]}{x_7(x_7+\phi)[l+(1+x_7\omega)^2]}\right.\\
&+ \left.\frac{e^{x_8 t}(1+x_8\omega)^2\left[p_0 x_8+\rho g\sin\gamma(x_8+\phi)\right]}{x_8(x_8+\phi)[l+(1+x_8\omega)^2]}\right] - \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\\
&\times \frac{\cosh(\beta n)}{\beta^2\cosh(\beta)}\left[\frac{e^{x_3 t}(1+x_3\omega)^2\left[p_0 x_3+\rho g\sin\gamma(x_3+\phi)\right]}{x_3(x_3+\phi)[l+(1+x_3\omega)^2]}\right.\\
&+ \left.\left.\frac{e^{x_4 t}(1+x_4\omega)^2\left[p_0 x_4+\rho g\sin\gamma(x_4+\phi)\right]}{x_4(x_4+\phi)[l+(1+x_4\omega)^2]}\right]\right\} - \frac{2J}{\nu\pi}\sum_{r_1=0}^{\infty}\frac{1}{r_1}
\end{aligned}
$$

$$
\begin{aligned}
&\times \quad \sin\left(\frac{r_1\pi}{r}s\right)\left\{\rho g\sin\gamma\left[\frac{\cosh(Yn)}{X^2\cosh(Y)}\right]+p_0e^{-\phi t}\left[\frac{\cosh(\alpha_3 n)}{\alpha_2^2\cosh(\alpha_3)}\right]\right.\\
&- \quad \nu\pi\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{\alpha_1^2}(2r_2+1)\cos\left[\frac{(2r_2+1)\pi}{2}n\right]\\
&\times \quad \left[\frac{e^{x_7t}(1+x_7\omega)^2\left[p_0x_7+\rho g\sin\gamma(x_7+\phi)\right]}{x_7(x_7+\phi)[l+(1+x_7\omega)^2]}\right.\\
&+ \quad \left.\left.\frac{e^{x_8t}(1+x_8\omega)^2\left[p_0x_8+\rho g\sin\gamma(x_8+\phi)\right]}{x_8(x_8+\phi)[l+(1+x_8\omega)^2]}\right]\right\}+\frac{2u_0}{\pi}\\
&\times \quad \sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\left\{e^{-\phi t}\left[\frac{\cosh(\alpha_3 n}{\cosh(\alpha_3)}\right]+\nu\pi\sum_{r_2=0}^{\infty}(-1)^{r_2}\right.\\
&\times \quad (2r_2+1)\cos\left[\frac{(2r_2+1)\pi}{2}n\right]\left[\frac{e^{x_7t}(1+x_7\omega)^2}{(x_7+\phi)[l+(1+x_7\omega)^2]}\right.\\
&+ \quad \left.\left.\frac{e^{x_8t}(1+x_8\omega)^2}{(x_8+\phi)[l+(1+x_8\omega)^2]}\right]\right\}+\frac{2u_1}{\pi}\sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\\
&\times \quad \left\{e^{-\phi t}\left[\frac{\sinh(\alpha_3(n-1))}{\alpha_3\cosh(\alpha_3)}\right]+2\nu\sum_{r_2=0}^{\infty}(-1)^{r_2}\sin\left[\frac{(2r_2+1)\pi}{2}(n-1)\right]\right.\\
&\times \quad \left.\left[\frac{e^{x_7t}(1+x_7\omega)^2}{(x_7+\phi)[l+(1+x_7\omega)^2]}+\frac{e^{x_8t}(1+x_8\omega)^2}{(x_8+\phi)[l+(1+x_8\omega)^2]}\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
v_b(s,n,t) \quad &= \quad \frac{J}{\nu}\left\{\rho g\sin\gamma\left[\frac{\cosh(Xr)-\cosh(Xs)}{X^2\cosh(Xr)}\right]+p_0e^{-\phi t}\right.\\
&\times \quad \left[\frac{\cosh(\alpha_2 r)-\cosh(\alpha_2 s)}{\alpha_2^2(1-\phi\omega)\cosh(\alpha_2 r)}\right]+\frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\cos\left[\frac{(2r_2+1)\pi}{2r}s\right]\\
&\times \quad \left[\frac{e^{x_3t}(1+x_3\omega)\left[p_0x_3+\rho g\sin\gamma(x_3+\phi)\right]}{(x_3+\phi)[l+(1+x_3\omega)^2]}\right.\\
&+ \quad \left.\left.\frac{e^{x_4t}(1+x_4\omega)\left[p_0x_4+\rho g\sin\gamma(x_4+\phi)\right]}{(x_4+\phi)[l+(1+x_4\omega)^2]}\right]\right\}+\frac{2J}{\pi\nu}\sum_{r_1=0}^{\infty}\frac{(-1)^{r_1}}{r_1}\\
&\times \quad \sin\left(\frac{r_1\pi}{r}s\right)\left\{\rho g\sin\gamma\left[\frac{\cosh(Yn)}{Y^2\cosh(Y)}\right]+p_0e^{-\phi t}\right.\\
&\times \quad \left[\frac{\cosh(\alpha_3 n}{\alpha_3^2(1-\phi\omega)\cosh(\alpha_3)}\right]-\frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\cos\left[\frac{(2r_2+1)\pi}{2}n\right]
\end{aligned}
$$

$$
\begin{aligned}
\times & \left[\frac{e^{x_7 t}(1+x_7\omega)\left[p_0 x_7+\rho g\sin\gamma(x_7+\phi)\right]}{x_7(x_7+\phi)[l+(1+x_7\omega)^2]}\right.\\
+ & \left.\left.\frac{e^{x_8 t}(1+x_8\omega)\left[p_0 x_8+\rho g\sin\gamma(x_8+\phi)\right]}{x_8(x_8+\phi)[l+(1+x_8\omega)^2]}\right]\right\} + \frac{2\pi J}{\nu r^2}\sum_{r_1=0}^{\infty} r_1\\
\times & \sin\left(\frac{r_1\pi}{r}s\right)\ \left\{\rho g\sin\gamma\left[\frac{\cosh(Yn)}{X^2Y^2\cosh(Y)\cosh(Xr)}\right]\right.\\
+ & p_0 e^{-\phi t}\left[\frac{\cosh(\alpha_3 n)}{\alpha_2^2\alpha_3^2(1-\phi\omega)\cosh(\alpha_3)\cosh(\alpha_2 r)}\right] + \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\\
\times & \frac{\cos\left[\frac{(2r_2+1)\pi}{2}n\right]}{\alpha_1^2\cos(\alpha_1 r)}\left[\frac{e^{x_7 t}(1+x_7\omega)\left[p_0 x_7+\rho g\sin\gamma(x_7+\phi)\right]}{x_7(x_7+\phi)[l+(1+x_7\omega)^2]}\right.\\
+ & \left.\frac{e^{x_8 t}(1+x_8\omega)\left[p_0 x_8+\rho g\sin\gamma(x_8+\phi)\right]}{x_8(x_8+\phi)[l+(1+x_8\omega)^2]}\right] - \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\\
\times & \frac{\cosh(\beta n)}{\beta^2\cosh(\beta)}\left[\frac{e^{x_3 t}(1+x_3\omega)\left[p_0 x_3+\rho g\sin\gamma(x_3+\phi)\right]}{x_3[(x_3+\phi)[l+(1+x_3\omega)^2]}\right.\\
+ & \left.\left.\frac{e^{x_4 t}(1+x_4\omega)\left[p_0 x_4+\rho g\sin\gamma(x_4+\phi)\right]}{x_4(x_4+\phi)[l+(1+x_4\omega)^2]}\right]\right\} - \frac{2J}{\nu\pi}\sum_{r_1=0}^{\infty}\frac{1}{r_1}\\
\times & \sin\left(\frac{r_1\pi}{r}s\right)\left\{\rho g\sin\gamma\left[\frac{\cosh(Yn)}{X^2\cosh(Y)}\right] + \frac{p_0 e^{-\phi t}}{\alpha_2^2(1-\phi\omega)}\left[\frac{\cosh(\alpha_3 n)}{\cosh(\alpha_3)}\right]\right.\\
- & \nu\pi\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{\alpha_1^2}(2r_2+1)\cos\left[\frac{(2r_2+1)\pi}{2}n\right]\\
\times & \left[\frac{e^{x_7 t}(1+x_7\omega)\left[p_0 x_7+\rho g\sin\gamma(x_7+\phi)\right]}{x_7(x_7+\phi)[l+(1+x_7\omega)^2]}\right.\\
+ & \left.\left.\frac{e^{x_8 t}(1+x_8\omega)\left[p_0 x_8+\rho g\sin\gamma(x_8+\phi)\right]}{x_8(x_8+\phi)[l+(1+x_8\omega)^2]}\right]\right\} + \frac{2u_0}{\pi}\\
\times & \sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\left\{e^{-\phi t}\left[\frac{\cosh(\alpha_3 n)}{(1-\phi\omega)\cosh(\alpha_3)}\right]\right.\\
+ & \nu\pi\sum_{r_2=0}^{\infty}(-1)^{r_2}(2r_2+1)\cos\left[\frac{(2r_2+1)\pi}{2}n\right]\\
\times & \left.\left[\frac{e^{x_7 t}(1+x_7\omega)}{(x_7+\phi)[l+(1+x_7\omega)^2]}+\frac{e^{x_8 t}(1+x_8\omega)}{(x_8+\phi)[l+(1+x_8\omega)^2]}\right]\right\} + \frac{2u_1}{\pi}
\end{aligned}
$$

$$\times \quad \sum_{r_1=0}^{\infty} \frac{[1-(-1)^{r_1}]}{r_1} \sin\left(\frac{r_1\pi}{r} s\right) \left\{ e^{-\phi t} \left[\frac{\sinh(\alpha_3(n-1)}{\alpha_3(1-\phi\omega)\cosh(\alpha_3)} \right] \right.$$
$$+ \quad 2\nu \sum_{r_2=0}^{\infty} (-1)^{r_2} \sin\left[\frac{(2r_2+1)\pi}{2}(n-1)\right]$$
$$\times \quad \left. \left[\frac{e^{x_7 t}(1+x_7\omega)}{(x_7+\phi)[l+(1+x_7\omega)^2]} + \frac{e^{x_8 t}(1+x_8\omega)}{(x_8+\phi)[l+(1+x_8\omega)^2]} \right] \right\}$$

Shearing Stress (Skin Friction):
The Shear stress at the boundaries $s = r, \quad s = -r$ and $n = 0, \quad n = 1$ are given by

$$D_{rn} = \frac{\mu J}{\nu}(\rho g \sin\gamma) \frac{\sin h(Xr)}{X \cos h(Xr)} + \frac{p_0 \mu J}{\nu} e^{-\phi t} \frac{\sinh(\alpha_2 r)}{\alpha_2 \cosh(\alpha_2 r)}$$
$$+ \quad \frac{2J\mu}{r} \left[\frac{e^{x_3 t}(1+x_3\omega)^2[p_0 x_3 + \rho g \sin\gamma(x_3+\phi)]}{x_3(x_3+\phi)[l+(1+x_3\omega)^2]} \right.$$
$$+ \quad \left. \frac{e^{x_4 t}(1+x_4\omega)^2[p_0 x_4 + \rho g \sin\gamma(x_4+\phi)]}{x_4(x_4+\phi)[l+(1+x_4\omega)^2]} \right] - \frac{2\mu J}{r\nu} \sum_{r_1=0}^{\infty}$$
$$\times \quad \left\{ (\rho g \sin\gamma) \frac{\cos h(Yn)}{Y^2 \cos h(Y)} + p_0 e^{-\phi t} \frac{\cos h(\alpha_3 n)}{\alpha_3^2 \cos h(\alpha_3)} - \frac{4\nu}{\pi} \sum_{r_2=0}^{\infty} \frac{(-1)^{r_2}}{(2r_2+1)} \right.$$
$$\times \quad \cos\left[\frac{(2r_2+1)\pi}{2} n\right] \left[\frac{e^{x_7 t}(1+x_7\omega)^2[p_0 x_7 + \rho g \sin\gamma(x_7+\phi)]}{x_7(x_7+\phi)[l+(1+x_7\omega)^2]} \right.$$
$$+ \quad \left. \left. \frac{e^{x_8 t}(1+x_8\omega)^2[p_0 x_8 + \rho g \sin\gamma(x_8+\phi)]}{x_8(x_8+\phi)[l+(1+x_8\omega)^2]} \right] \right\} - \frac{2J\pi^2\mu}{r^3\nu} \sum_{r_1=0}^{\infty} r_1^2(-1)^{r_1}$$
$$\times \quad \left\{ (\rho g \sin\gamma) \frac{\cos h(Yn)}{X^2 Y^2 \cos h(Y) \cos h(Xr)} + p_0 e^{-\phi t} \right.$$
$$\times \quad \frac{\cos h(\alpha_3 n)}{\alpha_2^2 \alpha_3^2 \cos h(\alpha_3) \cos h(\alpha_2 r)} + \frac{4\nu}{\pi} \sum_{r_2=0}^{\infty} \frac{(-1)^{r_2}}{(2r_2+1)} \frac{\cos\left[\frac{(2r_2+1)\pi}{2} n\right]}{\alpha_1^2 \cos(\alpha_1 r)}$$
$$\times \quad \left[\frac{e^{x_7 t}(1+x_7\omega)^2[p_0 x_7 + \rho g \sin\gamma(x_7+\phi)]}{x_7(x_7+\phi)[l+(1+x_7\omega)^2]} \right.$$
$$+ \quad \left. \frac{e^{x_8 t}(1+x_8\omega)^2[p_0 x_8 + \rho g \sin\gamma(x_8+\phi)]}{x_8(x_8+\phi)[l+(1+x_8\omega)^2]} \right] - \frac{4\nu}{\pi} \sum_{r_2=0}^{\infty} \frac{(-1)^{r_2}}{(2r_2+1)}$$

$$\begin{aligned}
&\times \quad \frac{\cosh(\beta n)}{\beta^2 \cosh(\beta)} \left[\frac{e^{x_3 t}(1+x_3\omega)^2[p_0 x_3 + \rho g \sin\gamma(x_3+\phi)]}{x_3(x_3+\phi)[l+(1+x_3\omega)^2]} \right.\\
&+ \quad \left.\left. \frac{e^{x_4 t}(1+x_4\omega)^2[p_0 x_4 + \rho g \sin\gamma(x_4+\phi)]}{(x_4+\phi)[l+(1+x_4\omega)^2]} \right] \right\} + \frac{2J\mu}{\nu r} \sum_{r_1=0}^{\infty} (-1)^{r_1}\\
&\times \quad \left\{ \rho g \sin\gamma \frac{\cos h(Yn)}{X^2 \cos h(Y)} + p_0 e^{-\phi t} \left[\frac{\cosh(\alpha_3 n)}{\alpha_2^2 \cosh(\alpha_3)} \right] - \nu\pi \sum_{r_2=0}^{\infty} (-1)^{r_2} \right.\\
&\times \quad (2r_2+1) \frac{\cos\left[\frac{(2r_2+1)\pi}{2} n\right]}{\alpha_1^2} \left[\frac{e^{x_7 t}(1+x_7\omega)^2[p_0 x_7 + \rho g \sin\gamma(x_7+\phi)]}{x_7(x_7+\phi)[l+(1+x_7\omega)^2]} \right.\\
&+ \quad \left.\left. \frac{e^{x_8 t}(1+x_8\omega)^2[p_0 x_8 + \rho g \sin\gamma(x_8+\phi)]}{x_8(x_8+\phi)[l+(1+x_8\omega)^2]} \right] \right\} - \frac{2u_0\mu}{r} \sum_{r_1=0}^{\infty} [(-1)^{r_1} - 1]\\
&\times \quad \left\{ e^{-\phi t} \frac{\cosh(\alpha_3 n)}{\cosh(\alpha_3)} + \nu\pi \sum_{r_2=0}^{\infty} (-1)^{r_2} (2r_2+1) \cos\left[\frac{(2r_2+1)\pi}{2} n \right] \right.\\
&\times \quad \left.\left[\frac{e^{x_7 t}(1+x_7\omega)^2}{(x_7+\phi)[l+(1+x_7\omega)^2]} + \frac{e^{x_8 t}(1+x_8\omega)^2}{(x_8+\phi)[l+(1+x_8\omega)^2]} \right] \right\}\\
&- \quad \frac{2u_1\mu}{r} \sum_{r_1=0}^{\infty} [(-1)^{r_1} - 1] \left\{ e^{-\phi t} \left[\frac{\sinh(\alpha_3(n-1)}{\alpha_3 \cosh(\alpha_3)} \right] \right.\\
&+ \quad 2\nu \sum_{r_2=0}^{\infty} (-1)^{r_2} \sin\left[\frac{(2r_2+1)\pi}{2} (n-1) \right]\\
&\times \quad \left.\left[\frac{e^{x_7 t}(1+x_7\omega)^2}{(x_7+\phi)[l+(1+x_7\omega)^2]} + \frac{e^{x_8 t}(1+x_8\omega)^2}{(x_8+\phi)[l+(1+x_8\omega)^2]} \right] \right\}
\end{aligned}$$

$$D_{-rn} = -D_{rn}$$

$$\begin{aligned}
D_{s0} \quad &= \quad \frac{2u_1\mu}{\pi} \sum_{r_1=0}^{\infty} \frac{[1-(-1)^{r_1}]}{r_1} \sin\left(\frac{r_1\pi}{r} s\right) \left\{ e^{-\phi t} + \nu\mu\pi \sum_{r_2=0}^{\infty} (-1)^{r_2} \right.\\
&\times \quad (2r_2+1) \cos\left[\frac{(2r_2+1)\pi}{2} \right] \left[\frac{e^{x_7 t}(1+x_7\omega)^2}{(x_7+\phi)[l+(1+x_7\omega)^2]} \right.\\
&+ \quad \left.\left. \frac{e^{x_8 t}(1+x_8\omega)^2}{(x_8+\phi)[l+(1+x_8\omega)^2]} \right] \right\} + \frac{\mu J}{\nu} (\rho g \sin\gamma) \frac{\sin h(Xs)}{X \cos h(Xr)}\\
&+ \quad \frac{p_0 J\mu}{\nu} e^{-\phi t} \frac{\sin h(\alpha_2 s)}{\alpha_2 \cos h(\alpha_2 r)} + \frac{2\mu J}{r} \sum_{r_2=0}^{\infty} (-1)^{r_2} \sin\left[\frac{(2r_2+1)\pi}{2r} s \right]
\end{aligned}$$

$$
\begin{aligned}
&\times \quad \left[\frac{e^{x_3 t}(1+x_3\omega)^2[p_0x_3+\rho g\sin\gamma(x_3+\phi)]}{x_3(x_3+\phi)[l+(1+x_3\omega)^2]}\right.\\
&+ \quad \left.\frac{e^{x_4 t}(1+x_4\omega)^2[p_0x_4+\rho g\sin\gamma(x_4+\phi)]}{x_4(x_4+\phi)[l+(1+x_4\omega)^2]}\right]-\frac{2J\mu}{r\nu}\sum_{r_1=0}^{\infty}(-1)^{r_1}\\
&\times \quad \cos\left(\frac{r_1\pi}{r}s\right)\left\{\frac{\rho g\sin\gamma}{Y^2\cos h(Y)}\frac{p_0e^{-\phi t}}{\alpha_3^2\cos h(\alpha_3)}-\frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\right.\\
&\times \quad \left[\frac{e^{x_7 t}(1+x_7\omega)^2[p_0x_7+\rho g\sin\gamma(x_7+\phi)]}{x_7(x_7+\phi)[l+(1+x_7\omega)^2]}\right.\\
&+ \quad \left.\left.\frac{e^{x_8 t}(1+x_8\omega)^2[p_0x_8+\rho g\sin\gamma(x_8+\phi)]}{x_8(x_8+\phi)[l+(1+x_8\omega)^2]}\right]\right\}-\frac{2J\pi^2\mu}{\nu r^3}\sum_{r_1=0}^{\infty}r_1^2\\
&\times \quad \cos\left(\frac{r_1\pi}{r}s\right)\left\{\frac{\rho g\sin\gamma}{X^2Y^2\cos h(Xr)}+\frac{p_0e^{-\phi t}}{\alpha_2^2\alpha_3^2\cos h(\alpha_3)\cos h(\alpha_2 r)}-\frac{4\nu}{\pi}\right.\\
&+ \quad \sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{1}{\alpha_1^2\cos(\alpha_1 r)}\left[\frac{e^{x_7 t}(1+x_7\omega)^2[p_0x_7+\rho g\sin\gamma(x_7+\phi)]}{x_7(x_7+\phi)[l+(1+x_7\omega)^2]}\right.\\
&+ \quad \left.\left.\frac{e^{x_8 t}(1+x_8\omega)^2[p_0x_8+\rho g\sin\gamma(x_8+\phi)]}{x_8(x_8+\phi)[l+(1+x_8\omega)^2]}\right]\right\}-\frac{2u_1\mu}{r}\sum_{r_1=0}^{\infty}[1-(-1)^{r_1}]\\
&\times \quad \cos\left(\frac{r_1\pi}{r}s\right)\left\{-e^{-\phi t}\frac{\sin h(\alpha_3)}{\alpha_3\cos h(\alpha_3)}-2\nu\sum_{r_2=0}^{\infty}(-1)^{r_1}\sin\left[\frac{(2r_2+1)\pi}{2}\right]\right.\\
&\times \quad \left.\left[\frac{e^{x_7 t}(1+x_7\omega)^2}{(x_7+\phi)[l+(1+x_7\omega)^2]}+\frac{e^{x_8 t}(1+x_8\omega)^2}{(x_8+\phi)[l+(1+x_8\omega)^2]}\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
D_{s1} \quad &= \quad \frac{2J\mu}{\nu\pi}\sum_{r_1=0}^{\infty}\frac{(-1)^{r_1}}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\left\{(\rho g\sin\gamma)\frac{\sin h(Y)}{Y\cos h(Y)}+p_0e^{-\phi t}\right.\\
&\times \quad \frac{\sin h(\alpha_3)}{\alpha_3\cos h(\alpha_3)}2\nu\sum_{r_2=0}^{\infty}\left[\frac{e^{x_7 t}(1+x_7\omega)^2[p_0x_7+\rho g\sin\gamma(x_7+\phi)]}{x_7(x_7+\phi)[l+(1+x_7\omega)^2]}\right.\\
&+ \quad \left.\left.\frac{e^{x_8 t}(1+x_8\omega)^2[p_0x_8+\rho g\sin\gamma(x_8+\phi)]}{x_8(x_8+\phi)[l+(1+x_8\omega)^2]}\right]\right\}+\frac{2J\mu\pi}{\nu r^2}\sum_{r_1=0}^{\infty}r_1\sin\left(\frac{r_1\pi}{r}s\right)\\
&\times \quad \left\{(\rho g\sin\gamma)\frac{\sin h(Y)}{X^2Y\cos h(Y)\cos h(Xr)}+p_0e^{-\phi t}\frac{\sin h(\alpha_3)}{\alpha_2^2\alpha_3\cos h(\alpha_3)\cos h(\alpha_2 r)}\right.\\
&- \quad 2\nu\sum_{r_2=0}^{\infty}\frac{1}{\alpha_1^2\cos(\alpha_1 r)}\left[\frac{e^{x_7 t}(1+x_7\omega)^2[p_0x_7+\rho g\sin\gamma(x_7+\phi)]}{x_7(x_7+\phi)[l+(1+x_7\omega)^2]}\right.
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \left.\frac{e^{x_8 t}(1+x_8\omega)^2[p_0x_8+\rho g\sin\gamma(x_8+\phi)]}{x_8(x_8+\phi)[l+(1+x_8\omega)^2]}\right] - \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)} \\
&\times \quad \frac{\sinh(\beta)}{\beta\cosh(\beta)}\left[\frac{e^{x_3 t}(1+x_3\omega)^2[p_0x_3+\rho g\sin\gamma(x_3+\phi)]}{x_3(x_3+\phi)[l+(1+x_3\omega)^2]}\right. \\
&+ \quad \left.\left.\frac{e^{x_4 t}(1+x_4\omega)^2[p_0x_4+\rho g\sin\gamma(x_4+\phi)]}{x_4(x_4+\phi)[l+(1+x_4\omega)^2]}\right]\right\} - \frac{2J\mu}{\nu\pi}\sum_{r_1=0}^{\infty}\frac{1}{r_1}\sin\left(\frac{r_1\pi}{r}s\right) \\
&\times \quad \left\{(\rho g\sin\gamma)\frac{Y\sin h(Y)}{X^2\cos h(Y)} + p_0e^{-\phi t}\frac{\alpha_3\sin h(\alpha_3)}{\alpha_2^2\cos h(\alpha_3)}\right. \\
&+ \quad \frac{\nu\pi^2}{2}\sum_{r_2=0}^{\infty}\frac{(2r_2+1)^2}{\alpha_1^2}\left[\frac{e^{x_7 t}(1+x_7\omega)^2[p_0x_7+\rho g\sin\gamma(x_7+\phi)]}{x_7(x_7+\phi)[l+(1+x_7\omega)^2]}\right. \\
&+ \quad \left.\left.\frac{e^{x_8 t}(1+x_8\omega)^2[p_0x_8+\rho g\sin\gamma(x_8+\phi)]}{x_8(x_8+\phi)[l+(1+x_8\omega)^2]}\right]\right\} + \frac{2\mu u_0}{\pi}\sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1} \\
&\times \quad \sin\left(\frac{r_1\pi}{r}s\right)\left\{e^{-\phi t}\alpha_3\frac{\sin h(\alpha_3)}{\cos h(\alpha_3)} - \frac{\nu\pi^2}{2}\sum_{r_2=0}^{\infty}(2r_2+1)^2\right. \\
&\times \quad \left.\left[\frac{e^{x_7 t}(1+x_7\omega)^2}{(x_7+\phi)[l+(1+x_7\omega)^2]} + \frac{e^{x_8 t}(1+x_8\omega)^2}{(x_8+\phi)[l+(1+x_8\omega)^2]}\right]\right\} \\
&+ \quad \frac{2\mu u_1}{\pi}\sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\left\{e^{-\phi t}\frac{\cos h(2\alpha_3)}{\cos h(\alpha_3)} + \nu\mu\pi\sum_{r_2=0}^{\infty}(-1)^{r_2}\right. \\
&\times \quad \left.(2r_2+1)\left[\frac{e^{x_7 t}(1+x_7\omega)^2}{(x_7+\phi)[l+(1+x_7\omega)^2]} + \frac{e^{x_8 t}(1+x_8\omega)^2}{(x_8+\phi)[l+(1+x_8\omega)^2]}\right]\right\} \\
&+ \quad \frac{\mu J}{\nu}(\rho g\sin\nu)\frac{\sinh(Xs)}{\cosh(Xr)} + \frac{p_0J\mu}{\nu}e^{-\phi t}\frac{\sinh(\alpha_2 s)}{\alpha_2\cosh(\alpha_2 r)} + \frac{2\mu J}{r}\sum_{r_2=0}^{\infty}(-1)^{r_2} \\
&\times \quad \sin\left[\frac{(2r_2+1)\pi}{2r}s\right]\left[\frac{e^{x_3 t}(1+x_3\omega)^2[p_0x_3+\rho g\sin\gamma(x_3+\phi)]}{x_3(x_3+\phi)[l+(1+x_3\omega)^2]}\right. \\
&+ \quad \left.\frac{e^{x_4 t}(1+x_4\omega)^2[p_0x_4+\rho g\sin\gamma(x_4+\phi)]}{x_4(x_4+\phi)[l+(1+x_4\omega)^2]}\right] - \frac{2\mu J}{\nu r}\sum_{r_1=0}^{\infty}(-1)^{r_1} \\
&\times \quad \cos\left(\frac{r_1\pi}{r}s\right)\left\{\frac{\rho g\sin\gamma}{Y^2} + \frac{p_0e^{-\phi t}}{\alpha_3^2}\right\} - \frac{2\mu J\pi^2}{\nu r^3}\sum_{r_1=0}^{\infty}{r_1}^2 \\
&\times \quad \cos\left(\frac{r_1\pi}{r}s\right)\left\{\frac{\rho g\sin\gamma}{X^2Y^2\cosh(Xr)} + \frac{p_0e^{-\phi t}}{\alpha_2^2\alpha_3^2\cosh(\alpha_2 r)} - \frac{4\nu}{\pi}\right.
\end{aligned}
$$

$$
\begin{aligned}
\times & \sum_{r_2=0}^{\infty} \frac{(-1)^{r_2}}{(2r_2+1)\beta^2} \left[\frac{e^{x_3 t}(1+x_3\omega)^2[p_0 x_3 + \rho g \sin\gamma (x_3+\phi)]}{x_3(x_3+\phi)[l+(1+x_3\omega)^2]} \right. \\
+ & \left. \left. \frac{e^{x_4 t}(1+x_4\omega)^2[p_0 x_4 + \rho g \sin\gamma (x_4+\phi)]}{x_4(x_4+\phi)[l+(1+x_4\omega)^2]} \right] \right\} + \frac{2\mu J \pi^2}{\nu r} \sum_{r_1=0}^{\infty} \cos\left(\frac{r_1 \pi}{r} s\right) \\
\times & \left\{ \frac{\rho g \sin\gamma}{X^2} + \frac{p_0 e^{-\phi t}}{\alpha_2^2} \right\} - \frac{2u_0\mu}{r} \sum_{r_1=0}^{\infty} [1-(-1)^{r_1}] \cos\left(\frac{r_1\pi}{r} s\right) \frac{e^{-\phi t}}{(1-\phi\omega)}
\end{aligned}
$$

CASE-3. Motion for a finite time:

In this case, consider the boundary conditions are

$$
\begin{aligned}
P(t) &= P_0\,[H(t) - H(t-T)] \\
u_b &= 0 \qquad at \quad s = \pm d \\
u_b &= u_0\,[H(t) - H(t-T)] \quad at \quad n = h \\
\frac{\partial u_b}{\partial n} &= u_1\,[H(t) - H(t-T)] \quad at \quad n = 0
\end{aligned}
$$

where $H(t)$ is the Heaviside unit step function and p_0,u_0 and u_1 are constants.

The expressions for the fluid phase velocity u_b and dust phase velocity v_b are given by

$$
\begin{aligned}
u_b(s,n,t) &= \frac{J}{\nu} \left\{ \rho g \sin\gamma \left[\frac{\cosh(Xr) - \cosh(Xs)}{X^2 \cosh(Xr)} \right] + \frac{4\nu}{\pi} \sum_{r_2=0}^{\infty} \frac{(-1)^{r_2}}{(2r_2+1)} \right. \\
&\times \cos\left[\frac{(2r_2+1)\pi}{2r} s \right] \left[\frac{e^{x_3 t}(1+x_3\omega)^2 \left[p_0(1-e^{-Tx_3}) + \rho g \sin\gamma\right]}{x_3[l+(1+x_3\omega)^2]} \right. \\
&+ \left. \left. \frac{e^{x_4 t}(1+x_4\omega)^2 \left[p_0(1-e^{-Tx_4}) + \rho g \sin\gamma\right]}{x_4[l+(1+x_4\omega)^2]} \right] \right\} + \frac{2J}{\pi\nu} \sum_{r_1=0}^{\infty} \frac{(-1)^{r_1}}{r_1} \\
&\times \sin\left(\frac{r_1\pi}{r} s\right) \left\{ \rho g \sin\gamma \left[\frac{\cosh(Yn)}{Y^2 \cosh(Y)} \right] - \frac{4\nu}{\pi} \sum_{r_2=0}^{\infty} \frac{(-1)^{r_2}}{(2r_2+1)} \right. \\
&\times \cos\left[\frac{(2r_2+1)\pi}{2} n \right] \left[\frac{e^{x_7 t}(1+x_7\omega)^2 \left[p_0(1-e^{-Tx_7}) + \rho g \sin\gamma\right]}{x_7[l+(1+x_7\omega)^2]} \right. \\
&+ \left. \left. \frac{e^{x_8 t}(1+x_8\omega)^2 \left[p_0(1-e^{-Tx_8}) + \rho g \sin\gamma\right]}{x_8[l+(1+x_8\omega)^2]} \right] \right\} + \frac{2\pi J}{\nu r^2} \sum_{r_1=0}^{\infty} r_1
\end{aligned}
$$

$$
\begin{aligned}
&\times \quad \sin\left(\frac{r_1\pi}{r}s\right)\left\{\rho g\sin\gamma\left[\frac{\cosh(Yn)}{X^2Y^2\cosh(Y)\cosh(Xr)}\right]\right.\\
&+ \quad \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{\cos\left[\frac{(2r_2+1)\pi}{2}n\right]}{\alpha_1^2\cos(\alpha_1 r)}\\
&\times \quad \left[\frac{e^{x_7t}(1+x_7\omega)^2\left[p_0(1-e^{-Tx_7})+\rho g\sin\gamma\right]}{x_7[l+(1+x_7\omega)^2]}\right.\\
&+ \quad \left.\frac{e^{x_8t}(1+x_8\omega)^2\left[p_0(1-e^{-Tx_8})+\rho g\sin\gamma\right]}{x_8[l+(1+x_8\omega)^2]}\right]-\frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\\
&\times \quad \frac{\cosh(\beta n)}{\beta^2\cosh(\beta)}\left[\frac{e^{x_3t}(1+x_3\omega)^2\left[p_0(1-e^{-Tx_3})+\rho g\sin\gamma\right]}{x_3[[l+(1+x_3\omega)^2]}\right.\\
&+ \quad \left.\left.\frac{e^{x_4t}(1+x_4\omega)^2\left[p_0(1-e^{-Tx_4})+\rho g\sin\gamma\right]}{x_4[l+(1+x_4\omega)^2]}\right]\right\}-\frac{2J}{\nu\pi}\sum_{r_1=0}^{\infty}\frac{1}{r_1}\\
&\times \quad \sin\left(\frac{r_1\pi}{r}s\right)\left\{\rho g\sin\gamma\left[\frac{\cosh(Yn)}{X^2\cosh(Y)}\right]-\nu\pi\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{\alpha_1^2}(2r_2+1)\right.\\
&\times \quad \cos\left[\frac{(2r_2+1)\pi}{2}n\right]\left[\frac{e^{x_7t}(1+x_7\omega)^2\left[p_0(1-e^{-Tx_7})+\rho g\sin\gamma\right]}{x_7[l+(1+x_7\omega)^2]}\right.\\
&+ \quad \left.\left.\frac{e^{x_8t}(1+x_8\omega)^2\left[p_0(1-e^{-Tx_8})+\rho g\sin\gamma\right]}{x_8[l+(1+x_8\omega)^2]}\right]\right\}+2u_0\nu\sum_{r_1=0}^{\infty}\\
&\times \quad \frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\sum_{r_2=0}^{\infty}(-1)^{r_2}(2r_2+1)\cos\left[\frac{(2r_2+1)\pi}{2}n\right]\\
&\times \quad \left[\frac{e^{x_7t}(1-e^{-Tx_7})(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8t}(1-e^{-Tx_8})(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right]+\frac{4u_1}{\pi}\\
&\times \quad \sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\sum_{r_2=0}^{\infty}(-1)^{r_2}\sin\left[\frac{(2r_2+1)\pi}{2}(n-1)\right]\\
&\times \quad \left[\frac{e^{x_7t}(1-e^{-Tx_7})(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8t}(1-e^{-Tx_8})(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right]
\end{aligned}
$$

$$
v_b(s,n,t) = \frac{J}{\nu}\left\{\rho g\sin\gamma\left[\frac{\cosh(Xr)-\cosh(Xs)}{X^2\cosh(Xr)}\right]+\frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\right.
$$

$$
\begin{aligned}
\times & \cos\left[\frac{(2r_2+1)\pi}{2r}s\right]\left[\frac{e^{x_3t}(1+x_3\omega)\left[p_0(1-e^{-Tx_3})+\rho g\sin\gamma\right]}{x_3[l+(1+x_3\omega)^2]}\right.\\
+ & \left.\left.\frac{e^{x_4t}(1+x_4\omega)\left[p_0(1-e^{-Tx_4})+\rho g\sin\gamma\right]}{x_4[l+(1+x_4\omega)^2]}\right]\right\} + \frac{2J}{\pi\nu}\sum_{r_1=0}^{\infty}\\
\times & \frac{(-1)^{r_1}}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\left\{\rho g\sin\gamma\left[\frac{\cosh(Yn)}{Y^2\cosh(Y)}\right] - \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\right.\\
\times & \cos\left[\frac{(2r_2+1)\pi}{2}n\right]\left[\frac{e^{x_7t}(1+x_7\omega)\left[p_0(1-e^{-Tx_7})+\rho g\sin\gamma\right]}{x_7[l+(1+x_7\omega)^2]}\right.\\
+ & \left.\left.\frac{e^{x_8t}(1+x_8\omega)\left[p_0(1-e^{-Tx_8})+\rho g\sin\gamma\right]}{x_8[l+(1+x_8\omega)^2]}\right]\right\} + \frac{2\pi J}{\nu r^2}\sum_{r_1=0}^{\infty}r_1\\
\times & \sin\left(\frac{r_1\pi}{r}s\right)\left\{\rho g\sin\gamma\left[\frac{\cosh(Yn)}{X^2Y^2\cosh(Y)\cosh(Xr)}\right]\right.\\
+ & \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{\cos\left[\frac{(2r_2+1)\pi}{2}n\right]}{\alpha_1^2\cos(\alpha_1 r)}\\
\times & \left[\frac{e^{x_7t}(1+x_7\omega)\left[p_0(1-e^{-Tx_7})+\rho g\sin\gamma\right]}{x_7[l+(1+x_7\omega)^2]}\right.\\
+ & \left.\frac{e^{x_8t}(1+x_8\omega)\left[p_0(1-e^{-Tx_8})+\rho g\sin\gamma\right]}{x_8[l+(1+x_8\omega)^2]}\right] - \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\\
\times & \frac{\cosh(\beta n)}{\beta^2\cosh(\beta)}\left[\frac{e^{x_3t}(1+x_3\omega)\left[p_0(1-e^{-Tx_3})+\rho g\sin\gamma\right]}{x_3[[l+(1+x_3\omega)^2]}\right.\\
+ & \left.\left.\frac{e^{x_4t}(1+x_4\omega)\left[p_0(1-e^{-Tx_4})+\rho g\sin\gamma\right]}{x_4[l+(1+x_4\omega)^2]}\right]\right\} - \frac{2J}{\nu\pi}\sum_{r_1=0}^{\infty}\frac{1}{r_1}\\
\times & \sin\left(\frac{r_1\pi}{r}s\right)\left\{\rho g\sin\gamma\left[\frac{\cosh(Yn)}{X^2\cosh(Y)}\right] - \nu\pi\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{\alpha_1^2}(2r_2+1)\right.\\
\times & \cos\left[\frac{(2r_2+1)\pi}{2}n\right]\left[\frac{e^{x_7t}(1+x_7\omega)\left[p_0(1-e^{-Tx_7})+\rho g\sin\gamma\right]}{x_7[l+(1+x_7\omega)^2]}\right.
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{e^{x_8 t}(1+x_8\omega)\left[p_0(1-e^{-Tx_8})+\rho g\sin\gamma\right]}{x_8[l+(1+x_8\omega)^2]}\Bigg]\Bigg\} + 2u_0\nu\sum_{r_1=0}^{\infty} \\
&\times \quad \frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\sum_{r_2=0}^{\infty}(-1)^{r_2}(2r_2+1)\cos\left[\frac{(2r_2+1)\pi}{2}n\right] \\
&\times \quad \left[\frac{e^{x_7 t}(1-e^{-Tx_7})(1+x_7\omega)}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8 t}(1-e^{-Tx_8})(1+x_8\omega)}{x_8[l+(1+x_8\omega)^2]}\right]+\frac{4u_1\nu}{\pi} \\
&\times \quad \sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\sum_{r_2=0}^{\infty}(-1)^{r_2}\sin\left[\frac{(2r_2+1)\pi}{2}(n-1)\right] \\
&\times \quad \left[\frac{e^{x_7 t}(1-e^{-Tx_7})(1+x_7\omega)}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8 t}(1-e^{-Tx_8})(1+x_8\omega)}{x_8[l+(1+x_8\omega)^2]}\right]
\end{aligned}
$$

Shearing Stress (Skin Friction):

The Shear stress at the boundaries $s = r$, $s = -r$ and $n = 0$, $n = 1$ are given by

$$
\begin{aligned}
D_{rn} \quad = \quad & \frac{\mu J}{\nu}(\rho g\sin\gamma)\frac{\sinh(Xr)}{X\cosh(Xr)}+\frac{2\mu J}{r}\sum_{r_2=0}^{\infty} \\
\times \quad & \Bigg[\frac{e^{x_3 t}(1+x_3\omega)^2\left[p_0(1-e^{-Tx_3})+\rho g\sin\gamma\right]}{x_3\left[l+(1+x_3\omega)^2\right]} \\
+ \quad & \frac{e^{x_4 t}(1+x_4\omega)^2\left[p_0(1-e^{-Tx_4})+\rho g\sin\gamma\right]}{x_4\left[l+(1+x_4\omega)^2\right]}\Bigg]-\frac{2\mu J}{\nu r} \\
\times \quad & \sum_{r_1=0}^{\infty}\Bigg\{(\rho g\sin\gamma)\frac{\cosh(Yn)}{Y^2\cosh(Y)}-\frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\cos\left[\frac{(2r_2+1)\pi}{2}n\right] \\
\times \quad & \Bigg[\frac{e^{x_7 t}(1+x_7\omega)^2\left[p_0(1-e^{-Tx_7})+\rho g\sin\gamma\right]}{x_7[l+(1+x_7\omega)^2]} \\
+ \quad & \frac{e^{x_8 t}(1+x_8\omega)^2\left[p_0(1-e^{-Tx_8})+\rho g\sin\gamma\right]}{x_8[l+(1+x_8\omega)^2]}\Bigg]\Bigg\}-\frac{2J\mu\pi^2}{r^3\nu}\sum_{r_1=0}^{\infty}r_1^2(-1)^{r_1} \\
\times \quad & \Bigg\{(\rho g\sin\gamma)\frac{\cosh(Yn)}{X^2Y^2\cosh(Y)\cosh(Xr)}+\frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}
\end{aligned}
$$

$$
\begin{aligned}
&\times \quad \frac{\cos\left[\frac{(2r_2+1)\pi}{2}n\right]}{\alpha_1^2\cos(\alpha_1 r)}\left[\frac{e^{x_7 t}(1+x_7\omega)^2\left[p_0(1-e^{-Tx_7})+\rho g\sin\gamma\right]}{x_7[l+(1+x_7\omega)^2]}\right.\\
&+ \quad \left.\frac{e^{x_8 t}(1+x_8\omega)^2\left[p_0(1-e^{-Tx_8})+\rho g\sin\gamma\right]}{x_8[l+(1+x_8\omega)^2]}\right] - \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\\
&\times \quad \frac{\cosh(\beta n)}{\beta^2\cosh(\beta)}\left[\frac{e^{x_3 t}(1+x_3\omega)^2\left[p_0(1-e^{-Tx_3})+\rho g\sin\gamma\right]}{x_3[l+(1+x_3\omega)^2]}\right.\\
&+ \quad \left.\left.\frac{e^{x_4 t}(1+x_4\omega)^2\left[p_0(1-e^{-Tx_4})+\rho g\sin\gamma\right]}{x_4[l+(1+x_4\omega)^2]}\right]\right\} + \frac{2\mu J}{\nu r}\sum_{r_1=0}^{\infty}\frac{(-1)^{r_1}}{r_1}\\
&\times \quad \left\{(\rho g\sin\gamma)\frac{\cosh(yn)}{X^2\cosh(y)} - \nu\pi\sum_{r_2=0}^{\infty}(-1)^{r_2}(2r_2+1)\frac{\cos\left[\frac{(2r_2+1)\pi}{2}n\right]}{\alpha_1^2}\right.\\
&\times \quad \left[\frac{e^{x_7 t}(1+x_7\omega)^2\left[p_0(1-e^{-Tx_7})+\rho g\sin\gamma\right]}{x_7[l+(1+x_7\omega)^2]}\right.\\
&+ \quad \left.\left.\frac{e^{x_8 t}(1+x_8\omega)^2\left[p_0(1-e^{-Tx_8})+\rho g\sin\gamma\right]}{x_8[l+(1+x_8\omega)^2]}\right]\right\} - \frac{2u_0\nu\pi\mu}{r}\\
&\times \quad \sum_{r_1=0}^{\infty}[(-1)^{r_1}-1]\sum_{r_2=0}^{\infty}(-1)^{r_2}(2r_2+1)\cos\left[\frac{(2r_2+1)\pi}{2}n\right]\\
&\times \quad \left[\frac{e^{x_7 t}(1-e^{-Tx_7})(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8 t}(1-e^{-Tx_8})(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right]\\
&- \quad \frac{4u_1\mu\nu}{r}\sum_{r_1=0}^{\infty}[(-1)^{r_1}-1]\sum_{r_2=0}^{\infty}(-1)^{r_2}\sin\left[(n-1)\frac{(2r_2+1)\pi}{2}\right]\\
&\times \quad \left[\frac{e^{x_7 t}(1-e^{-Tx_7})(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}+\frac{e^{x_8 t}(1-e^{-Tx_8})(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\right]
\end{aligned}
$$

$$
D_{-rn} = -D_{rn}
$$

$$
\begin{aligned}
D_{s_1} &= \frac{2\mu J}{\nu\pi}\sum_{r_1=0}^{\infty}\frac{(-1)^{r_1}}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\left\{(\rho g\sin\gamma)\frac{\sinh(y)}{Y\cosh(y)}\right.\\
&+ \quad 2\nu\sum_{r_2=0}^{\infty}\left[\frac{e^{x_7 t}(1+x_7\omega)^2\left[p_0(1-e^{-Tx_7})+\rho g\sin\gamma\right]}{x_7[l+(1+x_7\omega)^2]}\right.
\end{aligned}
$$

$$
\begin{aligned}
+ &\quad \frac{e^{x_8 t}(1+x_8\omega)^2\left[p_0(1-e^{-Tx_8})+\rho g\sin\gamma\right]}{x_8[l+(1+x_8\omega)^2]}\Bigg]\Bigg\} + \frac{2\mu\pi J}{r^2\nu} \\
\times &\quad \sum_{r_1=0}^{\infty} r_1 \sin\left(\frac{r_1\pi}{r}s\right)\left\{\frac{\rho g\sin\gamma\sinh(y)}{X^2 Y\cosh(y)\cosh(Xr)} - 2\nu\sum_{r_2=0}^{\infty}\frac{1}{\alpha_1^2\cos(\alpha_1 r)}\right. \\
\times &\quad \left[\frac{e^{x_7 t}(1+x_7\omega)^2\left[p_0(1-e^{-Tx_7})+\rho g\sin\gamma\right]}{x_7[l+(1+x_7\omega)^2]}\right. \\
+ &\quad \frac{e^{x_8 t}(1+x_8\omega)^2\left[p_0(1-e^{-Tx_8})+\rho g\sin\gamma\right]}{x_8[l+(1+x_8\omega)^2]}\Bigg]\Bigg\} - \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)} \\
\times &\quad \frac{\sin(\beta)}{\beta\cosh(\beta)}\left[\frac{e^{x_3 t}(1+x_3\omega)^2\left[p_0(1-e^{-Tx_3})+\rho g\sin\gamma\right]}{x_3[l+(1+x_3\omega)^2]}\right. \\
+ &\quad \frac{e^{x_4 t}(1+x_4\omega)^2\left[p_0(1-e^{-Tx_4})+\rho g\sin\gamma\right]}{x_4[l+(1+x_4\omega)^2]}\Bigg] - \frac{2\mu J}{\pi\nu}\sum_{r_1=0}^{\infty}\frac{1}{r_1} \\
\times &\quad \sin\left(\frac{r_1\pi}{r}s\right)\left\{(\rho g\sin\gamma)\frac{Y\sinh(y)}{X^2\cosh(y)} + \frac{\nu\pi^2}{2}\sum_{r_2=0}^{\infty}(2r_2+1)^2\frac{1}{{\alpha_1}^2}\right. \\
\times &\quad \left[\frac{e^{x_7 t}(1+x_7\omega)^2\left[p_0(1-e^{-Tx_7})+\rho g\sin\gamma\right]}{x_7[l+(1+x_7\omega)^2]}\right. \\
+ &\quad \frac{e^{x_8 t}(1+x_8\omega)^2\left[p_0(1-e^{-Tx_8})+\rho g\sin\gamma\right]}{x_8[l+(1+x_8\omega)^2]}\Bigg]\Bigg\} - u_0\mu\nu\pi \\
\times &\quad \sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right)\sum_{r_2=0}^{\infty}(2r_2+1)^2\left[\frac{e^{x_7 t}(1-e^{-Tx_7})(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}\right. \\
+ &\quad \frac{e^{x_8 t}(1-e^{-Tx_8})(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\Bigg] + 2u_1\mu\nu\sum_{r_1=0}^{\infty}\frac{[1-(-1)^{r_1}]}{r_1}\sin\left(\frac{r_1\pi}{r}s\right) \\
\times &\quad \sum_{r_2=0}^{\infty}(-1)^{r_2}(2r_2+1)\left[\frac{e^{x_7 t}(1-e^{-Tx_7})(1+x_7\omega)^2}{x_7[l+(1+x_7\omega)^2]}\right. \\
+ &\quad \frac{e^{x_8 t}(1-e^{-Tx_8})(1+x_8\omega)^2}{x_8[l+(1+x_8\omega)^2]}\Bigg] + \frac{\mu J}{\nu}(\rho g\sin\gamma)\frac{sinh(Xs)}{X\cosh(Xr)} + \frac{2\mu J}{r} \\
\times &\quad \sum_{r_2=0}^{\infty}\sin\left[\frac{(2r_2+1)\pi}{2r}s\right]\left[\frac{e^{x_3 t}(1+x_3\omega)^2\left[p_0(1-e^{-Tx_3})+\rho g\sin\gamma\right]}{x_3[l+(1+x_3\omega)^2]}\right.
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{e^{x_4 t}(1+x_4\omega)^2\left[p_0(1-e^{-Tx_4})+\rho g\sin\gamma\right]}{x_4[l+(1+x_4\omega)^2]}\Bigg] - \frac{2\mu J}{\nu r}\sum_{r_1=0}^{\infty}(-1)^{r_1} \\
&\times \quad \cos\left(\frac{r_1\pi}{r}s\right)\left(\frac{\rho g\sin\gamma}{Y^2}\right) - \frac{2\pi^2\mu J}{r^3\nu}\sum_{r_1=0}^{\infty} r_1^2\cos\left(\frac{r_1\pi}{r}s\right)\Bigg\{\frac{(\rho g\sin\gamma)}{X^2Y^2\cosh(Xr)} \\
&- \quad \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{1}{\rho^2}\Bigg[\frac{e^{x_3 t}(1+x_3\omega)^2\left[p_0(1-e^{-Tx_3})+\rho g\sin\gamma\right]}{x_3[l+(1+x_3\omega)^2]} \\
&+ \quad \frac{e^{x_4 t}(1+x_4\omega)^2\left[p_0(1-e^{-Tx_4})+\rho g\sin\gamma\right]}{x_4[l+(1+x_4\omega)^2]}\Bigg]\Bigg\} \\
&+ \quad \frac{2\mu J}{\nu r}\sum_{r_1=0}^{\infty}\frac{1}{r_1}\cos\left(\frac{r_1\pi}{r}s\right)\frac{(\rho g\sin\gamma)}{X^2}
\end{aligned}
$$

$$
\begin{aligned}
D_{s_0} \quad &= \quad \frac{J\mu}{\nu}(\rho g\sin\gamma)\frac{\sinh(Xs)}{X\cosh(Xr)} + \frac{2\mu J}{r}\sum_{r_2=0}^{\infty}(-1)^{r_2}\sin\left[\frac{(2r_2+1)\pi}{2r}s\right] \\
&\times \quad \Bigg[\frac{e^{x_3 t}(1+x_3\omega)^2\left[p_0(1-e^{-Tx_3})+\rho g\sin\gamma\right]}{x_3[l+(1+x_3\omega)^2]} \\
&+ \quad \frac{e^{x4t}(1+x_4\omega)^2\left[p_0(1-e^{-Tx_4})+\rho g\sin\gamma\right]}{x_4[l+(1+x_4\omega)^2]}\Bigg] - \frac{2\mu J}{\nu r}\sum_{r_1=0}^{\infty}(-1)^{r_1} \\
&\times \quad \cos\left(\frac{r_1\pi}{r}s\right)\Bigg\{\frac{\rho g\sin\gamma}{Y^2\cosh(y)} - \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)} \\
&\times \quad \Bigg[\frac{e^{x_7 t}(1+x_7\omega)^2\left[p_0(1-e^{-Tx_7})+\rho g\sin\gamma\right]}{x_7[l+(1+x_7\omega)^2]} \\
&+ \quad \frac{e^{x_8 t}(1+x_8\omega)^2\left[p_0(1-e^{-Tx_8})+\rho g\sin\gamma\right]}{x_8[l+(1+x_8\omega)^2]}\Bigg]\Bigg\} - \frac{2\pi^2\mu J}{\nu r^3}\sum_{r_1=0}^{\infty} r_1^2 \\
&\times \quad \cos\left(\frac{r_1\pi}{r}s\right)\Bigg\{\frac{\rho g\sin\gamma}{X^2Y^2\cosh(Xr)} + \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{1}{\alpha_1^2\cos(\alpha_1 r)} \\
&\times \quad \Bigg[\frac{e^{x_7 t}(1+x_7\omega)^2\left[p_0(1-e^{-Tx_7})+\rho g\sin\gamma\right]}{x_7[l+(1+x_7\omega)^2]}
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{e^{x_8 t}(1+x_8\omega)^2\left[p_0(1-e^{-Tx_8})+\rho g\sin\gamma\right]}{x_8[l+(1+x_8\omega)^2]}\Bigg] - \frac{4\nu}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)} \\
&\times \quad \frac{1}{\beta^2\cosh(\beta)}\Bigg[\frac{e^{x_3 t}(1+x_3\omega)^2\left[p_0(1-e^{-Tx_3})+\rho g\sin\gamma\right]}{x_3[l+(1+x_3\omega)^2]} \\
&+ \quad \frac{e^{x_4 t}(1+x_4\omega)^2\left[p_0(1-e^{-Tx_4})+\rho g\sin\gamma\right]}{x_4[l+(1+x_4\omega)^2]}\Bigg]\Bigg\} + \frac{2\mu J}{\nu r}\sum_{r_1=0}^{\infty}\frac{1}{r_1} \\
&\times \quad \cos\left(\frac{r_1\pi}{r}s\right)\Bigg\{\frac{\rho g\sin\gamma}{X^2\cosh(y)} - \nu\pi\sum_{r_2=0}^{\infty}(-1)^{r_2}\frac{(2r_2+1)}{\alpha_1^2} \\
&\times \quad \Bigg[\frac{e^{x_7 t}(1+x_7\omega)^2\left[p_0(1-e^{-Tx_7})+\rho g\sin\gamma\right]}{x_7[l+(1+x_7\omega)^2]} \\
&+ \quad \frac{e^{x_8 t}(1+x_8\omega)^2\left[p_0(1-e^{-Tx_8})+\rho g\sin\gamma\right]}{x_8[l+(1+x_8\omega)^2]}\Bigg]\Bigg\} - \frac{2u_0\mu\nu\pi}{r} \\
&\times \quad \sum_{r_1=0}^{\infty}[1-(-1)^{r_1}]\cos\left(\frac{r_1\pi}{r}s\right)\sum_{r_2=0}^{\infty}(-1)^{r_2}(2r_2+1) \\
&\times \quad \left[\frac{e^{x_7 t}(1+x_7\omega)^2(1-e^{-Tx_7})}{x_7[l+(1+x_7\omega)^2]} + \frac{e^{x_8 t}(1+x_8\omega)^2(1-e^{-Tx_8})}{x_8[l+(1+x_8\omega)^2]}\right] \\
&+ \quad \frac{4u_1\mu\nu}{r}\sum_{r_1=0}^{\infty}[1-(-1)^{r_1}]\cos\left(\frac{r_1\pi}{r}s\right) \\
&\times \quad \left[\frac{e^{x_7 t}(1+x_7\omega)^2(1-e^{-Tx_7})}{x_7[l+(1+x_7\omega)^2]} + \frac{e^{x_8 t}(1+x_8\omega)^2(1-e^{-Tx_8})}{x_8[l+(1+x_8\omega)^2]}\right]
\end{aligned}
$$

where

$$
\begin{aligned}
I &= \frac{M^2h^2}{u_0}\left(p_0+\rho g\sin\gamma\right), \quad J = \frac{M^2h^2}{u_0}, \quad a_1 = 4r^2\omega \\
b_1 &= \left[(C_r+M^2)\nu\omega+l+1\right]4r^2 + (2r_2+1)^2\pi^2\nu\omega \\
c_1 &= (C_r+M^2)4\nu r^2 + (2r_2+1)^2\pi^2\nu \\
x_3 &= \frac{-b_1+\sqrt{b_1^2-4a_1c_1}}{2a_1}, \quad x_4 = \frac{-b_1-\sqrt{b_1^2-4a_1c_1}}{2a_1}, \quad \phi = \frac{\omega\rho h^2}{\nu} \\
a_3 &= 4r^2\nu, \quad b_3 = \left[(C_r+M^2)\nu\omega+l+1\right]4r^2 + \left[4r_1^2+(2r_2+1)^2r^2\right]\pi^2\nu\omega
\end{aligned}
$$

$$
\begin{aligned}
c_3 &= (C_r + M^2)4\nu r^2 + \left[4r_1^2 + (2r_2+1)^2 r_2\right]\pi^2\nu \\
x_7 &= \frac{-b_3 + \sqrt{b_3^2 - 4a_3c_3}}{2a_3}, \quad x_8 = \frac{-b_3 - \sqrt{b_3^2 - 4a_3c_3}}{2a_3}, \quad \alpha^2 = \frac{r_1^2\pi^2}{r^2} \\
\beta^2 &= \frac{4r_1^2\pi^2 - (2r_2+1)^2\pi^2}{4r^2}, \quad X^2 = C_r + M^2, \quad Y^2 = X^2 + \alpha^2 \\
\alpha_1^2 &= \frac{4r_1^2\pi^2 + (2r_2+1)^2\pi^2}{4r^2}, \quad \alpha_2^2 = C_r + M^2 - \frac{\phi}{\nu} - \frac{\phi l}{\nu(1-\phi\omega)} \\
\alpha_3^2 &= \alpha_2^2 + \frac{r_1^2\pi^2}{r^2}
\end{aligned}
$$

4.Conclusion

One can observed, the paraboloid nature of both fluid and dust phase velocities which are drawn as in figures 3 to 8. From these graphs, it is evident that the flow of fluid particles is parallel to that of dust. Also, one can see that if the dust is very fine i.e., mass of the dust particles is negligibly small then the relaxation time of dust particle decreases and ultimately as $\tau \to 0$ the velocities of fluid and dust particles will be the same. Further, we can see the effect of inclined angle γ on the velocity fields of both fluid and dust phase i.e., as inclined angle increases the velocities of both fluid and dust particles increases. As a particular case if the angle $\gamma = 0$ then the results coincides with the previous result [5].

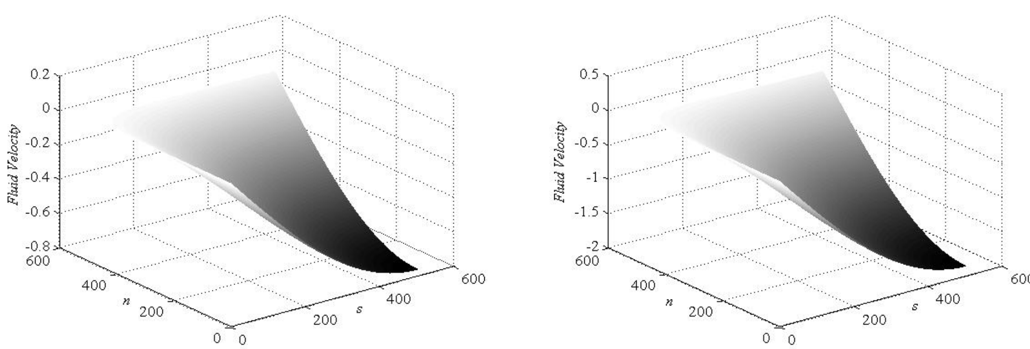

Figure-3: Variation of fluid velocity with s and n (for $\gamma = 10$ & 40, Case-1)

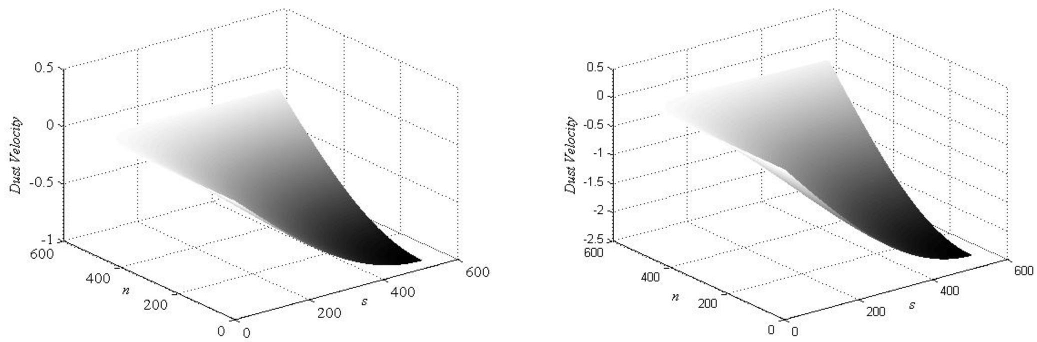

Figure-4: Variation of dust velocity with s and n (for $\gamma = 10$ & 40, Case-1)

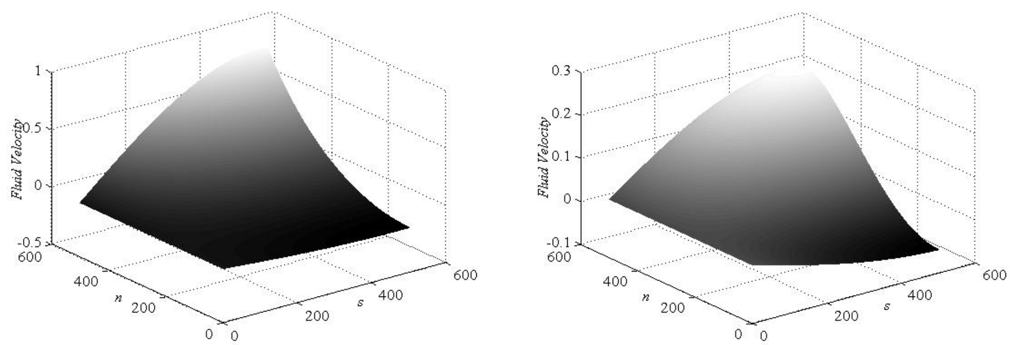

Figure-5: Variation of fluid velocity with s and n (for $\gamma = 10$ & 40, Case-2)

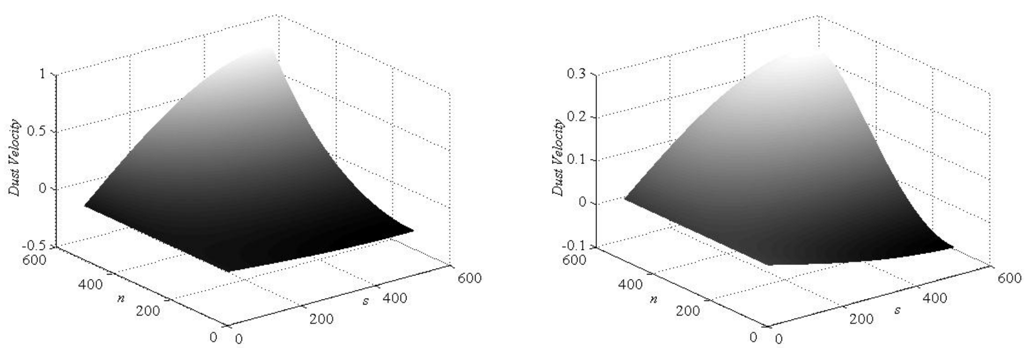

Figure-6: Variation of dust velocity with s and n (for $\gamma = 10$ & 40, Case-2)

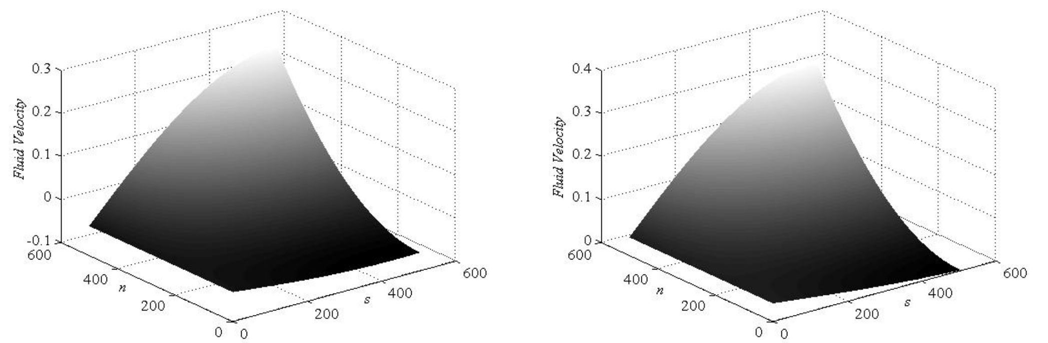

Figure-7: Variation of fluid velocity with s and n (for $\gamma = 10$ & 40, Case-3)

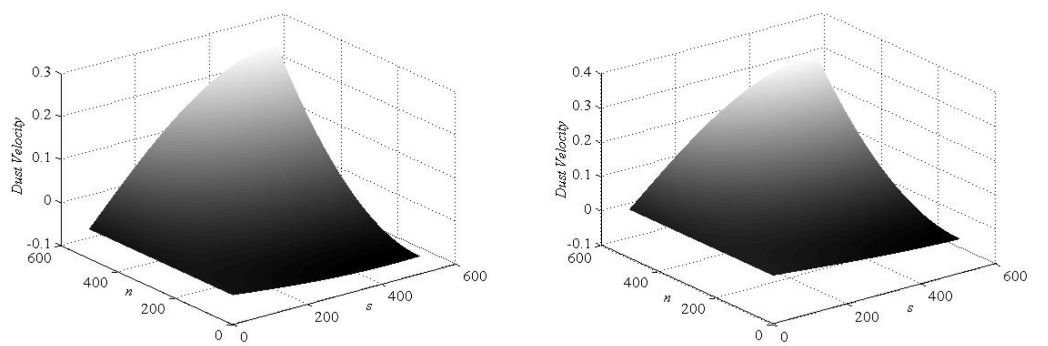

Figure-8: Variation of dust velocity with s and n (for $\gamma = 10$ & 40, Case-3)

References

[1] E.Amos, *Magnetic effect on pulsatile flow in a constricted axis-symmetric tube,* Indian Journal Pure Appl. Math., **34**(9) (2003) 1315-1326.

[2] S.G.Agrawal and N.K.Varshney, *MHD flow of a dusty gas induced by the motion of a semi-infinite flat plate moving with the velocity decreasing,* Jour.of.M.A.C.T.Com, **19**(97) (1986).

[3] C.S.Bagewadi and B.J.Gireesha, *A study of two dimensional steady dusty fluid flow under varying temperature,* Int. Journal of Appl. Mech. & Eng., **9**4 (2004) 647-653.

[4] C.S.Bagewadi and B.J.Gireesha, *A study of two dimensional unsteady dusty fluid flow under varying pressure gradient,* Tensor, N.S., **64** (2003) 232-240.

[5] C.S.Bagewadi, B.C.PrasannaKumara and B.J.Gireesha, *Transition Motion of an Unsteady Dusty Fluid Through an Open Rectangular Channel,* Anal. Uni. Oradea, fasc. Math., XVI (2009).

[6] Barret O' Nell, *Elementary Differential Geometry,* Academic Press, New York & London, 1966.

[7] G.S.Beavers and D.D.Joseph, *Boundary conditions at a naturally permeable wall,* Jour.Fluid.Mech., **30**(167) (1967)

[8] A.J.Chamkha, *Hydromagnetic two-phase flow in a channel,* Int. J. Engng. Sci., **33**(3), (1995) 437-446;

[9] N.Datta and D.C.Dalal, *Pulsatile flow and heat transfer of a dusty fluid through an infinite long annular pipe,* Int., J. Multiphase Flow, **21**(3), (1995) 515-528.

[10] B.J.Gireesha, C. S. Bagewadi & B.C.Prasannakumara, *Study of unsteady dusty fluid Flow through rectangular channel in Frenet Frame Field system,* **In press**, Int. J. Pure and App. Math.

[11] B.J.Gireesha, C. S. Bagewadi & B.C.Prasannakumara, *Flow of unsteady dusty fluid between two parallel plates under constant pressure gradient,* To be appear in Tensor.N.S. **68** (2007).

[12] Indrasena, *Steady rotating hydrodynamic-flows,* Tensor, N.S., **32** (1978) 350-354.

[13] R.P.Kanwal, *Variation of flow quantities along streamlines, principal normals and bi-normals in three-dimensional gas flow,* J.Math., **6** (1957) 621-628.

[14] J.T.C.Liu, *Flow induced by an oscillating infinite plat plate in a dusty gas,* Phys. Fluids, **9** (1966) 1716-1720.

[15] D.H.Michael and D.A.Miller, *Plane parallel flow of a dusty gas,* Mathematika, **13** (1966) 97-109.

[16] G.Purushotham and Indrasena, *On intrinsic properties of steady gas flows ,* Appl.Sci. Res., **A 15**(1965) 196-202.

[17] P.G.Saffman, *On the stability of laminar flow of a dusty gas,* Journal of Fluid Mechanics, **13**(1962) 120-128.

[18] Shri Ram, B.K.Guptha and N.P.Singh, *unsteady flow of a dusty viscous stratified fluid through an inclined open rectangular channel,* Jour,Math.Phy.Sci., **27**(2)(1993) 117-128.

[19] N.C.Ghosh, B.C.Ghosh and L.Debnath, *The hydromagnetic flow of a Dusty Visco-Elastic fluid between two infinite parallel plates,* Int. J. Computers and Math. with Applications, **39**(2000) 103-116.

[20] C.Truesdell, *Intrinsic equations of spatial gas flows,* Z.Angew. Math. Mech, **40** (1960) 9-14.

[21] Tyn Myint-U, *Partial Differential Equations of Mathematical Physics,* American Elsevier Publishing Company, **40** New York, 1965.

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