RISK MANAGEMENT USING VAR SIMULATION WITH APPLICATIONS TO BUCHAREST STOCK EXCHANGE

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Abstract. In a recent paper, we have proposed and analyzed, from a theoretical point of view, a multidimensional stock market model (see [?]). In this paper, we construct a portfolio of stocks for a particular case of this market model. We introduce the Value at Risk, as a powerful tool for managing risks, which follow from holding such a portfolio. We present a mathematical calculation of Value at Risk for our market model. Using this mathematical framework, we develop Monte Carlo, Quasi-Monte Carlo and Mixed Monte Carlo and Quasi-Monte Carlo algorithms for the estimation of Value at Risk. We apply the developed methods to portfolios from Bucharest Stock Exchange.

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1. Introduction

In a recent paper (see [9]), we have introduced a multidimensional stock market model and analyzed some important features of it. We have considered n stock prices $S_i(t)$, $i = \overline{1,n}$, driven by a multidimensional Brownian motion process $B(t) = (B_1(t), B_2(t), \ldots, B_n(t))_{0 \le t \le T}$ on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$, together with the filtration generated by $B(t)$, denoted by $\{\mathcal{F}(t)\}_{0 \le t \le T}$. The stock price processes were defined as follows:

$$
\begin{aligned}
dS_1(t) &= S_1(t)[\mu_1(t)dt + \sigma_1(t)dB_1(t)], \qquad (1)\\
dS_i(t) &= S_i(t)[\mu_i(t)dt + \lambda\sigma_i(t)dB_1(t) + \sqrt{1-\lambda^2}\sigma_i(t)dB_i(t)], i = \overline{2,n}, \qquad (2)
\end{aligned}
$$

where λ is a real parameter, such that $-1 \leq \lambda \leq 1$ and $\sigma(t) = (\sigma_1(t), \ldots, \sigma_n(t)) > 0$.

In this paper, we assume that $\lambda = 0$. We therefore get the following market model:

$$dS_i(t) = S_i(t)[\mu_i dt + \sigma_i dB_i(t)], \quad i = \overline{1,n}, \tag{3}$$

where the drifts $\mu_i, i = \overline{1,n}$, and the volatilities $\sigma_i, i = \overline{1,n}$, are assumed to be constant over time.

In what follows, we consider a portfolio consisting of n risky assets, which, in our case, are the n stocks defined in (3). If we denote by $p_i,\ i = \overline{1,n}$, the number of positions we hold on asset i, $i = \overline{1,n}$, then we can define the portfolio value at time t as

$$V(t) = \sum_{i=1}^{n} p_i S_i(t). \tag{4}$$

Holding such a portfolio of stocks is a risky business due the market fluctuations. As we can expect huge losses from having a portfolio, we need a powerful tool to measure financial risks. Value at Risk (VaR) is such a tool for managing risk in financial institutions. VaR traces his roots from the great financial disasters from early 1990s. The valuable lesson that we learned was that poor supervision and risk management can lead to huge losses in tens of millions of dollars. The VaR history is closely connected with the name of the Investment Bank J.P. Morgan. Its president Dennis Weatherstone, in intention to evaluate the total risk his firm is exposed to, asked to his directors to present him daily a briefing on the financial risk of the company. RiskMetrics Department developed such a risk measure, widely used today among financial institutions, which they called Value at Risk (VaR).

The definition of VaR is the maximum loss that will occur, over a target horizon, in normal market conditions, with a certain confidence level (see [4]). For example, a daily VaR of 10000 \$ at 99% confidence level suggests a

1 in 100 chance for a loss grater than 10000 to occur any single day. VaR is a very useful number, as it translates all the complicated market risk factors into a single number, in a currency, which everybody can understand.

As our portfolio is composed only from shares of stocks, it is important to make the following remark on the market model. The process $B(t)$ is the Brownian motion observed for the assets in the market under the measure $\mathbb{P}$, induced by the market. In [9], we defined a risk-neutral probability measure $\widehat{\mathbb{P}}$ and an n-dimensional Brownian motion under this risk-neutral probability measure, denoted by $\widehat{B}(t) = (\widehat{B}_1(t), \ldots, \widehat{B}_n(t))$. Using this new defined Brownian motion, the stock price dynamics can be expressed as

$$dS_i = S_i(rdt + \sigma_i d\widehat{B}_i), \quad i = \overline{1, n}, \tag{5}$$

where r is the risk-free rate. It is important to note that the risk-free interest rate is used only with option pricing. The future values of the stocks should be modelled using $\mu_i, i = \overline{1, n}$, and hence, the market model (3). The parameter μ_i is replaced with the risk-free rate r, only in risk-neutral valuation of options. However, we are not trying to create a martingale, but model the future behavior of our portfolio. This is true for Value at Risk models, where we are interested in the future state of the portfolio, not in the present value. Hence, in our Monte Carlo, Quasi-Monte Carlo and Mixed Monte Carlo and Quasi-Monte Carlo simulations, we are going to simulate the real prices of stocks, described in relations (3).

The remaining part of the paper is organized as follows. In Section 2, we present a detailed mathematical calculation of Value at Risk for our market model. Using this mathematical framework, we develop Monte Carlo (MC), Quasi-Monte Carlo (QMC) and Mixed MC and QMC algorithms for estimation of Value at Risk. In Section 3, we apply the developed methods to two portfolios of stocks from Bucharest Stock Exchange.

2. Monte Carlo Simulation of VaR

There are a variety of methods for computing Value at Risk. Three of them are shortly summarized bellow:

1. Delta-gamma approximation

   In this method (see [3]), it is assumed that all assets are lognormal distributed and relies on historical data, in order to estimate the parameters: means, standard deviations, correlations and portfolios sensitivities to each of the risk factors. This method is computationally efficient and easiest to implement. However, it gives a poor estimation for portfolios containing assets with highly non-linear response to risk factors.

2. Historical simulation

   Historical Simulation (see [3]) takes a portfolio of assets at a particular point in time and revalues the portfolio a number of times, using a history of prices for the assets in the portfolio. The portfolio revaluations produce a distribution of profit and losses, which can be examined in order to determine the VaR, with a chosen level of confidence. The main criticism of this approach is the assumption that the past can predict the future accurately. This method also relies heavily on the time horizon that is used to capture historical data.

3. Monte Carlo simulation

   Monte Carlo simulation (see [1], [3] and [8]) is a good alternative to the above two methods because it can handle any non-linear portfolios and can accommodate any type of distribution of risk factors. This approach simulates possible price paths, for each of the assets, and values the portfolio. After many simulations, VAR can be calculated directly from the simulated distribution of portfolio value change. However, this method is computationally intensive.

In this paper, we focus on the last method: MC simulation. We will also use the methods of QMC and Mixed MC and QMC to calculate VaR estimations.

The SDE equation (3) can be solved using Ito's theorem (see [10]). Ones obtains

$$S_i(t) = S_i(0)e^{(\mu_i - \frac{1}{2}\sigma_i^2)t + \sigma_i B_i(t)}, \quad i = \overline{1,n}. \tag{6}$$

This process is called a Geometric Brownian Motion (see [6]). If we want to simulate this stochastic process, then the stock price at time t is given by

$$S_i(t) = S_i(0)e^{(\mu_i - \frac{1}{2}\sigma_i^2)t + \sigma_i \sqrt{t} x^{(i)}}, \quad i = \overline{1,n}, \tag{7}$$

where $x^{(i)} \in N(0,1), i = \overline{1,n}$, are standard normal random variables and t is the holding time.

Let $S(t) = (S_1(t), S_2(t), \dots, S_n(t))^T$ be a vector that contains the values of the stocks at time t and $S(0) = (S_1(0), S_2(0), \dots, S_n(0))^T$ be a vector that contains the initial values of the stocks. Let $\sigma = (\sigma_1, \sigma_n, \ldots, \sigma_n)^T$ be the volatility vector and $\mu = (\mu_1, \mu_2, \ldots, \mu_n)^T$ the drift vector. With these notations, we can rewrite relations (7) in matrix form, as follows:

$$S(t) = S(0)e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma\sqrt{t}.*x}, \tag{8}$$

where the symbol $.*$ is used for element by element multiplication and $x = (x^{(1)}, x^{(2)}, \ldots, x^{(n)})^T$ is a vector of standard normal variables.

Clearly, if the stocks $S_i, i = \overline{1,n}$, are all independent, the collection of stocks can be generated directly, using formula (8). But instead of using n independent Wiener processes $B_i(t)$, to represent the returns, the market model requires correlated underlying processes. This is an important assumption, since in practice, the stock prices from the market are in general correlated.

Let us consider the correlated processes $Z_1, Z_2, \ldots, Z_n$, with the correlation matrix $C = (\rho_{ij})_{i,j=\overline{1,n}}$. We also consider the corresponding covariance matrix Σ. Hence, we are given with

$$dS_i = S_i(\mu_i dt + \sigma_i dZ_i), \quad i = \overline{1,n}, \tag{9}$$

where $Z_i, i = \overline{1,n}$, are correlated Brownian motions, with correlation matrix C. As a whole, the trends will be apparent, since the processes Z_i are correlated according to matrix C.

We rewrite the SDE from (9) in the following form:

$$dS_i = S_i\Big(\mu_i dt + \sum_{j=1}^{n} \sigma_{ij} dB_j\Big), \quad i = \overline{1,n}. \tag{10}$$

Our objective is to determine the matrix $A = (\sigma_{i,j})_{i,j=\overline{1,n}}$, such that $AA^T = \Sigma$.

Proposition 1 *If V is an n-dimensional diagonal matrix such that $(V)_{ii} = \sigma_i, i = \overline{1,n}$, and $AA^T = \Sigma$, then there is a lower triangular matrix L, such that $A = VL$.*

Proof. Because C is a correlation matrix, it follows that it is a symmetric, positive definite matrix. Hence, it has a Cholesky decomposition of the form $C = LL^T$, where L is a lower triangular matrix. We have

$$AA^T = \Sigma = VCV = VLL^TV = VL(VL)^T.$$

Hence, we obtain that $A = VL$.

From Proposition 1, we obtain

$$dS_i = S_i(\mu_i dt + \sigma_i dZ_i) = S_i\Big(\mu_i dt + \sigma_i \sum_{j=1}^{i} l_{ij} dB_j\Big), \quad i = \overline{1,n}, \tag{11}$$

where $(l_{ij})_{i,j=\overline{1,n}}$ are the components of the lower triangular matrix L.

Hence, in order to capture the correlations among the stocks, we will replace relation (8) with

$$S(t) = S(0)e^{(\mu - \frac{1}{2}V\sigma)t + VLx\sqrt{t}}. \tag{12}$$

If the value of the portfolio at time t is $V(t)$, the holding period is Δt, and the value of the portfolio at time $t + \Delta t$ is $V(t + \Delta t)$, then the loss in the portfolio value is defined as

$$Loss = V(t) - V(t + \Delta t). \tag{13}$$

Having defined the *Loss* random variable, we present the Value at Risk definition.

Definition 2 (Value at Risk) *For a given probability α, the VaR, denoted by δ_α, is defined by the following relation:*

$$P(Loss \geq \delta_\alpha) = \alpha. \tag{14}$$

Typically, the interval Δt is fixed to one day or two weeks, and the confidence level α is close to zero, often $\alpha = 0.01$ or $\alpha = 0.05$. In the statistical terminology, VaR is nothing but the $(1 - \alpha)$'th quantile of the *Loss* distribution.

In what follows, we present the mathematical framework for VaR estimation, based on MC method. The relation (14) can be written as

$$1 - P(Loss < \delta_\alpha) = \alpha, \tag{15}$$

or

$$F(\delta_\alpha) = 1 - \alpha = \beta, \tag{16}$$

where F denotes the (unknown) cumulative distribution function (cdf) of random variable *Loss*. The VaR can be expressed in terms of the inverse cdf, as follows:

$$\delta_\alpha = F^{-1}(\beta) = \inf\{y | F(y) \geq \beta\}. \tag{17}$$

For a given y, the cdf $F(y)$ can be expressed as an expectation

$$\begin{aligned} F(y) &= E[\mathbf{1}_{\{Loss \leq y\}}] = E[\mathbf{1}_{\{V(0)-V(t) \leq y\}}] \\ &= E[\mathbf{1}_{\{V(0)-\sum_{i=1}^{n} p_i S_i(t) \leq y\}}] \\ &= E[\mathbf{1}_{\{V(0)-\sum_{i=1}^{n} p_i S_i(0) e^{(\mu_i - \frac{1}{2}\sigma_i^2)t + \sigma_i \sqrt{t} \sum_{j=1}^{i} l_{ij} x_j} \leq y\}}], \end{aligned}$$

where $\mathbf{1}_{\{\cdot\}}$ is an indicator function, which returns 1 when the relation $\{\cdot\}$ is true and 0 otherwise.

We denote by $f(x^{(1)}, \ldots, x^{(n)})$ the term $\mathbf{1}_{\{V(0)-\sum_{i=1}^{n} p_i S_i(0) e^{(\mu_i - \frac{1}{2}\sigma_i^2)t + \sigma_i \sqrt{t} \sum_{j=1}^{i} l_{ij} x_j} \leq y\}}$ in the last equality.

It follows that

$$F(y) = \int_{\mathbb{R}^n} f(x^{(1)}, \ldots, x^{(n)}) d\Phi(x^{(1)}, \ldots, x^{(n)}) = I, \tag{18}$$

where $\Phi(x^{(1)}, \ldots, x^{(n)})$ is a distribution function on $\mathbb{R}^n$, which can be factored $\Phi(x^{(1)}, \ldots, x^{(n)}) = \Psi_1(x^{(1)}) \cdot \ldots \cdot \Psi_n(x^{(n)})$, and $\Psi_i(x^{(i)})$, $i = \overline{1, n}$, represents the standard normal cumulative distribution function, denoted by Ψ. We have denoted the last integral by I.

Using the MC method, I is estimated by sums of the form

$$\hat{I}_K^{MC} = \frac{1}{K} \sum_{i=1}^{K} f(x_i^{(1)}, \ldots, x_i^{(n)}), \tag{19}$$

where $x_i = (x_i^{(1)}, \ldots, x_i^{(n)})$, $i \geq 1$, are independent identically distributed random points on $\mathbb{R}^n$, with the common distribution function $\Phi(x^{(1)}, \ldots, x^{(n)})$. Another representation of this estimation, in terms of the $Loss$ distribution, is

$$\hat{I}_K = \frac{1}{K} \sum_{i=1}^{K} \mathbf{1}_{\{Loss_i \leq y\}}, \tag{20}$$

where $\{Loss_i,\ i = \overline{1, K}\}$ are samples from the $Loss$ distribution. Sorting the samples $\{Loss_i, i = \overline{1, K}\}$ in increasing order, we obtain

$$Loss_{(1)} \leq Loss_{(2)} \leq \ldots \leq Loss_{(K)}. \tag{21}$$

Then the corresponding sample cumulative distribution function is

$$\overline{F}_K(y) = \begin{cases} 0 & \text{if } y < Loss_{(1)} \\ \frac{i}{K} & \text{if } Loss_{(i)} \leq y < Loss_{(i+1)}, \qquad i = 1, \ldots, K-1. \\ 1 & \text{if } y \geq Loss_{(K)} \end{cases} \tag{22}$$

From relations (19), (20) and (22), we immediately deduce that $y = Loss_{([K\beta])}$ gives $\overline{F}_K(y) = \beta$, which satisfies the definition of VaR.

The algorithm which generates VaR is presented next.

Algorithm 3 *VaR Generation by Monte Carlo Simulation Method*

Input data: The initial stock prices vector $S(0) = (S_1(0), \ldots, S_n(0))^T$, the horizont time t, the number of simulations K and the confidence level α.
Step 1.
for $i = 1, \ldots, K$ **do**

1.1. Generate a random point $x_i = (x_i^{(1)}, \ldots, x_i^{(n)})^T$ on $\mathbb{R}^n$, with independent identically distributed components, each component having the common distribution function Ψ.

1.2. Generate the stock prices at time t, using formula (12)

$$S_i(t) = S(0)e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma\sqrt{t}.*x_i}, \tag{23}$$

where $S_i(t) = (S_{i,1}(t), \ldots, S_{i,n}(t))^T$.

1.3. Determine the portfolio value at time t, using formula (4)

$$V_i(t) = \sum_{l=1}^{n} p_l S_{i,l}(t).$$

1.4. Determine the *Loss* distribution sample as

$$Loss_i(t) = V(0) - V_i(t),$$

where $V(0)$ is the value of the portfolio at initial time.

end for

Step 2. Sort the vector $Loss = (Loss_1(t), \ldots, Loss_K(t))$ in ascending order, i.e.

$$Loss_{(1)}(t) \leq Loss_{(2)}(t) \leq \ldots \leq Loss_{(K)}(t).$$

Output data: $VaR = Loss_{[K(1-\alpha)]}(t)$.

In order to generate a point x_i from Step 1.1, we proceed as follows. We first generate a random point $\omega_i = (\omega_i^{(1)}, \ldots, \omega_i^{(n)})$, where $\omega_i^{(l)}$ is uniformly distributed on $[0,1]$, for each $l = 1, \ldots, n$. Then, for each component $\omega_i^{(l)}$, $l = 1, \ldots, n$, we apply the inversion method and obtain that $\Psi_l^{-1}(\omega_i^{(l)}) = x_i^{(l)}$ is a random point with the distribution function Ψ.

A similar algorithm can be obtained for the QMC simulation method. First, we have to transform the integration domain to $[0,1]^n$. For this, we use the substitution $\Psi_i^{-1}(z^{(i)}) = x^{(i)}$, $i = \overline{1,n}$, and we obtain

$$\begin{aligned} I &= \int_{\mathbb{R}^n} f(x^{(1)}, \ldots, x^{(n)}) d\Psi_1(x^{(1)}) \cdot \ldots \cdot d\Psi_n(x^{(n)}) \\ &= \int_{[0,1]^n} f(\Psi_1^{-1}(z^{(1)}), \ldots, \Psi_n^{-1}(z^{(n)})) dz^{(1)} \ldots dz^{(n)} \\ &= \int_{[0,1]^n} g(z^{(1)}, \ldots, z^{(n)}) dz^{(1)} \ldots dz^{(n)}. \end{aligned}$$

In the last equality, we have denoted

$$f(\Psi_1^{-1}(z^{(1)}), \ldots, \Psi_n^{-1}(z^{(n)})) \text{by} g(z^{(1)}, \ldots, z^{(n)}).$$

Using the QMC method, the integral I is estimated by sums of the form

$$\hat{I}_K^{QMC} = \frac{1}{K} \sum_{i=1}^{K} g(z_i^{(1)}, \ldots, z_i^{(n)}), \tag{24}$$

where $(z_i)_{i \geq 1} = (z_i^{(1)}, \ldots, z_i^{(n)})_{i \geq 1}$ is a low-discrepancy sequence on $[0,1]^n$. If we replace in Step 1.1 of the Algorithm (3) the random points $x_i, i = \overline{1,K}$, with the low-discrepancy sequence $(z_i)_{i \geq 1} = (z_i^{(1)}, \ldots, z_i^{(n)})_{i \geq 1}$ on $[0,1]^n$, and the points $x_i, i = \overline{1,K}$, from formula (23) with

$$(v_i)_{i \geq 1} = (\Psi_1^{-1}(z_i^{(1)}), \ldots, \Psi_n^{-1}(z_i^{(n)}))_{i \geq 1},$$

then we obtain a QMC Algorithm.

During our experiments, we employed as low-discrepancy sequences on $[0,1]^n$ the Halton sequences (see [2] and [5]).

The Mixed MC and QMC method gives the following estimate:

$$\hat{I}_K^{MIX} = \frac{1}{K} \sum_{i=1}^{K} g(q_i^{(1)}, \ldots, q_i^{(d)}, z_i^{(d+1)}, \ldots, z_i^{(n)}), \tag{25}$$

where $(m_i)_{i\geq 1} = (q_i, z_i)_{i\geq 1}$ is an n-dimensional *mixed sequence* on $[0,1]^n$ (see [7]). First, we generate a low-discrepancy sequence $(q_i)_{i\geq 1}$, on $[0,1]^d$, then we generate the independent and identically distributed random points $z_i,\ i \geq 1$, on $[0,1]^{n-d}$. Finally, we concatenate q_i and z_i, for each $i \geq 1$, and we get our mixed sequence on $[0,1]^n$.

In our experiments, we used as low-discrepancy sequences on $[0,1]^d$ for the mixed sequences, the Halton sequences (see [2] and [5]).

If we replace in Step 1.1 of the Algorithm (3) the random points $x_i, i = \overline{1,K}$, with the mixed sequence $(m_i)_{i\geq 1} = (q_i, z_i)_{i\geq 1}$ on $[0,1]^n$, and the points $x_i, i = \overline{1,K}$, from formula (23) with

$$(v_i)_{i\geq 1} = (\Psi_1^{-1}(q_i^{(1)}), \Psi_d^{-1}(q_i^{(d)}), \Psi_{d+1}^{-1}(z_i^{(d+1)}) \ldots, \Psi_n^{-1}(z_i^{(n)}))_{i\geq 1},$$

then we obtain a Mixed MC and QMC algorithm.

3. Application of VaR to portfolios from Bucharest Stock Exchange

In this section, we determine Value at Risk for two portfolios of stocks from Bucharest Stock Exchange. First, we estimate the market model parameters vectors: the drift vector μ and the volatility vector σ. Then, we estimate the correlation matrix C. All the estimations are obtained based on the log-returns series calculated as follows. For each stock $S_i, i = \overline{1,n}$, the j-th entry of the return serie R_i, $i = \overline{1,n}$, is

$$R_i^j = \frac{\log\left(\frac{S_i(t_{j+1})}{S_i(t_j)}\right)}{t_{j+1} - t_j}, \quad j = \overline{1, M-1}, \tag{26}$$

where M is the number of observations of each of the stock price series.

The data used for our estimations are the stock prices from 15.08.2007 until 8.02.2008. The closing prices for each stock are on daily base and can be obtained freely from the internet site of the Bucharest Stock Exchange, *www.bvb.ro*.

The "true" value for VaR is obtained from a long MC simulation of 200000 paths for the stock processes in our market model. As initial price for each stock path simulation, we consider the closing price from the day 8.02.2008. The VaR is calculated over a horizont time of 1 day, respectively 10 days, with $\alpha = 0.01$.

We denote by VaR^{MC}, VaR^{QMC} and VaR^{MIX} the outputs of our MC, QMC and Mixed MC and QMC algorithms, respectively. The estimations VaR^{MC} and VaR^{MIX} are calculated as an average of m simulation runs

$$\overline{VaR}^{MC(MIX)} = \frac{1}{m}\sum_{i=1}^{m} VaR_i^{MC(MIX)}. \tag{27}$$

We also give the sample standard deviation

$$\overline{s} = \Big(\frac{1}{m-1}\sum_{i=1}^{m}\big(VaR_i^{MC(MIX)} - \overline{VaR}^{MC(MIX)}\big)^2\Big)^{\frac{1}{2}}, \tag{28}$$

where $VaR_i^{MC(MIX)}$ represents the estimate from run i, $i = \overline{1,m}$. We use the sample standard deviation to analyze the variance reduction effects. We fix the number of independent runs to $m = 10$.

3.1. VaR estimation for Portfolio 1

We assume that Portfolio 1, denoted by Π_1, contains the stocks of two companies: BANCA TRANSILVANIA S.A. (Symbol TLV) and BRD - Groupe Societe Generale S.A. (Symbol BRD), two of the most liquid companies of the Bucharest Stock Exchange market. We hold 150 shares of each company. The parameters of the stock market model are estimated using Matlab built-in functions and are given bellow:

i	1(TLV)	2(BRD)
$S_i(0)$	0.89	28.20
μ_i	0.0016	0.0036
σ_i	0.0200	0.0235

Table 1: Parameters of Portfolio Π_1.

The estimated correlation matrix is

$$C = \begin{pmatrix} 1 & 0.6964 \\ 0.6964 & 1 \end{pmatrix}.$$

We consider $d = 1$ for the Mixed estimate (25).

The results of our simulations are compared in the following two tables, in terms of their relative errors and standard deviation.

Portfolio Π_1	True Value	**214.8091**	
Simulations	K=10000	K=15000	K=20000
Relative Error VaR^{MC}	0.0007	0.0016	0.0020
Std^{MC}	3.0979	2.6084	2.4380
Relative Error VaR^{MIX}	0.0067	0.0046	0.0031
Std^{MIX}	3.0796	2.6976	1.3741
Relative Error VaR^{QMC}	0.0152	0.0159	0.0122

Table 2: 1-day VaR simulation results.

Portfolio Π_1	True Value	**568.2147**	
Simulations	K=10000	K=15000	K=20000
Relative Error VaR^{MC}	0.0079	0.0071	0.0017
Std^{MC}	10.1200	9.5613	8.7312
Relative Error VaR^{MIX}	0.0046	0.0001	0.0081
Std^{MIX}	10.8072	7.5239	7.0556
Relative Error VaR^{QMC}	0.0110	0.0110	0.0076

Table 3: 10-day VaR simulation results.

We see that in all methods, the sample standard deviation Std decreases, as K increases from 10000 to 20000.

3.2. VaR estimation for Portfolio 2

We assume that Portfolio 2, denoted by Π_2, contains the stocks of 5 companies: BANCA TRANSILVANIA S.A. (Symbol TLV), BRD - GROUPE SOCIETE GENERALE S.A. (Symbol BRD), ROMPETROL RAFINARE S.A. (Symbol RRC), PETROM S.A. (Symbol SNP) and C.N.T.E.E. TRANSELECTRICA (Symbol TEL). We hold 100 shares of each company. The parameters of the stock market model are estimated using Matlab built-in functions and are as follows:

i	1(TLV)	2(BRD)	3(RRC)	4(SNP)	5(TEL)
$S_i(0)$	0.89	28.20	0.093	0.515	42.50
μ_i	0.0016	0.0036	0.0008	0.0024	0.0035
σ_i	0.0200	0.0235	0.0328	0.0232	0.0261

Table 4: Parameters of Portfolio Π_2.

The estimated correlation matrix is

$$C = \begin{pmatrix} 1 & 0.6964 & 0.4709 & 0.6928 & 0.6137 \\ 0.6964 & 1 & 0.4325 & 0.5069 & 0.7634 \\ 0.4709 & 0.4325 & 1 & 0.4977 & 0.3826 \\ 0.6928 & 0.5069 & 0.4977 & 1 & 0.3982 \\ 0.6137 & 0.7634 & 0.3826 & 0.3982 & 1 \end{pmatrix}.$$

We consider $d = 3$ for the Mixed estimate (25).

The results of our simulations are compared in the following two tables, in terms of their relative errors and standard deviation.

Portfolio Π_2	True Value	**369.8088**	
Simulations	K=10000	K=15000	K=20000
Relative Error VaR^{MC}	0.0064	0.0014	0.0049
Std^{MC}	5.8898	3.7585	4.1561
Relative Error VaR^{MIX}	0.0025	0.0043	0.0026
Std^{MIX}	3.9055	4.3919	2.7017
Relative Error VaR^{QMC}	0.0087	0.0042	0.0007

Table 5: 1-day VaR simulation results.

Portfolio Π_2	True Value	**1022.2**	
Simulations	K=10000	K=15000	K=20000
Relative Error VaR^{MC}	0.0029	0.0008	0.0019
Std^{MC}	22.6326	13.4508	7.6167
Relative Error VaR^{MIX}	0.0036	0.0005	0.0018
Std^{MIX}	13.5871	11.7398	4.9724
Relative Error VaR^{QMC}	0.0085	0.0031	0.0029

Table 6: 10-day VaR simulation results.

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