ON SPECIAL HYPERSURFACE OF A FINSLER SPACE WITH THE METRIC $\alpha + \frac{\beta^{n+1}}{\alpha^n}$

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Abstract. The purpose of the present paper is to investigate the various kinds of hypersurfaces of Finsler space with special (α, β) metric $\alpha + \frac{\beta^{n+1}}{\alpha^n}$ which is a generalization of the metric $\alpha + \frac{\beta^2}{\alpha}$ consider in [9].

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1. Introduction

Let $F^n = (M^n, L)$ be an n-dimensional Finsler space, i.e., a pair consisting of an n-dimensional differential manifold M^n equipped with a fundamental function $L(x, y)$. The concept of the (α, β)-metric $L(\alpha, \beta)$ was introduced by M. Matsumoto [5] and has been studied by many authors ([1],[2],[8]). A Finsler metric $L(x, y)$ is called an (α, β)-metric $L(\alpha, \beta)$ if L is a positively homogeneous function of α and β of degree one, where $\alpha^2 = a_{ij}(x)y^iy^j$ is a Riemannian metric and $\beta = b_i(x)y^i$ is a 1-form on M^n.

A hypersurface M^{n-1} of the M^n may be represented parametrically by the equation $x^i = x^i(u^\alpha)$, $\alpha = 1, \cdots, n-1$, where u^α are Gaussian coordinates on M^{n-1}. The following notations are also employed [3] : $B^i_{\alpha\beta} := \partial^2 x^i/\partial u^\alpha \partial u^\beta$, $B^i_{0\beta} := v^\alpha B^i_{\alpha\beta}$, $B^{ij\ldots}_{\alpha\beta\ldots} := B^i_\alpha B^j_\beta\ldots$, If the supporting element y^i at a point (u^α) of M^{n-1} is assumed to be tangential to M^{n-1}, we may then write $y^i = B^i_\alpha(u)v_\alpha$, so that v^α is thought of as the supporting element of M^{n-1} at the point (u^α).

Since the function $\underline{L}(u, v) := L(x(u), y(u, v))$ gives rise to a Finsler metric of M^{n-1}, we get an (n-1)-dimensional Finsler space $F^{n-1} = (M^{n-1}, \underline{L}(u, v))$.

In the present paper, we consider an n-dimensional Finsler space $F^n = (M^n, L)$ with (α, β)-metric $L(\alpha, \beta) = \alpha + \frac{\beta^{n+1}}{\alpha^n}$ and the hypersurface of F^n with $b_i(x) = \partial_i b$ being the gradient of a scalar function $b(x)$. We prove the conditions for this hypersurface to be a hyperplane of 1^{st} kind, 2^{nd} kind and 3^{rd} kind.

2. Preliminaries

Let $F^n = (M^n, L)$ be a special Finsler space with the metric

$$L(\alpha, \beta) = \alpha + \frac{\beta^{n+1}}{\alpha^n}. \quad (1)$$

The derivatives of the (1) with respect to α and β are given by

$$\begin{aligned} L_\alpha &= \frac{\alpha^{n+1} + \beta^{n+1}}{\alpha^n}, \\ L_\beta &= \frac{(n+1)\beta^n}{\alpha^n}, \\ L_{\alpha\alpha} &= \frac{n(n+1)\beta^{n+1}}{\alpha^{n+2}}, \\ L_{\beta\beta} &= \frac{n(n+1)\beta^{n-1}}{\alpha^n}, \\ L_{\alpha\beta} &= \frac{-n(n+1)\beta^n}{\alpha^{n+1}}, \end{aligned}$$

where $L_\alpha = \partial L/\partial\alpha$, $L_\beta = \partial L/\partial\beta$, $L(\alpha, \beta) = \partial L_\alpha/\partial\beta$, $L_{\beta\beta} = \partial L_\beta/\partial\beta$ and $L_{\alpha\beta} = \partial L_\alpha/\partial\beta$.

In the special Finsler space $F^n = (M^n, L)$ the normalized element of support $l_i = \dot{\partial} L$ and the angular metric tensor h_{ij} are given by [7]:

$$\begin{aligned} l_i &= \alpha^{-1} L_\alpha Y_i + L_\beta b_i, \\ h_{ij} &= p a_{ij} + q_0 b_i b_j + q_1(b_i Y_j + b_j Y_i) + q_2 Y_i Y_j, \end{aligned}$$

where

$$Y_i = a_{ij} y^j,$$

$$
\begin{aligned}
p &= LL_\alpha \alpha^{-1} = \frac{(\alpha^{n+1}+\beta^{n+1})(\alpha^{n+1}-n\beta^{n+1})}{\alpha^{2(n+1)}},\\
q_0 &= LL_{\beta\beta} = \frac{n(n+1)(\alpha^{n+1}+\beta^{n+1})\beta^{n-1}}{\alpha^{2n}},\\
q_1 &= LL_{\alpha\beta}\alpha^{-1} = \frac{-n(n+1)(\alpha^{n+1}+\beta^{n+1})\beta^{n}}{\alpha^{2(n+1)}},\\
q_2 &= L\alpha^{-2}(L_{\alpha\alpha}-L_\alpha\alpha^{-1})\\
&= \frac{(\alpha^{n+1}+\beta^{n+1})(n(n+2)\beta^{n+1}-\alpha^{n+1})}{\alpha^{2(n+2)}}.
\end{aligned}
\tag{2}
$$

The fundamental tensor $g_{ij} = \frac{1}{2}\dot{\partial}_i\dot{\partial}_j L^2$ and it's reciprocal tensor g_{ij} is given by [7]

$$
g_{ij} = pa_{ij} + p_0 b_i b_j + p_1(b_i Y_j + b_j Y_i) + p_2 Y_i Y_j,
$$

where

$$
\begin{aligned}
p_0 &= q_0 + L_\beta^2 = \frac{(n+1)[n\alpha^{n+1}\beta^{n-1} + (2n+1)\beta^{2n}]}{\alpha^{2n}},\\
p_1 &= q_1 + L^{-1}pL_\beta = \frac{(n+1)\beta^n}{\alpha^{2(n+1)}}[(1-n)\alpha^{n+1} - 2n\beta^{n+1}],\\
p_2 &= q_2 + p^2L^{-2},\\
p_2 &= \frac{(\alpha^{n+1}+\beta^{n+1})(n(n+2)\beta^{n+1}-\alpha^{n+1}) + (\alpha^{n+1}-n\beta^{n+1})^2}{\alpha^{2(n+1)}}.
\end{aligned}
\tag{3}
$$

$$
g^{ij} = p^{-1}a^{ij} + S_0 b^i b^j + S_1(b^i y^j + b^j y^i) + S_2 y^i y^j,
\tag{4}
$$

where

$$
\begin{aligned}
& b^i = a^{ij}b_j, \qquad S_0 = (pp_0 + (p_0p_2 - p_1^2)\alpha^2)/\zeta,\\
& S_1 = (pp_1 + (p_0p_2 - p_1^2)\beta)/\zeta p,\\
& S_2 = (pp_2 + (p_0p_2 - p_1^2)b^2)/\zeta p, \qquad b^2 = a_{ij}b^i b^j,\\
& \zeta = p(p + p_0 b^2 + p_1\beta) + (p_0p_2 - p_1^2)(\alpha^2 b^2 - \beta^2).
\end{aligned}
\tag{5}
$$

The hv-torsion tensor $C_{ijk} = \frac{1}{2}\dot{\partial}_k g_{ij}$ is given by [7]

$$
2pC_{ijk} = p_1(h_{ij}m_k + h_{jk}m_i + h_{ki}m_j) + \gamma_1 m_i m_j m_k,
$$

where

$$\gamma_1 = p\frac{\partial p_0}{\partial \beta} - 3p_1 q_0, \qquad m_i = b_i - \alpha^{-2}\beta Y_i. \tag{6}$$

Here m_i is a non-vanishing covariant vector orthogonal to the element of support y^i.

Let $\left\{ \begin{array}{c} i \\ jk \end{array} \right\}$ be the components of Christoffel symbols of the associated Riemannian space R^n and ∇_k be covariant differentiation with respect to x^k relative to this Christoffel symbols. We put

$$2E_{ij} = b_{ij} + b_{ji}, \qquad 2F_{ij} = b_{ij} - b_{ji}, \tag{7}$$

where $b_{ij} = \nabla_j b_i$.

Let $C\Gamma = (\Gamma^{*i}_{jk}, \Gamma^{*i}_{0k}, C^i_{jk})$ be the Cartan connection of F^n. The difference tensor $D^i_{jk} = \Gamma^{*i}_{jk} - \left\{ \begin{array}{c} i \\ jk \end{array} \right\}$ of the special Finsler space F^n is given by [4]

$$\begin{aligned} D^i_{jk} \;=\; & B^i E_{jk} + F^i_k B_j + F^i_j B_k + B^i_j b_{0k} + B^i_k b_{0j} \\ & -b_{0m} g^{im} B_{jk} - C^i_{jm} A^m_k - C^i_{km} A^m_j + C_{jkm} A^m_s g^{is} \\ & +\lambda^s (C^i_{jm} C^m_{sk} + C^i_{km} C^m_{sj} - C^m_{jk} C^i_{ms}), \end{aligned} \tag{8}$$

where

$$\begin{aligned} & B_k = p_0 b_k + p_1 Y_k, \qquad B^i = g^{ij} B_j, \qquad F^k_i = g^{kj} F_{ji} \\ & B_{ij} = \left\{ p_1(a_{ij} - \alpha^{-2} Y_i Y_j) + \frac{\partial p_0}{\partial \beta} m_i m_j \right\} \Big/ 2, \\ & B^k_i = g^{kj} B_{ji}, \\ & A^m_k = b^m_k E_{00} + B^m E_{k0} + B_k F^m_0 + B_0 F^m_k, \\ & \lambda^m = B^m E_{00} + 2B_0 F^m_0, \qquad B_0 = B_i y^i. \end{aligned} \tag{9}$$

where '0' denote contraction with y^i except for the quantities p_0, q_0 and S_0.

3. Induced Cartan connection

Let F^{n-1} be a hypersurface of F^n given by the equations $x^i = x^i(u^\alpha)$. The element of support y^i of F^n is to be taken tangential to F^{n-1}, that is

$$y^i = B^i_\alpha(u) v^\alpha. \tag{10}$$

The metric tensor $g_{\alpha\beta}$ and v-torsion tensor $C_{\alpha\beta\gamma}$ of F^{n-1} are given by

$$g_{\alpha\beta} = g_{ij}B^i_\alpha B^j_\beta, \qquad C_{\alpha\beta\gamma} = C_{ijk}B^i_\alpha B^j_\beta B^k_\gamma.$$

At each point u^α of F^{n-1}, a unit normal vector $N^i(u,v)$ is defined by

$$g_{ij}(x(u,v), y(u,v))B^i_\alpha N^j = 0, \qquad g_{ij}(x(u,v), y(u,v))N^i N^j = 1.$$

As for the angular metric tensor h_{ij}, we have

$$h_{\alpha\beta} = h_{ij}B^i_\alpha B^j_\beta, \qquad h_{ij}B^i_\alpha N^j = 0 \qquad h_{ij}N^i N^j = 1. \tag{11}$$

If (B^α_i, N_i) denote the inverse of (B^i_α, N^i), then we have

$$\begin{array}{l} B^\alpha_i = g^{\alpha\beta} g_{ij} B^j_\beta, \qquad B^i_\alpha B^\beta_i = \delta^\beta_\alpha, \\ B^\alpha_i N^i = 0, \qquad B^i_\alpha N_i = 0, \qquad N_i = g_{ij}N^j, \\ B^k_i = g^{kj}B_{ji}, \\ B^i_\alpha B^\alpha_j + N^i N_j = \delta^i_j. \end{array}$$

The induced connection $IC\Gamma = (\Gamma^{*\alpha}_{\beta\gamma}, G^\alpha_\beta, C^\alpha_{\beta\gamma})$ of F^{n-1} induced from the Cartan's connection $(\Gamma^{*i}_{jk}, \Gamma^{*i}_{0k}, C^i_{jk})$ is given by [6]

$$\begin{array}{rcl} \Gamma^{*\alpha}_{\beta\gamma} &=& B^\alpha_i(B^i_{\beta\gamma} + \Gamma^{*i}_{jk}B^j_\beta B^k_\gamma) + M^\alpha_\beta H_\gamma, \\ G^\alpha_\beta &=& B^\alpha_i(B^i_{0\beta} + \Gamma^{*i}_{0j}B^j_\beta), \\ C^\alpha_{\beta\gamma} &=& B^\alpha_i C^i_{jk}B^j_\beta B^k_\gamma, \end{array}$$

where

$$\begin{array}{rcl} M_{\beta\gamma} &=& N_i C^i_{jk} B^j_\beta B^k_\gamma, \qquad M^\alpha_\beta = g^{\alpha\gamma}M_{\beta\gamma}, \\ H_\beta &=& N_i(B^i_{0\beta} + \Gamma^{*i}_{oj}B^j_\beta), \end{array} \tag{12}$$

and $B^i_{\beta\gamma} = \partial B^i_\beta / \partial U^r$, $B^i_{0\beta} = B^i_{\alpha\beta}v^\alpha$. The quantities $M_{\beta\gamma}$ and H_β are called the second fundamental v-tensor and normal curvature vector respectively [6]. The second fundamental h-tensor $H_{\beta\gamma}$ is defined as [6]

$$H_{\beta\gamma} = N_i(B^i_{\beta\gamma} + \Gamma^{*i}_{jk}B^j_\beta B^k_\gamma) + M_\beta H_\gamma, \tag{13}$$

where

$$M_\beta = N_i C^i_{jk} B^j_\beta N^k. \tag{14}$$

The relative h and v-covariant derivatives of projection factor B^i_α with respect to $IC\Gamma$ are given by

$$B^i_{\alpha|\beta} = H_{\alpha\beta}N^i, \qquad B^i_\alpha|_\beta = M_{\alpha\beta}N^i. \tag{15}$$

The equation (13) shows that $H_{\beta\gamma}$ is generally not symmetric and

$$H_{\beta\gamma} - H_{\gamma\beta} = M_\beta H_\gamma - M_\gamma H_\beta. \tag{16}$$

The above equations yield

$$H_{0\gamma} = H_\gamma, \qquad H_{\gamma 0} = H_\gamma + M_\gamma H_0. \tag{17}$$

We use following lemmas which are due to Matsumoto [6]:

Lemma 1 *The normal curvature $H_0 = H_\beta v^\beta$ vanishes if and only if the normal curvature vector H_β vanishes.*

Lemma 2 *A hypersurface F^{n-1} is a hyperplane of the 1^{st} kind if and only if $H_\alpha = 0$.*

Lemma 3 *A hypersurface F^{n-1} is a hyperplane of the 2^{nd} kind with respect to the connection $C\Gamma$ if and only if $H_\alpha = 0$ and $H_{\alpha\beta} = 0$.*

Lemma 4 *A hyperplane of the 3^{rd} kind is characterized by $H_{\alpha\beta} = 0$ and $M_{\alpha\beta} = 0$.*

4. Hypersurface $F^{n-1}(c)$ of the special Finsler space

Let us consider special Finsler metric $L = \alpha + \frac{\beta^{n+1}}{\alpha^n}$ with a gradient $b_i(x) = \partial_i b$ for a scalar function $b(x)$ and a hypersurface $F^{n-1}(c)$ given by the equation $b(x) = c$(constant) [9].
From parametric equations $x^i = x^i(u^\alpha)$ of $F^{n-1}(c)$, we get $\partial_\alpha b(x(u)) = 0 = b_i B^i_\alpha$, so that $b_i(x)$ are regarded as covariant components of a normal vector field of $F^{n-1}(c)$. Therefore, along the $F^{n-1}(c)$ we have

$$b_i B^i_\alpha = 0 \quad \textit{and} \quad b_i y^i = 0. \tag{18}$$

The induced metric $L(u, v)$ of $F^{n-1}(c)$ is given by

$$L(u, v) = a_{\alpha\beta}v^\alpha v^\beta, \qquad a_{\alpha\beta} = a_{ij}B^i_\alpha B^j_\beta \tag{19}$$

which is the Riemannian metric.

At a point of $F^{n-1}(c)$, from (2), (3) and (5), we have

$$p=1,\quad q_0=0,\quad q_1=0,\quad q_2=-\alpha^{-2},\quad p_0=0,\quad p_1=0 \qquad (20)$$
$$p_2=0,\quad \zeta=1,\quad S_0=0,\quad S_1=0,\quad S_2=0.$$

Therefore, from (4) we get

$$g^{ij}=a^{ij}. \qquad (21)$$

Thus along $F^{n-1}(c)$, (21) and (18) lead to $g^{ij}b_ib_j=b^2$.
Therefore, we get

$$b_i(x(u))=\sqrt{b^2}N_i,\qquad b^2=a^{ij}b_ib_j. \qquad (22)$$

i.e., $b_i(x(u))=bN_i$, where b is the length of the vector b^i.

Again from (21) and (22) we get

$$b^i=bN_i. \qquad (23)$$

Thus we have

Theorem 1 *In the special Finsler hypersurface $F^{n-1}(c)$,the induced metric is a Riemannian metric given by (19) and the scalar function $b(x)$ is given by (22) and (23).*

The angular metric tensor and metric tensor of F^n are given by

$$\begin{aligned} h_{ij} &= a_{ij}-\frac{Y_iY_j}{\alpha^2}, \\ g_{ij} &= a_{ij}. \end{aligned} \qquad (24)$$

Form (18), (24) and (11) it follows that if $h^{(a)}_{\alpha\beta}$ denote the angular metric tensor of the Riemannian $a_{ij}(x)$, then along $F^{n-1}(c)$, $h_{\alpha\beta}=h^{(a)}_{\alpha\beta}$.
From (3), we get

$$\frac{\partial p_0}{\partial\beta}=\frac{(n+1)[n(n-1)\alpha^{n+1}\beta^{n-2}+2n(2n+1)\beta^{2n-1}]}{\alpha^{2n}}.$$

Thus along $F^{n-1}(c)$, $\frac{\partial p_0}{\partial\beta}=0$ and therefore (6) gives $\gamma_1=0$, $m_i=b_i$.
Therefore the hv-torsion tensor becomes

$$C_{ijk}=0 \qquad (25)$$

in a special Finsler hypersurface $F^{n-1}(c)$.

Therefore, (12), (14) and (25) give

$$M_{\alpha\beta} = 0 \ \ and \ \ M_{\alpha} = 0. \tag{26}$$

From (16) it follows that $H_{\alpha\beta}$ is symmetric. Thus we have

Theorem 2 *The second fundamental v-tensor of special Finsler hypersurface $F^{n-1}(c)$ vanishes and the second fundamental h-tensor $H_{\alpha\beta}$ is symmetric.*

Next from (18), we get $b_{i|\beta}B^i_\alpha + b_i B^i_{\alpha|\beta} = 0$. Therefore, from (15) and using $b_{i|\beta} = b_{i|j}B^j_\beta + b_i \mid_j N^j H_\beta$, we get

$$b_{i|j}B^i_\alpha B^j_\beta + b_i \mid_j B^i_\alpha N^j H_\beta + b_i H_{\alpha\beta} N^i = 0. \tag{27}$$

Since $b_i \mid_j = -b_h C^h_{ij}$, we get

$$b_i \mid_j B^i_\alpha N^j = 0.$$

Thus (27) gives

$$bH_{\alpha\beta} + b_{i|j}B^i_\alpha B^j_\beta = 0. \tag{28}$$

It is noted that $b_{i|j}$ is symmetric. Furthermore, contracting (28) with v^β and then with v^α and using (10), (17) and (26) we get

$$bH_\alpha + b_{i|j}B^i_\alpha y^j = 0, \tag{29}$$

$$bH_0 + b_{i|j}y^i y^j = 0. \tag{30}$$

In view of Lemmas (1) and (2), the hypersurface $F^{n-1}(c)$ is hyperplane of the first kind if and only if $H_0 = 0$. Thus from (30) it follows that $F^{n-1}(c)$ is a hyperplane of the first kind if and only if $b_{i|j}y^i y^j = 0$. Here $b_{i|j}$ being the covariant derivative with respect to $C\Gamma$ of F^n depends on y^i.

Since b_i is a gradient vector, from (7) we have $E_{ij} = b_{ij}$, $F_{ij} = 0$ and $F^i_j = 0$. Thus (8) reduces to

$$\begin{aligned} D^i_{jk} \ = \ & B^i b_{jk} + B^i_j b_{0k} + B^i_k b_{0j} - b_{0m} g^{im} B_{jk} \\ & - C^i_{jm} A^m_k - C^i_{km} A^m_j + C_{jkm} A^m_s g^{is} \\ & + \lambda^s (C^i_{jm} C^m_{sk} + C^i_{km} C^m_{sj} - C^m_{jk} C^i_{ms}). \end{aligned} \tag{31}$$

In view of (20) and (21), the relations in (9) become to

$$B_i = 0, \quad B^i = 0, \quad B_{ij} = 0, \qquad (32)$$
$$B^i_j = 0, \quad A^m_k = 0, \quad \lambda^m = 0.$$

By virtue of (32) we have $B^i_0 = 0$, $B_{i0} = 0$ which leads $A^m_0 = 0$.
Therefore we have

$$D^i_{j0} = 0,$$
$$D^i_{00} = 0.$$

Thus from the relation (18), we get

$$b_i D^i_{j0} = 0, \qquad (33)$$
$$b_i D^i_{00} = 0. \qquad (34)$$

From (25) it follows that

$$b^m b_i C^i_{jm} B^j_\alpha = b^2 M_\alpha = 0.$$

Therefore, the relation $b_{i|j} = b_{ij} - b_r D^r_{ij}$ and equations (33), (34) give

$$b_{i|j} y^i y^j = b_{00}.$$

Consequently, (29) and (30) may be written as

$$bH_\alpha + b_{i|0} B^i_\alpha = 0,$$
$$bH_0 + b_{00} = 0.$$

Thus the condition $H_0 = 0$ is equivalent to $b_{00} = 0$, where b_{ij} does not depend on y^i. Since y^i is to satisfy (18), the condition is written as $b_{ij} y^i y^j = (b_i y^i)(c_j y^j)$ for some $c_j(x)$, so that we have

$$2b_{ij} = b_i c_j + b_j c_i. \qquad (35)$$

Thus we have

Theorem 3 *The special Finsler hypersurface $F^{n-1}(c)$ is hyperplane of 1^{st} kind if and only if (35) holds.*

Using (25), (31) and (32), we have $b_r D^r_{ij} = 0$. Substituting (35) in (28) and using (18), we get

$$H_{\alpha\beta} = 0. \tag{36}$$

Thus, from Lemmas (1), (2) (3) and Theorem 3, we have the following:

Theorem 4 *If the special Finsler hypersurface $F^{n-1}(c)$ is a hyperplane of the 1^{st} kind then it becomes a hyperplane of the 2^{nd} kind too.*

Hence from (17), (36), Theorem 2, and Lemma (4) we have

Theorem 5 *The special Finsler hypersurface $F^{n-1}(c)$ is a hyperplane of the 3^{rd} kind if and only if it is a hyperplane of 1^{st} kind.*

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