INFORMATION AND THE BAYESIAN-NASH EQUILIBRIUM IN A GAME WITH INCOMPLETE INFORMATION

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Abstract. This paper presents a new model of finding the Bayesian – Nash equilibrium in a game with incomplete information. New information is modeled with information sources. The proposed model shows a new way of estimating the predictive distribution of the Bayesian- Nash equilibrium. The information source is used as a way of measuring new information received by an information consumer. The theoretical example corresponds to the situation when there is a small number of firms in the market; each firm's output likely having a large impact on the market price.

Keywords and phrases: Bayesian-Nash equilibrium, information source, conditional probability distribution;

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1. Introduction

Bayesian decision theory is concerned with the question of how a decision maker should choose a particular action from a set of possible choices if the outcome of the choice also depends on some unknown state (from the states of the world). The decision maker models the received information (new information) as an information source (Florea, I., Parpucea, I., 1995). A decision problem involves one or several information sources. We assume that each person is able to represent his beliefs, as the likelihood of the different n states of the information source, by a subjective discrete probability distribution (Hirshleifer, J., Riley, J.G., 1995). The primary objective of this paper is answering the following question: How does a new information source influence the optimal decision?

"My aim is to present the game theory and information economics that currently exists in journal articles and oral tradition in a way that shows how to build simple models using a standard format." (Rasmusen, E., 2001).

Finding the Bayesian – Nash equilibrium in games with incomplete information is based on a given probability distribution, noted μ , on the set $T = T_1 \times T_2 \times ... \times T_n$, where T_i , $i = \overline{1, n}$ is the set of possible choices of player i. The probability $\mu(t), t \in T$, that type combination $t = (t_1, t_2, ..., t_n)$ will be chosen in a future period depending on the information received. How does the received information influence the Bayesian – Nash equilibrium in a game with incomplete information? The theoretical model presented in the next section, allows us to build a discrete probability distribution of the Bayesian – Nash equilibrium, conditioned by an information source. The model shows us how to find the predictive distribution of the Bayesian-Nash equilibrium.

New information is modeled with the help of information sources. References on the mathematical modeling of information sources can be found in (Parpucea, I., Florea, I., 1995 and Ventsel, H., 1973). It seems natural to ask the following question: how can an information source influence a given probability distribution? The use of some fundamental Bayesian methods (Bolstad, W.M., 2004 and Box, G., Tias, G., 1992) is the starting point in defining the probability distribution conditioned by an information source.

"The game-theoretic viewpoint is more useful in settings with a small number of players, for then each player's choice is more likely to matter to his opponents." (Fudenberg, D., Tirole, J., 1991). The example presented in the third section fits perfectly this idea. When the number of firms in a market is small, each firm's output is likely to have a large impact on the market price. Thus, it is not reasonable for each firm to accept the market price as given.

2. Theoretical framework

According to the principles of cybernetics, efficiency in managing systems in general and socio-economic systems in particular, depends on the management of information. By information we understand a message about an event that has occurred, will occur or is likely to occur. Extremely important is the information received regarding a possible realization of an event. Information is a particular case of reflection, as an interaction between two processes; one's properties (the process that generates information) will be reproduced in another or several other process (processes that consume information). Interaction between two or several processes involves an exchange of information.

A process (information generator or information consumer) is a set of variables, noted $V = \{V_1, V_2, ..., V_n\}$. The state of a process at a certain moment is given by the vector $v = (v_1, v_2, ..., v_n)$, $v_i \in V_i$, $i = \overline{1,n}$. Future values of a variable $V_i \in V$ have a random character. The realization of the states depends on received information. Information received as a message can decrease or increase the degree of incertitude as to the realization of an event.

An information source represents a way of specifying the states of a process, regarding one or several variables V_i. We will note by S the information source assigned to variable V_i. The set of distinct values $v_i \in V_i$, represent a complete space of events. The simultaneous realization of two events is impossible, and the sum of the events represents a certain event. We assign to each event, v_i, a state s_i of the information source. Next, we will consider only information sources with a finite number of states.

We assume that each person is able to represent his beliefs, as to the likelihood of the different n states of the information source, by a subjective discrete probability distribution. Assuming discrete states of the source, the individual is supposed to be able to assign to each state, s_i a degree of belief, in the form of numerical weights p_i , between one and zero and whose sum is one:

$$0 \leq p_i \leq 1; \forall i = \overline{1,n};$$
$$\sum_{i=1}^{n} p_i = 1.$$

Let us assume the following: p_i-the probability that s_i occurs. If the information source has n states, the set of states and probabilities defined above, form a discrete random variable:

$$S : \begin{pmatrix} s_i \\ p_i \end{pmatrix}_{i=\overline{1,n}} .$$

A simple information source is an information source defined with regard to one variable V_i . A complex information source is an information source defined in regard to two or several variables V_i. These variables can be dependent or independent. The existence of a relation between these variables can influence the mathematical model of the complex information source. We consider two independent variables $V_1 \in V$ and $V_2 \in V$. We assign to each variable a simple information source, S_1 and S_2:

$$S_1 : \begin{pmatrix} s_i \\ p_i \end{pmatrix}_{i=\overline{1,n_1}} \qquad S_2 : \begin{pmatrix} \tau_j \\ q_j \end{pmatrix}_{j=\overline{1,n_2}}$$

where $s_i, i = \overline{1, n_1}$ are the states of source S_1 and $\tau_j, j = \overline{1, n_2}$ are the states of source S_2. The complex information source built with regard to variables V_1 and V_2 , noted by S_c has the following mathematical model:

$$S_c : \begin{pmatrix} s_i\tau_j \\ p_iq_j \end{pmatrix}_{i=\overline{1,n_1},j=\overline{1,n_2}}$$

where $s_i\tau_j, i = \overline{1, n_1,}j = \overline{1, n_2}$ represent the states of the complex information source. Assuming the independence of the variables, V_1 and V_2 , then:

$$P\left(S_1 = s_i; S_2 = \tau_j\right) = P\left(S_1 = s_i\right) \cdot P\left(S_2 = \tau_j\right) = p_iq_j$$
$$\sum_{i=1}^{n_1} \sum_{j=1}^{n_2} p_iq_j = 1$$

where $P(\cdot)$ is the probability assigned to the event between brackets.

For example, when it comes to prices the market can be considered an information generator and consumer at the same time. In a past period prices are known, in a future period prices are considered random variables. Received information can condition the probabilities of these variables.

Let us consider the case of two variables among which there is a dependency relation: the simple information source, S_1 , assigned to the independent variable and the simple information source, S_2, of the dependent variable. The discrete random variable S_1 is:

$$S_1 : \begin{pmatrix} s_j \\ p_j \end{pmatrix}_{j=\overline{1,n_1}}$$

For a random state of the source $S_1, S_1 = s_j$, we will define the source S_2 conditioned by the state s_j:

$$S_2|S_1 = s_j : \begin{pmatrix} \tau_i \\ p_{ij} \end{pmatrix}_{i=\overline{1,n_2}}$$

where $p_{ij} = P(S_2 = \tau_i | S_1 = s_j)$ is the probability of state τ_i conditioned by state s_j. We will introduce the conditional probability matrix, noted by $M(S_2|S_1) = \{p_{ij}\}$, $i = \overline{1, n_2}, j = \overline{1, n_1}$. The probabilities of the states of the information source S_2 , will be calculated by using the formula:

$$q_i = \sum_{j=1}^{n_1} p_{ij} p_j, \quad \forall i = \overline{1, n_2}$$

where $q_i = P(S_2 = \tau_i)$ and $p_j = P(S_1 = s_j)$. If we arrange the probabilities of the two information sources S_1 and S_2 , as column - vectors, noted by $P(1)$ and $P(2)$, we have:

$$P(2) = M(S_2|S_1) \cdot P(1)$$

The column-vector $P(2)$ forms a probability distribution of the states of the simple information source S_2 . The complex information source S_c, constructed by considering the two variables, has the following form:

$$S_c : \begin{pmatrix} s_j \tau_i \\ p_j p_{ij} \end{pmatrix}_{i=\overline{1,n_2}, j=\overline{1,n_1}}$$

where the probability of state $s_j \tau_i$ is:

$$P(S_1 = s_j; S_2 = \tau_i) = P(S_1 = s_j) \cdot P(S_2 = \tau_i | S_1 = s_j) = p_i \cdot p_{ij}$$

If S_2 is a discrete probability distribution and S_1 is an information source then according to the above presentation we can say that S_2 is a distribution conditioned by information source S_1. Therefore we have a probability distribution actualized by an information source.

Next we will show the role of information sources in estimating the most probable Bayesian–Nash equilibrium, in a game with incomplete information. In order to do so I consider necessary to present some theoretical elements regarding the Bayesian-Nash equilibrium in a game with incomplete information (Eichberger, I., 1993). Consider a game with incomplete information, noted by:

$$\Gamma = \left(I, (S_i)_{i \in I}, (p_i(s,t))_{i \in I}, (T_i)_{i \in I}, \mu\right).$$

We introduce the following notations:

I – players;

S_i - player's i strategy set;

$p_i(s,t)$ - player's i payoff function if the strategy s is chosen by players and type combination t is chosen. Note that player's i payoff may depend not only on his type t_i but also on the other player's type, denoted by t_{-i} ;

T_i - player's i set of possible types;

μ - the probability distribution on the set $T = T_1 \times T_2 \times ... \times T_n$ of type combination.

In our exposition we assume that type sets T_i are finite. If all T_i are finite, then $T = T_1 \times T_2 \times ... \times T_n$ is a finite set. We denote by $\mu(t), t \in T$ the probability that type combination $t = (t_1, t_2, ..., t_n)$ will be chosen. Without loss of generality (Harsanyi, J. C., 1967), we assume that players have incomplete information about their opponents' payoffs but they have complete information about the strategies of all players.

In order to find the Bayesian – Nash equilibrium in a game with incomplete information we have to know the probability distribution μ on the set T. It is important to see how the probability distribution μ will be modified after receiving new information under the form of information source. Let us consider the probability distribution μ defined on the set T and an information source S. The information source S is a common information for all players. According to the probability distribution conditioned by an information source, we have:

$$P(\mu = t; S = s_j) = P(S = s_j) \cdot P(\mu = t | S = s_j)$$

where $t \in T$, $s_j \in S$. The probability distribution μ conditioned by the information source S, denoted by $\mu^{'}$, is the probability distribution μ actualized by the information source S.

If a player receives information about her own type then she chooses a particular strategy to maximize her expected payoff. We noted by $s_i(.)$ a decision function of player $i \in I$ that, for each type $t_i \in T_i$, specifies the strategy $s_i(t_i) \in S_i$ this player will choose if her type turns out to be t_i . Let $\mu_i^{'}(t_{-i}|t_i)$ be the updated probability, obtained by using Bayesian updating rule, of a particular type combination for the opponents t_{-i} , given that player i has type t_i . For each type profile $t \in T$, there are updated beliefs for each player, that is, a list of conditional probability distribution $\left(\mu_1^{'}(t_{-1}|t_1), ..., \mu_i^{'}(t_{-I}|t_I)\right)$. Players' beliefs after they have received information about their types, are no longer identical.

A list of decision functions $s_{-i}(.) = (s_1(.), ..., s_{i-1}(.), s_{i+1}(.), ..., s_I(.))$ for all players (other than player i) and $t_{-i} = (t_1, ..., t_{i-1}, t_{i+1}, ..., t_I)$, a type combination for the other players; $s_{-i}(t_{-i})$ denotes the strategy combination of all players except player i , that will be played according to the decision functions $s_{-i}(.)$, if type combination t_{-i} occurs, that is:

$$s_{-i}(t_i) = (s_1(t_1), ..., s_{i-1}(t_{i-1}), s_{i+1}(t_{i+1}), ..., s_I(t_I))$$

With this notation, a Bayesian – Nash equilibrium is a list of decision functions $(s_1^*(.), ..., s_I^*(.))$, such that for all possible players $i \in I$ and all types $t_i \in T_i$:

$$\sum_{t_{-i} \in T_{-i}} p_i\left(s_i^*(t_i), s_{-i}^*(t_{-i}), t_i, t_{-i}\right) \cdot \mu_i^{'}(t_{-i}|t_i) \geq \sum_{t_{-i} \in T_{-i}} p_i\left(s_i, s_{-i}^*(t_{-i}), t_i, t_{-i}\right) \cdot \mu_i^{'}(t_{-i}|t_i)$$

holds for all strategies $s_i \in S_i$.

3. Example

Consider two firms supplying slightly different products produced with zero production costs. Denote with p_1 and p_2 the prices of the two products. As a result of new received information, the variation of each price around the average price can be modeled using two sources and, as it follows:

$$S_1 : \begin{pmatrix} \overline{p_1} - \epsilon_1 & \overline{p_1} + \epsilon_2 \\ p & 1-p \end{pmatrix}; \quad S_2 : \begin{pmatrix} \overline{p_2} - \eta_1 & \overline{p_2} + \eta_2 \\ q & 1-q \end{pmatrix}$$

where $\overline{p_1}$ represents the average price of the first firm, $\overline{p_2}$ the average price of the second firm and and is the deviation from the average price. Each information source has two states. For example, of the source is realized if the price $p_1 \in (\overline{p_1} - \epsilon_1; \overline{p_1}]$, and the state if $p_1 \in (\overline{p_1}; \overline{p_1} + \epsilon_2]$. Suchlike, we obtain the two states τ_1 and τ_2 for the information source. The appearance of a state in a future period as a result of new information received is uncertain. For simplifying this presentation, we will rewrite the two sources as it follows:

$$S_1 : \begin{pmatrix} s_1 & s_2 \\ p & 1-p \end{pmatrix}; \quad S_2 : \begin{pmatrix} \tau_1 & \tau_2 \\ q & 1-q \end{pmatrix}.$$

where p and q are the probabilities that the two prices p_1 and p_2 to go under the average price in the next period as a result of the new received information. The complex information source is the next one:

$$S_c : \begin{pmatrix} s_1\tau_1 & s_1\tau_2 & s_2\tau_1 & s_2\tau_2 \\ pq & p(1-q) & (1-p)q & (1-p)(1-q) \end{pmatrix}.$$

The information source S_c describes the behavior of the market of the both products considering the variation of their prices, as a result of the received information.

The demand functions for the goods of the two firms are:

$$d_1(p_1, p_2) = a \cdot \Delta p_1 + b \cdot \Delta p_2, \quad d_2(p_1, p_2) = c \cdot \Delta p_1 + d \cdot \Delta p_2$$

where $\Delta p_i = p_i - \overline{p_i}$ represents the deviation of price p_i from the average price $\overline{p_i}$. The demand functions $d_1(p_1, p_2), d_2(p_1.p_2)$ quantify the deviation from the average demand. Firm one does not know parameters c and d; firm two does not know parameters a and b. We can easily see that the Bayesian-Nash equilibrium requirements are fulfilled. In order to simplify the presentation we will consider two significant values, different for each parameter. The sets of possible types were assumed to be:

$$T_1 = \{(a_i, b_j); i = 1, 2; j = 1, 2\}, \quad T_2 = \{(c_i, d_j); i = 1, 2; j = 1, 2\}.$$

The payoff functions are:

$$\begin{array}{l} \Pi_1(p_1, p_2, (a_i, b_j)) = (a_i \cdot \Delta p_1 + b_j \cdot \Delta p_2) \cdot p_1, \\ \Pi_2(p_1, p_2, (c_i, d_j)) = (c_i \cdot \Delta p_1 + d_j \cdot \Delta p_2) \cdot p_2. \end{array}$$

In a Bayesian – Nash equilibrium, each firm is supposed to choose a type contingent strategy, that is decision functions $p_1(\cdot)$ and $p_2(\cdot)$ respectively, which is the best response to the opponent's decision function. In this example μ' is a probability distribution defined on $T_1 \times T_2$ conditioned by the information source S_c . For the most probable state $s_i\tau_j$ of source S_c we build two conditional distributions, μ_1 and μ_2 as can be seen next.

First, take $p_2(\cdot) = (p_2(c_1, d_1), p_2(c_1, d_2), p_2(c_2, d_1), p_2(c_2, d_2))$ as given and suppose that firm 1 has just learned that it has the demand parameters

(a_1, b_1). Firm 1's expected payoff can be written as:

$$\begin{aligned} & \Pi_1\left(p_1\left(a_1, b_1\right), p_2\left(c_1, d_1\right)\right) \cdot \mu_1\left(\left(c_1, d_1\right) \mid \left(a_1, b_1\right)\right) \\ + & \Pi_1\left(p_1\left(a_1, b_1\right), p_2\left(c_1, d_2\right)\right) \cdot \mu_1\left(\left(c_1, d_2\right) \mid \left(a_1, b_1\right)\right) \\ + & \Pi_1\left(p_1\left(a_1, b_1\right), p_2\left(c_2, d_1\right)\right) \cdot \mu_1\left(\left(c_2, d_1\right) \mid \left(a_1, b_1\right)\right) \\ + & \Pi_1\left(p_1\left(a_1, b_1\right), p_2\left(c_2, d_2\right)\right) \cdot \mu_1\left(\left(c_2, d_2\right) \mid \left(a_1, b_1\right)\right) \\ = & a_1 \cdot p_1^2\left(a_1, b_1\right) + p_1\left(a_1, b_1\right)\left(b_1 \cdot \overline{p_2}\left(c_i d_j | a_1 b_1\right) - a_1 \overline{p_1} - b_1 \overline{p_2}\right) \end{aligned}$$

We used the following notation for the average price p_2 conditioned by state (a_1, b_1):

$$\begin{aligned} \overline{p_2}\left(c_i d_j | a_1 b_1\right) = & \; p_2\left(c_1, d_1\right) \cdot \mu_1\left(\left(c_1, d_1\right) \mid \left(a_1, b_1\right)\right) \\ + & \; p_2\left(c_1, d_2\right) \cdot \mu_1\left(\left(c_1, d_2\right)\left(a_1, b_1\right)\right) \\ + & \; p_2\left(c_2, d_1\right) \cdot \mu_1\left(\left(c_2, d_1\right) \mid \left(a_1, b_1\right)\right) \\ + & \; p_2\left(c_2, d_2\right) \cdot \mu_1\left(\left(c_2, d_2\right) \mid \left(a_1, b_1\right)\right) \end{aligned}$$

This payoff function is continuously differentiable in firm's 1 strategy $p_1(\cdot)$.

Therefore, any p_1 satisfying the first - order condition for a maximum will be the best response to the type- contingent strategy $p_2(\cdot)$, previously considered. Solving the first - order condition for the maximum of the expected payoff function:

$$a_1 \cdot p_1^2\left(a_1, b_1\right) + p_1\left(a_1, b_1\right)\left(b_1 \cdot \overline{p_2}\left(c_i d_j | a_1 b_1\right) - a_1 \overline{p_1} - b_1 \overline{p_2}\right)$$

one obtains the following best response $p_1(a_1, b_1)$ for firm 1 of type (a_1, b_1) to the decision functions $(p_2(c_1, d_1), p_2(c_1, d_2), p_2(c_2, d_1), p_2(c_2, d_2))$ of firm 2:

$$p_1\left(a_1, b_1\right) = \frac{1}{2}\left(\overline{p_1} + \frac{b_1}{a_1}\left(\overline{p_2} - \overline{p_2}\left(c_i d_j | a_1 b_1\right)\right)\right)$$

Similarly, one obtains for all types of firm 1:

$$\begin{aligned} p_1\left(a_1, b_2\right) &= \frac{1}{2}\left(\overline{p_1} + \frac{b_2}{a_1}\left(\overline{p_2} - \overline{p_2}\left(c_i d_j | a_1 b_2\right)\right)\right); \\ p_1\left(a_2, b_1\right) &= \frac{1}{2}\left(\overline{p_1} + \frac{b_1}{a_2}\left(\overline{p_2} - \overline{p_2}\left(c_i d_j | a_2 b_1\right)\right)\right); \\ p_1\left(a_2, b_2\right) &= \frac{1}{2}\left(\overline{p_1} + \frac{b_2}{a_2}\left(\overline{p_2} - \overline{p_2}\left(c_i d_j | a_2 b_2\right)\right)\right). \end{aligned}$$

Now consider that firm 2 learns that its type is (c_1, d_1). For a type- contingent strategy of firm 1 $p_1(\cdot) = (p_1(a_1, b_1), p_1(a_1, b_2), p_1(a_2, b_1), p_1(a_2, b_2))$ the expected payoff of firm 2 will be as follows:

$$\begin{aligned}
& \Pi_2(p_1(a_1, b_1), p_2(c_1, d_1)) \cdot \mu_2((a_1, b_1) \,|\, (c_1, d_1)) \\
+ \; & \Pi_2(p_1(a_1, b_2), p_2(c_1, d_1)) \cdot \mu_2((a_1, b_2) \,|\, (c_1, d_1)) \\
+ \; & \Pi_2(p_1(a_2, b_1), p_2(c_1, d_1)) \cdot \mu_2((a_2, b_1) \,|\, (c_1, d_1)) \\
+ \; & \Pi_2(p_1(a_2, b_2), p_2(c_1, d_1)) \cdot \mu_2((a_2, b_2) \,|\, (c_1, d_1)) \\
= \; & d_1 \cdot p_2^2(c_1, d_1) + p_2(c_1, d_1)\left(c_1 \cdot \overline{p_1}(a_i b_j | c_1 d_1) - c_1 \overline{p_1} - d_1 \overline{p_2}\right)
\end{aligned}$$

In this representation of the payoff function we used a notation for the average price p_1 conditioned by the state (c_1, d_1):

$$\begin{aligned}
\overline{p_1}(a_i b_j | c_1 d_1) &= p_1(a_1, b_1) \cdot \mu_2((a_1, b_1) \,|\, (c_1, d_1)) \\
&+ p_1(a_1, b_2) \cdot \mu_2((a_1, b_2)(c_1, d_1)) \\
&+ p_1(a_2, b_1) \cdot \mu_2((a_2, b_1) \,|\, (c_1, d_1)) \\
&+ p_1(a_2, b_2) \cdot \mu_2((a_2, b_2) \,|\, (c_1, d_1))
\end{aligned}$$

The first-order condition gives the best response function for a firm of type (c_1, d_1):

$$p_2(c_1, d_1) = \frac{1}{2}\left(\overline{p_2} + \frac{c_1}{d_1}\left(\overline{p_1} - \overline{p_1}(a_i b_j | c_1 d_1)\right)\right).$$

A similar calculation yields firm's 2 best response for all types of the following type-contingent strategies:

$$\begin{aligned}
p_2(c_1, d_2) &= \frac{1}{2}\left(\overline{p_2} + \frac{c_1}{d_2}\left(\overline{p_1} - \overline{p_1}(a_i b_j | c_1 d_2)\right)\right); \\
p_2(c_2, d_1) &= \frac{1}{2}\left(\overline{p_2} + \frac{c_2}{d_1}\left(\overline{p_1} - \overline{p_1}(a_i b_j | c_2 d_1)\right)\right); \\
p_2(c_2, d_2) &= \frac{1}{2}\left(\overline{p_2} + \frac{c_2}{d_2}\left(\overline{p_1} - \overline{p_1}(a_i b_j | c_2 d_2)\right)\right).
\end{aligned}$$

In order to find a Bayesian-Nash equilibrium one has to solve the equation system given by the best response functions. With two players and four types for each player, this involves eight equations. The solution, $(p_1^*(\cdot), p_2^*(\cdot))$, is the Bayesian –Nash equilibrium. The probability of realization of equilibrium prices $(p_1^*(\cdot), p_2^*(\cdot))$, as a result of information received, is equal to the probability of realization of state $s_i\tau_j$ of complex source S_c.

4. Remarks

In some problems involving optimums, conditional probability distributions must have a dynamic character. A variable or an economical phenomenon can follow a probability law. If regarding a following period we receive new information, it is natural to ask ourselves how this new information influences the probability distribution. Because of the dynamic character of the probability distribution, the optimal decision in the next period will depend on the new information received. Some final remarks need to be made:

- The probabilities of an information source states have a subjective character. Estimating the states' probabilities depends on the experience of the researcher.
- An information source is a way to quantify new received information. It remains to be seen if it is the best solution.
- The proposed model allows us to predict, for the next period, the equilibrium solution of some conflict situations.

In this context we can elaborate new strategies regarding the equilibrium evolution for a conflict situation, considering the newly received information.

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