ON α-LEVEL TOPOLOGICAL GROUPS

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Abstract. In this paper by using the notion of fuzzy topological group we introduced the notion of α-level topological groups and extend the results of [2] to the corresponding theorems in α-level topological groups. We stated and proved some theorems which determine the properties of this notion.

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1. Introduction

In 1965, Zadeh introduced the notion of fuzzy sets and fuzzy set operation [9]. Subsequently, Chang [1], applied basic concepts of general topology to fuzzy sets and introduced fuzzy topology. Also studied the theory of fuzzy topological spaces. In [4], Foster introduced the notion of fuzzy topological groups.

In this paper by using the notion of fuzzy topological group we introduced the notion of α-level topological group and we characterize some basic properties of α-level topological groups and proved that if $\widetilde{A}_\alpha$ is a subgroup of α-level topological group G and $cl(\widetilde{A}_\alpha) \times cl(\widetilde{A}_\alpha) \subseteq cl(\widetilde{A}_\alpha \times \widetilde{A}_\alpha)$, then $cl(\widetilde{A}_\alpha)$ is a subgroup of G and if $\widetilde{A}_\alpha$ is a normal subgroup of α-level topological group G and $cl(\widetilde{A}_\alpha) \times cl(\widetilde{A}_\alpha) \subseteq cl(\widetilde{A}_\alpha \times \widetilde{A}_\alpha)$, then $cl(\widetilde{A}_\alpha)$ is a normal subgroup of G.

2. Preliminaries Notes

In this paper, we used some notations in order to simplify our work. As G is a group with multiplication and e is identity element.

We consider the set of all fuzzy subset of X is denoted by $FP(X)$. A fuzzy set $\widetilde{k}_c$ is called constant if for all $c \in [0,1]$, the membership function of it, is defined $M_{\tilde{k}_c}(x) = c$, for all $x \in X$.

Given $\widetilde{A} \in FP(X)$ and $\alpha \in I$ (where $I = [0,1]$), the α-level set of fuzzy set $\widetilde{A}$ is the subset of X which is defined by

$$\widetilde{A}_\alpha = \{x \in X \mid M_{\widetilde{A}}(x) > \alpha\}.$$

We recall the Lowen's definitions of a fuzzy topological space.

Definition 2.1. [8] *A fuzzy topology is a family $\widetilde{T}$ of fuzzy sets in X, which satisfies the following conditions:*
1- $\tilde{k}_c \in \widetilde{T}$, for all $c \in [0,1]$,
2- If $\widetilde{A}, \widetilde{B} \in \widetilde{T}$, then $\widetilde{A} \cap \widetilde{B} \in \widetilde{T}$,
3- If $\widetilde{A}_i \in \widetilde{T}$, for all $i \in \Lambda$, then $\bigcup_{i\in\Lambda} \widetilde{A}_i \in \widetilde{T}$.
The pair $(X, \widetilde{T})$ is a fuzzy topological space (FTS). Every member of $\widetilde{T}$ is called a $\widetilde{T}$-open fuzzy set in $(X, \widetilde{T})$ (or simply open fuzzy set) and complement of an open fuzzy set is called a closed fuzzy set.

Definition 2.2. [7] *The topological space $(X, l_\alpha(\widetilde{T}))$ is called α-level space of X, where $l_\alpha(\widetilde{T}) = \{\widetilde{A}_\alpha \mid \widetilde{A} \in \widetilde{T}\} \subseteq 2^X$. The α-level topology of fuzzy topological space $(X, \widetilde{T})$, where $\alpha \in [0,1]$, is a topology on X.*

Example 2.3. Suppose that $(G, .)$ is a group where $G = \{-1, 1\}$. Let $\widetilde{T} = \{\widetilde{\emptyset}, \widetilde{A}, \widetilde{B}, \widetilde{A} \cap \widetilde{B}, \widetilde{A} \cup \widetilde{B}, G\}$, where $\widetilde{A} = \{(1, 0.4), (-1, 0.6)\}$ and $\widetilde{B} = \{(1, 0.6), (-1, 0.5)\}$. Since $\widetilde{A}_\alpha = \{x \in X \mid M_{\widetilde{A}}(x) > \alpha\}$, we get that $\widetilde{A}_{0.5} = \{-1\}$, $\widetilde{B}_{0.5} = \{1\}$, $(\widetilde{A} \cup \widetilde{B})_{0.5} = G$ and $(\widetilde{A} \cap \widetilde{B})_{0.5} = \emptyset$ from above definition we have $l_{0.5}(\widetilde{T}) = \{\emptyset, \{-1\}, \{1\}, G\}$ is 0.5-level space.

Definition 2.4. [3] *A fuzzy topology $\widetilde{T}$ on a group G is said to be fuzzy topological group if the mappings:*

$$g : (G \times G, \widetilde{T} \times \widetilde{T}) \to (G, \widetilde{T})$$

$$g(x, y) = xy$$

and

$$g : (G, \widetilde{T}) \to (G, \widetilde{T})$$

$$h(x) = x^{-1}$$

are fuzzy continuous.

Definition 2.5. [6] *A subset B of a group G is called symmetric if $B = B^{-1}$.*

3. α-Level Topological Groups

Definition 3.1. *Let $(G, \widetilde{T})$ be a fuzzy topological group. $(G, l_\alpha(\widetilde{T}))$ is called α-level topological group if the mapping*

$$g : (G \times G, l_\alpha(\widetilde{T}) \times l_\alpha(\widetilde{T})) \to (G, l_\alpha(\widetilde{T}))$$

$$g(x, y) = xy$$

and

$$g : (G, l_\alpha(\widetilde{T})) \to (G, l_\alpha(\widetilde{T}))$$
$$h(x) = x^{-1}$$

are continuous.

Example 3.2. In Example 2.3, $(G, l_{0.5}(\widetilde{T}))$ is 0.5-level topological group.

We state some equivalent condition for definition of α-level topological group

Theorem 3.3. *Let G be a group having α-level topology $\widetilde{T}$. Then $(G, l_\alpha(\widetilde{T}))$ is α-level topological group if and only if the mapping*

$$l : (G \times G, l_\alpha(\widetilde{T}) \times l_\alpha(\widetilde{T})) \to (G, l_\alpha(\widetilde{T}))$$

$$l(x, y) = xy^{-1}$$

is α-level continuous.

Proof. Let $l(x, y) = xy^{-1}$. Then continuity of l follows from the continuity of f and g. The converse follows from the fact that $x = xe^{-1}$ and $xy = x(y^{-1})^{-1}$.

Theorem 3.4. *Let G be a group having α-level topology $\widetilde{T}$. Then $(G, l_\alpha(\widetilde{T}))$ is α-level topological group if and only if*

1- For every $x, y \in G$ and each open set $\widetilde{W}_\alpha$ containing xy, there exist open sets $\widetilde{U}_\alpha$ containing x and $\widetilde{V}_\alpha$ containing y such that $\widetilde{U}_\alpha \widetilde{V}_\alpha \subseteq \widetilde{W}_\alpha$

2- For every $x \in G$ and each open set $\widetilde{V}_\alpha$ contains x^{-1}, there exists an open set $\widetilde{U}_\alpha$ contains x such that $\widetilde{U}_\alpha^{-1} \subseteq \widetilde{V}_\alpha$.

Proof. Obvious.

Theorem 3.5. *Let $(G, l_\alpha(\widetilde{T}))$ be an α-level topological group and $a, b \in G$. Then*

1- The translation maps

$$r_a : (G, l_\alpha(\widetilde{T})) \rightarrow (G, l_\alpha(\widetilde{T}))$$

$$r_a(x) = xa$$

and

$$l_a : (G, l_\alpha(\widetilde{T})) \rightarrow (G, l_\alpha(\widetilde{T}))$$

$$l_a(x) = ax$$

2- The inversion map

$$f : (G, l_\alpha(\widetilde{T})) \rightarrow (G, l_\alpha(\widetilde{T}))$$

$$f(x) = x^{-1}$$

3- The map

$$\phi : (G, l_\alpha(\widetilde{T})) \rightarrow (G, l_\alpha(\widetilde{T}))$$

$$\phi(x) = axb$$

are homeomorphisms.

Proof. Obvious.

Corollary 3.6. *Let $(G, l_\alpha(\widetilde{T}))$ be an α-level topological group, $\widetilde{A}_\alpha, \widetilde{B}_\alpha \subseteq G$ and $g \in G$. Then*

1. If $\widetilde{A}_\alpha$ is an open set, then $\widetilde{A}_\alpha g$, $g\widetilde{A}_\alpha$, $g\widetilde{A}_\alpha g^{-1}$ and $\widetilde{A}_\alpha^{-1}$ are open sets.

2. If $\widetilde{A}_\alpha$ is a closed set, then $\widetilde{A}_\alpha g$, $g\widetilde{A}_\alpha$, $g\widetilde{A}_\alpha g^{-1}$ and $\widetilde{A}_\alpha^{-1}$ are closed sets.

3. If $\widetilde{A}_\alpha$ is an open set, then $\widetilde{A}_\alpha \widetilde{B}_\alpha$ and $\widetilde{B}_\alpha \widetilde{A}_\alpha$ are open set.

4. If $\widetilde{A}_\alpha$ is a closed set and $\widetilde{B}_\alpha$ is a finite set, then $\widetilde{A}_\alpha \widetilde{B}_\alpha$ and $\widetilde{B}_\alpha \widetilde{A}_\alpha$ are closed set.

Proof. (1, 2) Since r_a, l_a, f and ϕ are homeomorphism, then each of them is α-open and α-closed mapping.

(3, 4) $\widetilde{A}_\alpha \widetilde{B}_\alpha = \cup\{\widetilde{A}_\alpha b \mid b \in \widetilde{B}_\alpha\}$ is a union of open sets and hence $\widetilde{A}_\alpha \widetilde{B}_\alpha$ is an open set similarly for $\widetilde{B}_\alpha \widetilde{A}_\alpha$.

Definition 3.7. *An α-level topological group $(G, l_\alpha(\widetilde{T}))$ is called an α-homogeneous if for any $a, b \in G$, there exists an α-level homeomorphism*

$$f : G \rightarrow G$$

$$f(a) = b.$$

Theorem 3.8. *An α-level topological group is an α-homogeneous space.*

Proof. Let $(G, l_\alpha(\widetilde{T}))$ be an α-level topological group and $x_1, x_2 \in G$ take $a = x_1^{-1}x_2$, then $f(x) = r_a(x) = xa = xx_1^{-1}x_2$ implies $f(x_1) = x_2$.

Theorem 3.9. *A non trivial α-level topological group has no fixed point properties.*

Proof. Let $(G, l_\alpha(\widetilde{T}))$ be an α-level topological group and $a \in G$ with $a \neq e$. Now the map $r_a : G \to G$ is an α-level continuous. In contrary, suppose that $r_a(x) = x$, for some $x \in G$. Then $xa = x$ we can conclude that $a = e$, which is a contradiction, then r_a has no fixed point, hence G has no fixed point properties

Theorem 3.10. *Every open subgroup of α-level topological group is a closed set.*

Proof. Let $(G, l_\alpha(\widetilde{T}))$ be an α-level topological group and $\widetilde{H}_\alpha$ be an open subgroup of G. Then $G - \widetilde{H}_\alpha = \cup\{g\widetilde{H}_\alpha \mid g \notin \widetilde{H}_\alpha\} = \cap\{r_g(x) \mid g \notin \widetilde{H}_\alpha\}$, which is an open set, therefore $\widetilde{H}_\alpha$ is a closed set.

Theorem 3.11. *Every closed subgroup of finite index of an α-level topological group is an open set.*

Proof. If $\widetilde{H}_\alpha$ is a closed set of finite index, then its complement is the union of finite number of coset, each of them is closed set, hence $\widetilde{H}_\alpha$ is an open set.

Theorem 3.12. *Every subgroup of an α-level topological group is α-level topological group.*

Proof. Let $\widetilde{H}_\alpha$ be a subgroup of an α-level topological group $(G, l_\alpha(\widetilde{T}))$. It is clear that $\widetilde{H}_\alpha$ is also group, $(\widetilde{H}_\alpha, l_\alpha(\widetilde{T})_{\widetilde{H}_\alpha})$ is relative α-level space. It is enough to show that

$$\alpha : (\widetilde{H}_\alpha \times \widetilde{H}_\alpha, l_\alpha(\widetilde{T})_{\widetilde{H}_\alpha} \times l_\alpha(\widetilde{T})_{\widetilde{H}_\alpha}) \to (\widetilde{H}_\alpha, l_\alpha(\widetilde{T})_{\widetilde{H}_\alpha})$$

defined by $\alpha(x, y) = xy$ and

$$h : (\widetilde{H}_\alpha, l_\alpha(\widetilde{T})_{\widetilde{H}_\alpha}) \to (\widetilde{H}_\alpha, l_\alpha(\widetilde{T})_{\widetilde{H}_\alpha})$$

define by $h(y) = y^{-1}$ are α-level continuous.

Let $\widetilde{W}_{\widetilde{H}_\alpha}$ be any open set containing xy. Then $\widetilde{W}_{\widetilde{H}_\alpha} = \widetilde{H}_\alpha \cap \widetilde{W}_\alpha$, for some $\widetilde{W}_\alpha \in l_\alpha(\widetilde{T})$. Then $xy \in \widetilde{W}_\alpha$, since G is an α-level topological group, there exist open sets $\widetilde{U}_\alpha$ and $\widetilde{V}_\alpha$ of x and y respectively such that $\widetilde{U}_\alpha \widetilde{V}_\alpha \subseteq \widetilde{W}_\alpha$, then the intersection $\widetilde{U}_{\widetilde{H}_\alpha} = \widetilde{H}_\alpha \cap \widetilde{U}_\alpha$ and $\widetilde{V}_{\widetilde{H}_\alpha} = \widetilde{H}_\alpha \cap \widetilde{V}_\alpha$ are open sets containing x and y respectively in the space $\widetilde{H}_\alpha$.

Note that $\widetilde{U}_{\widetilde{H}_\alpha} \widetilde{V}_{\widetilde{H}_\alpha} = (\widetilde{H}_\alpha \cap \widetilde{U}_\alpha)(\widetilde{H}_\alpha \cap \widetilde{U}_\alpha) \subseteq \widetilde{W}_\alpha$ as well as $\widetilde{U}_{\widetilde{H}_\alpha} \widetilde{V}_{\widetilde{H}_\alpha} \subseteq \widetilde{H}_\alpha$ so that $\widetilde{U}_{\widetilde{H}_\alpha} \widetilde{V}_{\widetilde{H}_\alpha} \subseteq \widetilde{H}_\alpha \cap \widetilde{W}_\alpha = \widetilde{W}_\alpha$.

Similarly we can prove that

$$h : (\widetilde{H}_\alpha, l_\alpha(\widetilde{T})_{\widetilde{H}_\alpha}) \to (\widetilde{H}_\alpha, l_\alpha(\widetilde{T})_{\widetilde{H}_\alpha})$$

defined by $h(y) = y^{-1}$ is continuous. Therefore $(\widetilde{H}_\alpha, l_\alpha(\widetilde{T})_{\widetilde{H}_\alpha})$ is an α-level topological group.

Theorem 3.13. *Let $(G, l_\alpha(\widetilde{T}))$ be an α-level topological group, $\widetilde{A}_\alpha$ and $\widetilde{B}_\alpha$ are subset of G. Then*

1- $cl(a\widetilde{A}_\alpha a^{-1}) = acl(\widetilde{A}_\alpha)a^{-1}$, where $a \in G$,

2- If $cl(\widetilde{A}_\alpha) \times cl(\widetilde{B}_\alpha) \subseteq cl(\widetilde{A}_\alpha \times \widetilde{B}_\alpha)$, then $cl(\widetilde{A}_\alpha)cl(\widetilde{B}_\alpha) \subseteq cl(\widetilde{A}_\alpha \widetilde{B}_\alpha)$ and $cl(\widetilde{A}_\alpha)cl(\widetilde{B}_\alpha^{-1}) \subseteq cl(\widetilde{A}_\alpha \widetilde{B}_\alpha^{-1})$.

Proof. 1) From Corollary 3.6, we know that $acl(\widetilde{A}_\alpha)a^{-1}$ is a closed set, since $cl(a\widetilde{A}_\alpha a^{-1})$ is the smallest closed set containing $a\widetilde{A}_\alpha a^{-1}$, then $cl(a\widetilde{A}_\alpha a^{-1}) \subseteq acl(\widetilde{A}_\alpha)a^{-1}$.

Consider $f : (G, l_\alpha(\widetilde{T})) \to (G, l_\alpha(\widetilde{T}))$ which is defined by $f(x) = axa^{-1}$, then f is α-level homeomorphism, implies $f(cl(\widetilde{A}_\alpha)) \subseteq cl(f(\widetilde{A}_\alpha))$, thus $cl(a\widetilde{A}_\alpha a^{-1}) = acl(\widetilde{A}_\alpha)a^{-1}$.

2) Since the map $g : (G \times G, l_\alpha(\widetilde{T}) \times l_\alpha(\widetilde{T})) \to (G, l_\alpha(\widetilde{T}))$ which is defined by $g(x, y) = xy^{-1}$ is α-level continuous. By hypothesis $cl(\widetilde{A}_\alpha) \times cl(\widetilde{B}_\alpha) \subseteq cl(\widetilde{A}_\alpha \times \widetilde{B}_\alpha)$, then $f(cl(\widetilde{A}_\alpha), cl(\widetilde{B}_\alpha)) \subseteq f(cl(\widetilde{A}_\alpha \times \widetilde{B}_\alpha))$. Since f is α-level continuous, $f(cl(\widetilde{A}_\alpha \times \widetilde{B}_\alpha)) \subseteq cl(f(\widetilde{A}_\alpha, \widetilde{B}_\alpha))$, thus $cl(\widetilde{A}_\alpha)cl(\widetilde{B}_\alpha)^{-1} \subseteq cl(\widetilde{A}_\alpha \widetilde{B}_\alpha^{-1})$,

$cl(\widetilde{B}_\alpha^{-1}) = \cap\{\widetilde{F}_\alpha \mid \widetilde{F}_\alpha \text{ is closed and } \widetilde{B}_\alpha^{-1} \subseteq \widetilde{F}_\alpha\}$

$= \cap\{\widetilde{F}_\alpha \mid \widetilde{F}_\alpha^{-1} \text{ is closed and } \widetilde{F}_\alpha^{-1} \subseteq \widetilde{B}_\alpha\} = cl(\widetilde{B}_\alpha)^{-1}$.

We get that $cl(\widetilde{B}_\alpha^{-1}) = cl(\widetilde{B}_\alpha)^{-1}$, hence $cl(\widetilde{A}_\alpha)cl(\widetilde{B}_\alpha)^{-1} \subseteq cl(\widetilde{A}_\alpha \widetilde{B}_\alpha)^{-1}$. Similarly, we have $cl(\widetilde{A}_\alpha)cl(\widetilde{B}_\alpha) \subseteq cl(\widetilde{A}_\alpha \widetilde{B}_\alpha))$.

Theorem 3.14.

1- If $\widetilde{H}_\alpha$ is a subgroup of an α-level topological group $(G, l_\alpha(\widetilde{T}))$ and $cl(\widetilde{H}_\alpha) \times cl(\widetilde{H}_\alpha) \subseteq cl(\widetilde{H}_\alpha \times \widetilde{H}_\alpha))$, then $cl(\widetilde{H}_\alpha)$ is a subgroup.

2- If $\widetilde{H}_\alpha$ is a normal subgroup of an α-level topological group $(G, l_\alpha(\widetilde{T}))$ and $cl(\widetilde{H}_\alpha) \times cl(\widetilde{H}_\alpha) \subseteq cl(\widetilde{H}_\alpha \times \widetilde{H}_\alpha))$, then $cl(\widetilde{H}_\alpha)$ is a normal subgroup.

Proof.

(1) Since $\widetilde{H}_\alpha$ is subgroup, then $\widetilde{H}_\alpha\widetilde{H}_\alpha \subseteq \widetilde{H}_\alpha$, thus $cl(\widetilde{H}_\alpha\widetilde{H}_\alpha) \subseteq cl(\widetilde{H}_\alpha)$. By Theorem 3.13, $cl(\widetilde{H}_\alpha)cl(\widetilde{H}_\alpha) \subseteq cl(\widetilde{H}_\alpha\widetilde{H}_\alpha))$, we get that

$$cl(\widetilde{H}_\alpha)cl(\widetilde{H}_\alpha) \subseteq cl(\widetilde{H}_\alpha)) \quad (1)$$

since $\widetilde{H}_\alpha$ is a subgroup $\widetilde{H}_\alpha = \widetilde{H}_\alpha^{-1}$ and hence $cl(\widetilde{H}_\alpha) = cl(\widetilde{H}_\alpha^{-1})$, also we get that

$$cl(\widetilde{H}_\alpha^{-1}) = cl(\widetilde{H}_\alpha)^{-1} \quad (2)$$

from (1) and (2) we get that $cl(\widetilde{H}_\alpha)$ is a subgroup of G.

(2) Let $\widetilde{H}_\alpha$ be a normal subgroup of G. Then $x\widetilde{H}_\alpha x^{-1} = \widetilde{H}_\alpha$, therefore $cl(x\widetilde{H}_\alpha x^{-1}) = cl(\widetilde{H}_\alpha)$, hence $xcl(\widetilde{H}_\alpha)x^{-1} = cl(\widetilde{H}_\alpha)$, for every $x \in G$. We get that $cl(\widetilde{H}_\alpha)$ is a normal subgroup of G.

Lemma 3.15. *Let $(G, l_\alpha(\widetilde{T}))$ and $(H, l_\alpha(\widetilde{T}))$ be two α-level topological groups and f a homomorphism of G into H. Then*

1- For any subsets $\widetilde{A}_\alpha$ and $\widetilde{B}_\alpha$ of H,

$$cl(f^{-1}(\widetilde{A}_\alpha))cl(f^{-1}(\widetilde{B}_\alpha)) \subseteq cl(f^{-1}(\widetilde{A}_\alpha\widetilde{B}_\alpha)).$$

2- For any subsets $\widetilde{A}_\alpha$ and $\widetilde{B}_\alpha$ of G,

$$cl(f(\widetilde{A}_\alpha))cl(f(\widetilde{B}_\alpha)) \subseteq cl(f(\widetilde{A}_\alpha\widetilde{B}_\alpha)).$$

3- For any symmetric subset $\widetilde{A}_\alpha$ of G, $cl(f(\widetilde{A}_\alpha))$ is symmetric in H and hence

$$cl(f(\widetilde{A}_\alpha^{-1})) = (cl(f(\widetilde{A}_\alpha)))^{-1}.$$

4- For any symmetric subset $\widetilde{A}_\alpha$ of H, $cl(f(\widetilde{A}_\alpha^{-1}))$ is symmetric in G and hence

$$cl(f(\widetilde{A}_\alpha^{-1})) = (cl(f(\widetilde{A}_\alpha^{-1})))^{-1}.$$

Proof. Obvious.

Theorem 3.16. *Let $(G, l_\alpha(\widetilde{T}))$ be an α-level topological group and $\widetilde{A}_\alpha$ be compact subset of G. Then $\widetilde{A}_\alpha^{-1}$, $a\widetilde{A}_\alpha$, $\widetilde{A}_\alpha a$ and $a\widetilde{A}_\alpha a^{-1}$ are also compact.*

Proof. Obvious.

Remark 3.17. It is clear every α-level topological group is topological group. We can obtain from α-level topological space fuzzy topological space, in [7] Lowen, shows that if (X, T) is a topological space, then $(X, \omega(T))$ is a fuzzy topological space where $\omega(T) = \{A \mid A : X \to [0, 1]$ is lower semi-continues $\}$.

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References

[1] C. L. Chang, *Fuzzy topological spaces*, J. Math, Anal Appl., 24 (1968), 182-190.

[2] I. Chon, *Some properties of fuzzy topological group*, Fuzzy sets and Systems, 123 (2001), 197-201.

[3] N. R. Das, P. Das, *Neighborhood systems in fuzzy topological space*, Fuzzy sets and Systems, 116 (2001), 401-408.

[4] D. H. Foster, *Fuzzy topological group*, J. Math. Anal. Appl., 67 (1979), 549-564.

[5] P. J. Higgin, *An introduction to topological groups*, London Math. Soc., Lecture note ser. Vol.15, Cambridge Univ. Press, London, 1974.

[6] Taqdir Husain, *Introduction to topological groups*, R. E. Kreger pub Co. Philadelphia and London, 1981.

[7] R. Lowen, *A comparison of different compactness notions in fuzzy topological spaces*, J. Math. Annl. Appl., 64 (1978), 446-454.

[8] R. Lowen, *Fuzzy topological spaces and fuzzy compactness*, J. Math, Anal. Appl., 56 (1976), 621-633.

[9] L. A. Zadeh, *Fuzzy sets*, Infrom. Control, 8 (1965), 338-353.

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