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SOME RESULTS ON THE COMPLEX OSCILLATION THEORY OF SOME DIFFERENTIAL POLYNOMIALS

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Abstract. In this paper, we investigate the complex oscillation of the differential polynomial $g_f = d_2 f^{''} + d_1 f^{'} + d_0 f$, where d_j $(j = 0,1,2)$ are meromorphic functions with finite iterated $p-$order not all equal to zero generated by solutions of the differential equation $f^{''} + A(z) f = 0$, where $A(z)$ is a transcendental meromorphic function with finite iterated $p-$order $\rho_p(A) = \rho > 0$.

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1. Introduction and main results

Throughout this paper, we assume that the reader is familiar with the fundamental results and the standard notations of the Nevanlinna's value distribution theory (see [5, 7, 10]). In addition, we will use $\lambda(f)$ and $\lambda(1/f)$ to denote respectively the exponents of convergence of the zero-sequence and the pole-sequence of a meromorphic function f, $\rho(f)$ to denote the order of growth of f, $\overline{\lambda}(f)$ and $\overline{\lambda}(1/f)$ to denote respectively the exponents of convergence of the sequence of distinct zeros and distinct poles of f. In order to express the rate of growth of meromorphic solutions of infinite order, we recall the following definition.

Definition 1.1 [9, 11, 13] *Let f be a meromorphic function. Then the hyper order $\rho_2(f)$ of $f(z)$ is defined by*

$$\rho_2(f) = \overline{\lim_{r\to+\infty}} \frac{\log\log T(r,f)}{\log r}, \tag{1.1}$$

where $T(r, f)$ is the Nevanlinna characteristic function of f (see [5, 10]).

Definition 1.2 [6, 9, 11] *Let f be a meromorphic function. Then the hyper exponent of convergence of the sequence of distinct zeros of $f(z)$ is defined by*

$$\overline{\lambda}_2(f) = \overline{\lim_{r\to+\infty}} \frac{\log\log \overline{N}\left(r, \frac{1}{f}\right)}{\log r}, \tag{1.2}$$

where $\overline{N}\left(r, \frac{1}{f}\right)$ *is the counting function of distinct zeros of* $f(z)$ *in* $\{|z| < r\}$.

Consider the linear differential equation

$$f'' + A(z) f = 0, \tag{1.3}$$

where $A(z)$ is a transcendental meromorphic function. Many important results have been obtained on the fixed points of general transcendental meromorphic functions for almost four decades (see [14]). However, there are a few studies on the fixed points of solutions of differential equations. In [12], Wang and Lü have investigated the fixed points and hyper order of solutions of second order linear differential equations with meromorphic coefficients and their derivatives and have obtained the following result:

Theorem A [12] *Suppose that* $A(z)$ *is a transcendental meromorphic function satisfying* $\delta(\infty, A) = \underline{\lim}_{r\to+\infty} \frac{m(r,A)}{T(r,A)} > 0$, $\rho(A) = \rho < +\infty$. *Then every meromorphic solution* $f(z) \not\equiv 0$ *of the equation* (1.3) *satisfies that* f *and* f', f'' *all have infinitely many fixed points and*

$$\overline{\lambda}(f - z) = \overline{\lambda}\left(f' - z\right) = \overline{\lambda}\left(f'' - z\right) = \rho(f) = +\infty, \tag{1.4}$$

$$\overline{\lambda}_2(f - z) = \overline{\lambda}_2\left(f' - z\right) = \overline{\lambda}_2\left(f'' - z\right) = \rho_2(f) = \rho. \tag{1.5}$$

In [9], Theorem A has been generalized to higher order linear differential equations by Liu Ming-Sheng and Zhang Xiao-Mei as follows :

Theorem B [9] *Suppose that* $k \geq 2$ *and* $A(z)$ *is a transcendental meromorphic function satisfying* $\delta(\infty, A) = \delta > 0$, $\rho(A) = \rho < +\infty$. *Then every meromorphic solution* $f(z) \not\equiv 0$ *of*

$$f^{(k)} + A(z) f = 0 \tag{1.6}$$

satisfies that f *and* $f', f'', ..., f^{(k)}$ *all have infinitely many fixed points and*

$$\overline{\lambda}(f - z) = \overline{\lambda}\left(f' - z\right) = ... = \overline{\lambda}\left(f^{(k)} - z\right) = \rho(f) = +\infty, \tag{1.7}$$

$$\overline{\lambda}_2(f - z) = \overline{\lambda}_2\left(f' - z\right) = ... = \overline{\lambda}_2\left(f^{(k)} - z\right) = \rho_2(f) = \rho. \tag{1.8}$$

Recently, Theorem A has been generalized to differential polynomials by the first author as follows:

Theorem C [2] *Let $A(z)$ be a transcendental meromorphic function of finite order $\rho(A) = \rho > 0$ such that $\delta(\infty, A) = \delta > 0$. Suppose, moreover, that either:*
(i) *all poles of f are of uniformly bounded multiplicity or that*
(ii) $\delta(\infty, f) > 0$.
Let d_j $(j = 0,1,2)$ be polynomials that are not all equal to zero, and let $\varphi(z) \not\equiv 0$ be a meromorphic function of finite order. If $f(z) \not\equiv 0$ is a meromorphic solution of (1.3) with $\lambda\left(\frac{1}{f}\right) < +\infty$, then the differential polynomial $g_f = d_2 f'' + d_1 f' + d_0 f$ satisfies $\overline{\lambda}(g_f - \varphi) = +\infty$.

The first main purpose of this paper is to improve Theorem C. The first theorem considers the case " d_j $(j = 0,1,2)$ are meromorphic functions and $h \not\equiv 0$" which is different from " d_j $(j = 0,1,2)$ are polynomials and $\lambda\left(\frac{1}{f}\right) < +\infty$" in Theorem C.

Let us denote by

$$\alpha_1 = d_1, \quad \alpha_0 = d_0 - d_2 A, \quad \beta_1 = -d_2 A + d_0 + d_1', \tag{1.9}$$

$$\beta_0 = -d_1 A - (d_2 A)' + d_0', \tag{1.10}$$

$$h = \alpha_1 \beta_0 - \alpha_0 \beta_1, \tag{1.11}$$

where A , d_j $(j = 0.1.2)$, φ are meromorphic functions. We first obtain:

Theorem 1.1 *Let $A(z)$ be a transcendental meromorphic function of finite order $\rho(A) = \rho > 0$ such that $\delta(\infty, A) = \delta > 0$. Let d_j $(j = 0,1,2)$ be meromorphic functions with finite order not all equal to zero such that $h \not\equiv 0$ and let $\varphi(z)\ (\not\equiv 0)$ be a meromorphic function of finite order such that $\alpha_1 \varphi' - \beta_1 \varphi \not\equiv 0$. Suppose, moreover, that either:*
(i) *all poles of f are of uniformly bounded multiplicity or that*
(ii) $\delta(\infty, f) > 0$.
If $f(z) \not\equiv 0$ is a meromorphic solution of (1.3), then the differential polynomial $g_f = d_2 f'' + d_1 f' + d_0 f$ satisfies $\overline{\lambda}(g_f - \varphi) = \rho(f) = +\infty$ and $\overline{\lambda}_2(g_f - \varphi) = \rho_2(f) = \rho(A) = \rho$.

The second main purpose of this paper is to study the relation between differential polynomials generated by solutions of the differential equation (1.3) and meromorphic functions of finite iterated order.

Before we can state our second result, we need to give some definitions.

For the definition of the iterated order of a meromorphic function, we use the same definition as in [6], [3, p. 317], [7, p. 129]. For all $r \in \mathbb{R}$, we define $\exp_1 r := e^r$ and $\exp_{p+1} r := \exp\left(\exp_p r\right)$, $p \in \mathbb{N}$. We also define for all r sufficiently large $\log_1 r := \log r$ and $\log_{p+1} r := \log\left(\log_p r\right)$, $p \in \mathbb{N}$. Moreover, we denote by $\exp_0 r := r$, $\log_0 r := r$, $\log_{-1} r := \exp_1 r$ and $\exp_{-1} r := \log_1 r$.

Definition 1.3 (see [6, 7]) *Let f be a meromorphic function. Then the iterated $p-$order $\rho_p(f)$ of f is defined by*

$$\rho_p(f) = \overline{\lim_{r\to+\infty}} \frac{log_p T(r,\ f)}{log\ r} \quad (p \geq 1 \quad \text{is an integer}). \tag{1.12}$$

For $p = 1$, this notation is called order and for $p = 2$ hyper-order.

Definition 1.4 (see [6, 7]) *The finiteness degree of the order of a meromorphic function f is defined by*

$$i(f) = \begin{cases} 0, & \text{for } f \text{ rational,} \\ \min\left\{j \in \mathbb{N} : \rho_j(f) < +\infty\right\}, & \text{for } f \text{ transcendental for which} \\ & \text{some } j \in \mathbb{N} \text{ with } \rho_j(f) < +\infty \text{ exists,} \\ +\infty, & \text{for } f \text{ with } \rho_j(f) = +\infty \text{ for all } j \in \mathbb{N}. \end{cases} \tag{1.13}$$

Definition 1.5 (see [8]) *Let f be a meromorphic function. Then the iterated exponent of convergence of the sequence of distinct zeros of $f(z)$ is defined by*

$$\overline{\lambda}_p(f) = \overline{\lim_{r\to+\infty}} \frac{\log_p \overline{N}\left(r, \frac{1}{f}\right)}{\log r} \quad (p \geq 1 \quad \text{is an integer}). \tag{1.14}$$

For $p = 1$, this notation is called exponent of convergence of the sequence of distinct zeros and for $p = 2$ hyper-exponent of convergence of the sequence of distinct zeros (see [9]*).*

Definition 1.6 (see [8]) *Let f be a meromorphic function. Then the iterated exponent of convergence of the sequence of distinct fixed points of $f(z)$ is defined by*

$$\overline{\tau}_p(f) = \overline{\lambda}_p(f - z) = \overline{\lim_{r\to+\infty}} \frac{\log_p \overline{N}\left(r, \frac{1}{f-z}\right)}{\log r} \quad (p \geq 1 \quad \text{is an integer}). \tag{1.15}$$

For $p = 1$, this notation is called exponent of convergence of the sequence of distinct fixed points and for $p = 2$ hyper-exponent of convergence of the sequence of distinct *fixed points (see [9]). Thus* $\overline{\tau}_p(f) = \overline{\lambda}_p(f-z)$ *is an indication of oscillation of distinct fixed points of* $f(z)$.

We obtain the following result:

Theorem 1.2 *Let* $A(z)$ *be a transcendental meromorphic function of finite iterated* $p-$*order* $\rho_p(A) = \rho > 0$ *such that* $\delta(\infty, A) = \delta > 0$. *Let* d_j $(j = 0,1,2)$ *be meromorphic functions with finite iterated* $p-$*order not all equal to zero such that* $h \not\equiv 0$ *and let* $\varphi(z) (\not\equiv 0)$ *be a meromorphic function of finite iterated* $p-$*order such that* $\alpha_1\varphi' - \beta_1\varphi \not\equiv 0$. *Suppose, moreover, that either:*
(i) *all poles of* f *are of uniformly bounded multiplicity or that*
(ii) $\delta(\infty, f) > 0$.
If $f(z) \not\equiv 0$ *is a meromorphic solution of* (1.3), *then the differential polynomial* $g_f = d_2f'' + d_1f' + d_0f$ *satisfies* $\overline{\lambda}_p(g_f - \varphi) = \rho_p(f) = +\infty$ *and* $\overline{\lambda}_{p+1}(g_f - \varphi) = \rho_{p+1}(f) = \rho_p(A) = \rho$.

Setting $p = 1$ and $\varphi(z) = z$ in Theorem 1.2, we obtain the following corollary:

Corollary 1.1 *Let* $A(z)$ *be a transcendental meromorphic function of finite order* $\rho(A) = \rho > 0$ *such that* $\delta(\infty, A) = \delta > 0$, *let* d_j $(j = 0,1,2)$ *be meromorphic functions with finite order not all equal to zero such that* $h \not\equiv 0$ *and* $\alpha_1 - \beta_1 z \not\equiv 0$. *Suppose, moreover, that either:*
(i) *all poles of* f *are of uniformly bounded multiplicity or that*
(ii) $\delta(\infty, f) > 0$.
If $f(z) \not\equiv 0$ *is a meromorphic solution of* (1.3), *then the differential polynomial* $g_f = d_2f'' + d_1f' + d_0f$ *has infinitely many fixed points and satisfies* $\overline{\tau}(g_f) = \rho(f) = +\infty$, $\overline{\tau}_2(g_f) = \rho_2(f) = \rho(A) = \rho$.

2. Several Lemmas

We need the following lemmas in the proofs of our theorems.

Lemma 2.1 (see Remark 1.3 of [6]). *If* f *is a meromorphic function with* $i(f) = p \geq 1$, *then* $\rho_p(f) = \rho_p\left(f'\right)$.

Lemma 2.2 [8] *If* f *is a meromorphic function with* $0 < \rho_p(f) < \rho$ $(p \geq 1)$, *then* $\rho_{p+1}(f) = 0$.

Lemma 2.3 [4] *Let* $A_0, A_1, ..., A_{k-1}, F \not\equiv 0$ *be finite order meromorphic functions. If* f *is a meromorphic solution with* $\rho(f) = +\infty$ *of the equation*

$$f^{(k)} + A_{k-1}f^{(k-1)} + ... + A_1f' + A_0f = F, \tag{2.1}$$

then $\overline{\lambda}(f) = \lambda(f) = \rho(f) = +\infty$.

Lemma 2.4 [2] *Let* $A_0, A_1, ..., A_{k-1}, F\ (\not\equiv 0)$ *be finite order meromorphic functions. If* f *is a meromorphic solution of the equation* (2.1) *with* $\rho(f) = +\infty$ *and* $\rho_2(f) = \rho$, *then* f *satisfies* $\overline{\lambda}_2(f) = \lambda_2(f) = \rho_2(f) = \rho$.

Lemma 2.5 [2] *Let* $k \geq 2$ *and* $A(z)$ *be a transcendental meromorphic function of finite order* $\rho(A) = \rho > 0$ *such that* $\delta(\infty, A) = \delta > 0$. *Suppose, moreover, that either:*
(i) *all poles of* f *are of uniformly bounded multiplicity or that*
(ii) $\delta(\infty, f) > 0$.
Then every meromorphic solution $f(z) \not\equiv 0$ *of* (1.6) *satisfies* $\rho(f) = +\infty$ *and* $\rho_2(f) = \rho(A) = \rho$.

Lemma 2.6 [1] *Let* $k \geq 2$ *and* $A(z)$ *be a transcendental meromorphic function of finite iterated* $p-$*order* $\rho_p(A) = \rho > 0$ *such that* $\delta(\infty, A) = \delta > 0$. *Suppose, moreover, that either:*
(i) *all poles of* f *are of uniformly bounded multiplicity or that*
(ii) $\delta(\infty, f) > 0$.
Then every meromorphic solution $f(z) \not\equiv 0$ *of* (1.6) *satisfies* $i(f) = p + 1$ *and* $\rho_p(f) = +\infty$, $\rho_{p+1}(f) = \rho_p(A) = \rho$.

Lemma 2.7 [1] *Let* $A_0, A_1, ..., A_{k-1}, F \not\equiv 0$ *be finite iterated* $p-$*order meromorphic functions. If* f *is a meromorphic solution with* $\rho_p(f) = +\infty$ *and* $\rho_{p+1}(f) = \rho < +\infty$ *of the equation* (2.1), *then* $\overline{\lambda}_p(f) = \rho_p(f) = +\infty$ *and* $\overline{\lambda}_{p+1}(f) = \rho_{p+1}(f) = \rho$.

Lemma 2.8 *Let* $A(z)$ *be a transcendental meromorphic function with finite iterated* $p-$*order* $\rho_p(A) = \rho > 0$ *such that* $\delta(\infty, A) = \delta > 0$, *let* d_j $(j = 0,1,2)$ *be meromorphic functions with finite iterated* $p-$*order not all equal to zero such that* $h \not\equiv 0$. *Suppose, moreover, that either:*
(i) *all poles of* f *are of uniformly bounded multiplicity or that*
(ii) $\delta(\infty, f) > 0$.
If $f(z) \not\equiv 0$ *is a meromorphic solution of* (1.3), *then the differential polynomial* $g_f = d_2 f'' + d_1 f' + d_0 f$ *satisfies* $i(g_f) = p+1$ *and* $\rho_p(g_f) = \rho_p(f) = +\infty$, $\rho_{p+1}(g_f) = \rho_{p+1}(f) = \rho_p(A) = \rho$.

Proof. Suppose that $f\ (\not\equiv 0)$ is a meromorphic solution of equation (1.3). Then by Lemma 2.6, we have $\rho_p(f) = +\infty$ and $\rho_{p+1}(f) = \rho_p(A) = \rho$. Set $g_f = d_2 f'' + d_1 f' + d_0 f$, we need to prove $\rho_p(g_f) = +\infty$ and $\rho_{p+1}(g_f) = \rho$. By substituting $f'' = -Af$ into g_f, we get

$$g_f = d_1 f' + (d_0 - d_2 A) f. \qquad (2.18)$$

Differentiating both a sides of equation (2.18), we obtain

$$g_f' = (-d_2 A + d_0 + d_1')f' + (-d_1 A - (d_2 A)' + d_0')f. \qquad (2.19)$$

Then by $(1.9), (1.10)$, (2.18) and (2.19), we have

$$\alpha_1 f' + \alpha_0 f = g_f, \tag{2.20}$$

$$\beta_1 f' + \beta_0 f = g_f'. \tag{2.21}$$

Set

$$h = \alpha_1 \beta_0 - \beta_1 \alpha_0. \tag{2.22}$$

By the condition $h \not\equiv 0$ and $(2.20) - (2.22)$, we get

$$f = \frac{\alpha_1 g_f' - \beta_1 g_f}{h}. \tag{2.23}$$

If $\rho_p(g_f) < +\infty$, then by (2.23) and Lemma 2.1 we obtain $\rho_p(f) < +\infty$, and this is a contradiction. Hence $\rho_p(g_f) = \rho_p(f) = +\infty$.

Now we prove that $\rho_{p+1}(g_f) = \rho_{p+1}(f) = \rho_p(A) = \rho$. By (2.18), Lemma 2.1 and Lemma 2.2 we have $\rho_{p+1}(g_f) \leqslant \rho_{p+1}(f)$ and by (2.23) we get $\rho_{p+1}(f) \leqslant \rho_{p+1}(g_f)$. Hence $\rho_{p+1}(g_f) = \rho_{p+1}(f) = \rho_p(A) = \rho$.

Lemma 2.9 *Let $A(z)$ be a transcendental meromorphic function with finite order $\rho(A) = \rho > 0$ such that $\delta(\infty, A) = \delta > 0$, let d_j $(j = 0,1,2)$ be meromorphic functions with finite order not all equal to zero such that $h \not\equiv 0$. Suppose, moreover, that either:*
(i) *all poles of f are of uniformly bounded multiplicity or that*
(ii) $\delta(\infty, f) > 0$.
If $f(z) \not\equiv 0$ is a meromorphic solution of (1.3), then the differential polynomial $g_f = d_2 f'' + d_1 f' + d_0 f$ satisfies $\rho(g_f) = \rho(f) = +\infty$ and $\rho_2(g_f) = \rho_2(f) = \rho(A) = \rho$.

Proof. Suppose that f $(\not\equiv 0)$ is a meromorphic solution of equation (1.3). Then by Lemma 2.5, we have $\rho(f) = +\infty$ and $\rho_2(f) = \rho(A) = \rho$. By using a similar arguments as in the proof of Lemma 2.8, we can prove Lemma 2.9.

3. Proof of Theorem 1.1

Suppose that $f(z) \not\equiv 0$ is a meromorphic solution of (1.3). Then by Lemma 2.5 we have $\rho(f) = +\infty$ and $\rho_2(f) = \rho(A) = \rho$. Set $w(z) = d_2 f'' + d_1 f' + d_0 f - \varphi$. Since $\rho(\varphi) < +\infty$, then by Lemma 2.9 we have $\rho(w) = \rho(g_f) = \rho(f) = +\infty$ and

$\rho_2(w) = \rho_2(g_f) = \rho_2(f) = \rho(A) = \rho$. In order to prove $\overline{\lambda}(g_f - \varphi) = +\infty$ and $\overline{\lambda}_2(g_f - \varphi) = \rho(A) = \rho$ we need to prove only $\overline{\lambda}(w) = +\infty$ and $\overline{\lambda}_2(w) = \rho(A) = \rho$. Substituting $g_f = w + \varphi$ into (2.23), we get

$$f = \frac{\alpha_1 w' - \beta_1 w}{h} + \psi, \tag{3.1}$$

where

$$\psi = \frac{\alpha_1 \varphi' - \beta_1 \varphi}{h}. \tag{3.2}$$

Substituting (3.1) into equation (1.3), we obtain

$$\frac{\alpha_1}{h} w''' + \phi_2 w'' + \phi_1 w' + \phi_0 w = -\left(\psi'' + A(z)\psi\right) = W, \tag{3.3}$$

where ϕ_j $(j = 0, 1, 2)$ are meromorphic functions with $\rho(\phi_j) < +\infty$ $(j = 0, 1, 2)$. By $\rho(\psi) < +\infty$ and the condition $\psi \not\equiv 0$, it follows by Lemma 2.5 that $W \not\equiv 0$. Then by Lemma 2.3 and Lemma 2.4, we obtain $\overline{\lambda}(w) = \lambda(w) = \rho(w) = +\infty$ and $\overline{\lambda}_2(w) = \lambda_2(w) = \rho_2(w) = \rho$, i.e., $\overline{\lambda}(g_f - \varphi) = \rho(f) = +\infty$ and $\overline{\lambda}_2(g_f - \varphi) = \rho_2(f) = \rho(A) = \rho$.

4. Proof of Theorem 1.2

Suppose that $f(z) \not\equiv 0$ is a meromorphic solution of (1.3). Then by Lemma 2.6 we have $\rho_p(f) = +\infty$ and $\rho_{p+1}(f) = \rho_p(A) = \rho$. Set $w(z) = d_2 f'' + d_1 f' + d_0 f - \varphi$. Since $\rho_p(\varphi) < +\infty$, then by Lemma 2.8 we have $\rho_p(w) = \rho_p(g_f) = \rho_p(f) = +\infty$ and $\rho_{p+1}(w) = \rho_{p+1}(g_f) = \rho_{p+1}(f) = \rho_p(A) = \rho$. In order to prove $\overline{\lambda}_p(g_f - \varphi) = +\infty$ and $\overline{\lambda}_{p+1}(g_f - \varphi) = \rho_p(A) = \rho$, we need to prove only $\overline{\lambda}_p(w) = +\infty$ and $\overline{\lambda}_{p+1}(w) = \rho_p(A) = \rho$. Substituting $g_f = w + \varphi$ into (2.23) and using a similar reasoning as in the proof of Theorem 1.1, we get that

$$\frac{\alpha_1}{h} w''' + \phi_2 w'' + \phi_1 w' + \phi_0 w = -\left(\psi'' + A(z)\psi\right) = W, \tag{4.1}$$

where ϕ_j $(j = 0, 1, 2)$ are meromorphic functions with $\rho_p(\phi_j) < \infty$ $(j = 0, 1, 2)$. By $\rho_p(\psi) < +\infty$ and the condition $\psi \not\equiv 0$, it follows by Lemma 2.6 that $W \not\equiv 0$. By Lemma 2.7, we obtain $\overline{\lambda}_p(w) = \rho_p(w) = +\infty$ and $\overline{\lambda}_{p+1}(w) = \rho_{p+1}(w) = \rho$, i.e., $\overline{\lambda}_p(g_f - \varphi) = \rho_p(f) = +\infty$ and $\overline{\lambda}_{p+1}(g_f - \varphi) = \rho_{p+1}(f) = \rho_p(A) = \rho$.

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