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ON NEIGHBORHOODS OF CERTAIN CLASSES OF P-VALENT FUNCTIONS WITH NEGATIVE COEFFICIENTS DEFINED BY A NEW EXTENDED GENERALIZED DERIVATIVE OPERATOR

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Abstract. In this paper, we give the extended definition of the generalized derivative operator $\mathrm{I}^{\mathrm{m}}(\lambda_1, \lambda_2, \mathrm{l}, \mathrm{n})f(\mathrm{z})$ for new classes of p-valent functions by means of the extended operator. In addition, important properties of the neighborhoods of the defined class are presented.

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1. Introduction

Geometric function theory (GFT) (which is considered as a branch of complex analysis) was started around the 20^{th} century and it is still an active research field. The geometric properties studies of analytical complex functions are included in the main aims of GFT, in addition to that GFT is applied widely in many mathematical fields. Recently, a huge progress has been remarked in GFT and its applications.

Many advanced concepts which are closely related to this specific field were introduced owing to the studies on neighborhoods properties. Following the highly appreciated efforts of Goodman[34] and Ruscheweyh [38], defined the (n,δ)-neighborhoods by [16],[39] ,[40].

After that (n, ρ)-neighborhoods properties were extended and applied to families of analytically $p-$ functions by Altintas, et al.[16], and families of meromorphically $p-$ functions by Liu and Srivastava [41]. Then by using the same strategy, other authors solved this type of problems via several subclasses, see from [21] to [38].

The various studies done on neighborhoods and properties by many authors due to its significance , see [10],[11] and [13].

We first denote by K the class of analytic functions in the open unit disk

$$\mathbb{U} = \{z \in C : |z| < 1\},$$

which have the following form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \quad (z \in \mathbb{U}) \quad . \tag{1}$$

Let A denote the subclass of K consisting of functions $f(z)$ in $\mathbb{U}$ and satisfy the normalization condition:

$$f\,(0) = f\,(0) - 1 = 0.$$

For two functions $f(z)$ given by (1) and $g(z)$ given by

$$g\,(z) = z + \sum_{k=2}^{\infty} b_k z^k, z \in U,$$

the convolution (or, equivalently, the Hadamard product) is defined below

$$(f * g)\,(z) = z + \sum_{k=2}^{\infty} a_k b_k z^k.$$

Let $(x)_k$ denote the Pochhammer symbol defined by

$$(x)_k = \begin{cases} 1 \quad\quad for \quad k = 0, \\ x(x+1)(x+2)...(x+k-1) \quad\quad for \quad k \in N = \{1, 2, 3, ...\}. \end{cases}$$

Using the Hadamarad product, Amer and Darus [1] and [2] had introduced a new generalized derivative operator $\mathrm{I^m}(\lambda_1, \lambda_2, \mathrm{l}, \mathrm{n})\mathrm{f}\,(\mathrm{z})$ As follows:

For $m, n \in \mathrm{N_0}{=}\mathrm{N}\bigcup\{0\}$, $\lambda_2 \geq \lambda_1 \geq 0, l \geq 0$. Let

$$\phi^m\,(\lambda_1, \lambda_2, l)\,(z) = z + \sum_{k=2}^{\infty} \frac{(1 + \lambda_1\,(k-1) + l)^{m-1}}{(1+l)^{m-1}(1 + \lambda_2(k-1))^m}\; z^k.$$

We define the generalized derivative operator $I^m(\lambda_1, \lambda_2, l, n)f\,(z) : A \to A$ by:

$$I^m\,(\lambda_1, \lambda_2, l, n)\,f\,(z) = \phi^m\,(\lambda_1, \lambda_2, l)\,(z) * R^n f\,(z)$$

$$= z + \sum_{k=2}^{\infty} \frac{(1 + \lambda_1\,(k-1) + l)^{m-1}}{(1+l)^{m-1}(1 + \lambda_2\,(k-1))^m}\; c\,(n, k)\;\; a_k z^k.$$

Here $c(n,k) = \frac{(n+1)_{k-1}}{(1)_{k-1}}$, and $R^n f(z)$ is the Ruscheweyh derivative operator given by

$$R^n f(z) = z + \sum_{k=2}^{\infty} c(n,k)\ a_k z^k.$$

Let $A(p)$ denote the class of functions of the form:

$$f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k, \qquad (p \in \mathrm{N}),$$

which are analytic multivalent (p-valent) in the open unit disk $\mathbb{U}$.

Now, we extended the generalized derivative operator $I^m(\lambda_1, \lambda_2, l, n) f(z)$ for a class $A(p)$ as follows:

Definition 1. *For a function $f(z) \in A(p)$ we define the extended generalized derivative operator $I_p^m(\lambda_1, \lambda_2, l, n) f(z)$ by*

$$I_p^m(\lambda_1, \lambda_2, l, n) f(z) = z^p + \sum_{k=p+1}^{\infty} \frac{(p + \lambda_1(k-p) + l)^{m-1}}{(p+l)^{m-1}(1 + \lambda_2(k-p))^m}\ c(n,k)\ a_k z^k. \quad (2)$$

Where $c(n,k) = z^p + \sum_{k=p+1}^{\infty} \frac{\Gamma(k+n)}{(k-p)!\Gamma(p+n)} z^k$, *and* $m, n \in \mathrm{N}_0, p \in \mathrm{N}$.

Special cases of this operator includes:

- $\mathrm{I}_1^{\mathrm{m}}(\lambda_1, \lambda_2, \mathrm{l}, \mathrm{n}) \mathrm{f}(\mathrm{z}) = \mathrm{I}^{\mathrm{m}}(\lambda_1, \lambda_2, \mathrm{l}, \mathrm{n}) \mathrm{f}(\mathrm{z})$,*defined by Amer and Darus[14].*
- $\mathrm{I}_1^{\mathrm{m}+1}(0,0,0,\mathrm{n}) \mathrm{f}(\mathrm{z}) = \mathrm{R}^{\mathrm{n}} \mathrm{f}(\mathrm{z})$, *the Ruscheweyh derivative operator [4].*
- $\mathrm{I}_1^{\mathrm{m}+1}(1,0,0,0) \mathrm{f}(\mathrm{z}) = \mathrm{S}^{\mathrm{n}} \mathrm{f}(\mathrm{z})$, *the Salagean derivative operator [5].*
- $\mathrm{I}^2(\lambda_1, 0, 0, \mathrm{n}) \mathrm{f}(\mathrm{z}) = \mathrm{R}_\lambda^{\mathrm{n}} \mathrm{f}(\mathrm{z})$, *the generalized Ruscheweyh derivative operator [6].*
- $\mathrm{I}^{\mathrm{m}+1}(\lambda_1, 0, 0, 0) \mathrm{f}(\mathrm{z}) = \mathrm{S}_\beta^{\mathrm{n}} \mathrm{f}(\mathrm{z})$, *the generalized Salagean derivative operator introduced by Al-aboudi [9].*
- $\mathrm{I}^{\mathrm{m}}(\lambda_1, 0, \mathrm{l}, \mathrm{n}) \mathrm{f}(\mathrm{z}) = \mathrm{I}^{\mathrm{m}}(\lambda, \beta, \mathrm{l}) \mathrm{f}(\mathrm{z})$, *defined by Catas [15].*
- $\mathrm{I}_1^{\mathrm{m}}(\lambda_1, \lambda_2, 0, \mathrm{n}) \mathrm{f}(\mathrm{z}) = \mu_{\lambda_1, \lambda_2}^{\mathrm{n,m}} \mathrm{f}(\mathrm{z})$, *defined by AL-abbadi and Darus[7].*
- $\mathrm{I}_{\mathrm{p}}^{\mathrm{m}+1}(1,0,0,0) \mathrm{f}(\mathrm{z}) = \mathrm{S}_{\mathrm{p}}^{\mathrm{n}} \mathrm{f}(\mathrm{z})$, *defined by Eker and Seker [17].*
- $\mathrm{I}_{\mathrm{p}}^{\mathrm{m}+1}(0,0,0,\mathrm{m}+\mathrm{p}-1) \mathrm{f}(\mathrm{z}) = \mathrm{D}^{\mathrm{n}+\mathrm{p}-1} \mathrm{f}(\mathrm{z})$, *defined by Kumar and Shukla [18].*
- $\mathrm{I}_{\mathrm{p}}^{\mathrm{m}+1}(\lambda_1, 0, 0, 0) \mathrm{f}(\mathrm{z}) = \mathrm{D}_{\delta,\mathrm{p}}^{\mathrm{n}} \mathrm{f}(\mathrm{z})$, *introduced by Bulut[19].*
- $\mathrm{I}_{\mathrm{p}}^{\mathrm{m}}(\lambda_1, 0, \mathrm{l}, 0) \mathrm{f}(\mathrm{z}) = \mathrm{I}_{\mathrm{p}}^{\mathrm{n}}(\lambda, \mathrm{l}) \mathrm{f}(\mathrm{z})$, *defined by Catas [20].*

Additionally, we let $J(\mathrm{p})$ denote the subclass of $A(\mathrm{p})$, which consists of functions $f(z)$ with the following power-series expansion:

$$f(z) = z^p - \sum_{k=p+1}^{\infty} a_k z^k, (a_k \geq 0, \quad p \in \mathrm{N}). \tag{3}$$

Let $f(z)$ be given by (3), then from (2) we get

$$I_p^m(\lambda_1, \lambda_2, l, n) f(z) = z^p - \sum_{k=p+1}^{\infty} \frac{(p + \lambda_1 (k-p) + l)^{m-1}}{(p+l)^{m-1}(1 + \lambda_2 (k-p))^m} c(n,k) \ a_k z^k \ , \tag{4}$$

where $c(n,k) = z^p - \sum_{k=p+1}^{\infty} \frac{\Gamma(k+n)}{(k-p)!\Gamma(p+n)} z^k$, and $m, n \in \mathrm{N}_0 \ , p \in \mathrm{N}$.

Definition 2. *A function $f(z) \in J(p)$ belongs to a class $S_{\mathrm{p}}^{\mathrm{m}}(\vartheta,\beta,\gamma,\varphi)$ if and only if*

$$\left| \frac{(I_p^m(\lambda_1, \lambda_2, l, n) f(z))' - pz^{p-1}}{\vartheta(I_p^m(\lambda_1, \lambda_2, l, n) f(z))' + (\beta - \gamma)} \right| < \varphi \ . \tag{5}$$

For $0 \leq \lambda_2 \geq \lambda_1 \leq 1$, $0 \leq \vartheta < 1$, $0 \leq \gamma < 1$, $0 < \varphi < 1$, $0 < \beta \leq 1, p \in \mathrm{N}$ and $I_p^m(\lambda_1, \lambda_2, l, n) f(z)$ as in (4).

Definition 3. *A functionf $\in J(p)$ belongs to a class $S_p^{m,\eta}(\vartheta, \beta, \gamma, \varphi)$ if there exists a functiong $(z) \in S_p^m(\vartheta, \beta, \gamma, \varphi)$ such that*

$$\left| \frac{f(z)}{g(z)} - 1 \right| < 1 - \eta; \qquad z \in \mathbb{U}, \ 0 \leq \eta < 1.$$

Definition 4. *For a function $f \in J(p)$,we define the $\delta-$neighborhood by*

$$N_{p,\delta}(f) = \left\{ g : g \in T(n), g(z) = z^p - \sum_{k=p+1}^{\infty} b_k z^k and \sum_{k=p+1}^{\infty} k |a_k - b_k| \leq \delta \right\}.$$

For $e(z) = z^p$, we have

$$N_{p,\delta}(e) = \left\{ g : g \in T(n), g(z) = z^p - \sum_{k=p+1}^{\infty} b_k z^k and \sum_{k=p+1}^{\infty} k |b_k| \leq \delta \right\}.$$

2. The coefficient inequality of the class $S_p^m(\vartheta,\beta,\gamma,\varphi)$.

Firstly, we give the following theorem related to coefficient inequality:

Theorem 1. *A function $f\in J(p)$ is in a class $S_p^m(\vartheta,\beta,\gamma,\varphi)$ if and only if*

$$\sum_{k=p+1}^{\infty} k\left[\frac{(p+\lambda_1(k-p)+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2(k-p))^m}\ \frac{\Gamma(k+n)}{(k-p)!\Gamma(p+n)}\right](1+\varphi\vartheta)\,a_k\le\varphi(\vartheta p+\beta-\gamma).\tag{6}$$

For

$0\le\lambda_2\ge\lambda_1\le 1,\quad 0\le\vartheta<1,\quad 0\le\gamma<1,\quad 0<\varphi<1,\quad 0<\beta\le 1,\quad p\in\mathrm{N}.$

Proof:
Suppose that $f\in S_p^m(\vartheta,\beta,\gamma,\varphi)$, then from inequality (5), we get

$$\left|\frac{\left(I_p^m(\lambda_1,\lambda_2,l,n)f(z)\right)'-pz^{p-1}}{\vartheta\left(I_p^m(\lambda_1,\lambda_2,l,n)f(z)\right)'+(\beta-\gamma)}\right|=$$

$$\left|\frac{pz^{p-1}-\sum_{k=p+1}^{\infty}k\left[\frac{(p+\lambda_1(k-p)+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2(k-p))^m}\ \frac{\Gamma(k+n)}{(k-p)!\Gamma(p+n)}\right]a_kz^{k-1}-pz^{p-1}}{\vartheta\left(pz^{p-1}-\sum_{k=p+1}^{\infty}k\left[\frac{(p+\lambda_1(k-p)+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2(k-p))^m}\ \frac{\Gamma(k+n)}{(k-p)!\Gamma(p+n)}\right]a_kz^{k-1}\right)+(\beta-\gamma)}\right|=$$

$$\left|\frac{\sum_{k=p+1}^{\infty}k\left[\frac{(p+\lambda_1(k-p)+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2(k-p))^m}\ \frac{\Gamma(k+n)}{(k-p)!\Gamma(p+n)}\right]a_kz^{k-1}}{\vartheta(pz^{p-1}-\sum_{k=p+1}^{\infty}k\left[\frac{(p+\lambda_1(k-p)+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2(k-p))^m}\ \frac{\Gamma(k+n)}{(k-p)!\Gamma(p+n)}\right]a_kz^{k-1})+(\beta-\gamma)}\right|<\varphi,$$

it is well known that $\mathtt{R}(z)\le|z|$, therefore we obtain

$$\mathtt{R}\left\{\frac{\sum_{k=p+1}^{\infty}k\left[\frac{(p+\lambda_1(k-p)+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2(k-p))^m}\ \frac{\Gamma(k+n)}{(k-p)!\Gamma(p+n)}\right]a_kz^{k-1}}{\vartheta(pz^{p-1}-\sum_{k=p+1}^{\infty}k\left[\frac{(p+\lambda_1(k-p)+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2(k-p))^m}\ \frac{\Gamma(k+n)}{(k-p)!\Gamma(p+n)}\right]a_kz^{k-1}-pz^{p-1})+(\beta-\gamma)}\right\}<\varphi.$$

If we choose z real and let $\mathrm{z}\to 1^-$, then we get

$$\sum_{k=p+1}^{\infty}k\left[\frac{(p+\lambda_1(k-p)+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2(k-p))^m}\ \frac{\Gamma(k+n)}{(k-p)!\Gamma(p+n)}\right]a_k$$

$$\le\varphi\left\{\vartheta\left(p-\sum_{k=p+1}^{\infty}k\left[\frac{(p+\lambda_1(k-p)+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2(k-p))^m}\ \frac{\Gamma(k+n)}{(k-p)!\Gamma(p+n)}\right]a_k\right)+(\beta-\gamma)\right\},$$

and thus the inequality (6).

On the other hand, suppose that the inequality (6) holds true and let $|z| = 1$.

Then from inequality (5), we get

$$\left|(I_p^m(\lambda_1,\lambda_2,l,n)f(z))' - pz^{p-1}\right| - \varphi\left|\vartheta(I_p^m(\lambda_1,\lambda_2,l,n)f(z))' + (\beta-\gamma)\right| \le$$

$$\sum_{k=p+1}^{\infty} k\left[\frac{(p+\lambda_1(k-p)+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2(k-p))^m}\;\frac{\Gamma(k+n)}{(k-p)!\Gamma(p+n)}\right]a_k|z|^{k-1}$$

$$-\varphi(\vartheta p+\beta-\gamma) + \varphi\vartheta\sum_{k=p+1}^{\infty} k\left[\frac{(p+\lambda_1(k-p)+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2(k-p))^m}\;\frac{\Gamma(k+n)}{(k-p)!\Gamma(p+n)}\right]a_k|z|^{k-1},$$

$$= \sum_{k=p+1}^{\infty} k\left[\frac{(p+\lambda_1(k-p)+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2(k-p))^m}\;\frac{\Gamma(k+n)}{(k-p)!\Gamma(p+n)}\right]a_k|z|^{k-1}\;(1+\varphi\vartheta)a_k\; - \varphi(\vartheta p+\beta-\gamma) \le 0.$$

By maximum modulus theorem, $f \in S_p^m(\vartheta,\beta,\gamma,\varphi)$.

Corollary 2. *If* $f \in S_p^m(\vartheta,\beta,\gamma,\varphi)$, *then*

$$a_{p+1} \le \frac{\varphi(\vartheta p+\beta-\gamma)}{(p+1)(1+\varphi\vartheta)\left[\frac{(p+\lambda_1+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2)^m}\;\frac{\Gamma(n+p+1)}{\Gamma(p+n)}\right]}\quad .$$

3. Neighborhood properties for $S_p^m(\vartheta,\beta,\gamma,\varphi)$ and $S_p^{m,\eta}(\vartheta,\beta,\gamma,\varphi)$.

Theorem 3. *If* $g(z) \in S_p^m(\vartheta,\beta,\gamma,\varphi)$ *and*

$$\delta = \frac{\varphi(\vartheta p+\beta-\gamma)}{(1+\varphi\vartheta)\left[\frac{(p+\lambda_1+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2)^m}\;\frac{\Gamma(n+p+1)}{\Gamma(p+n)}\right]}$$

Then $S_p^m(\vartheta,\beta,\gamma,\varphi) \subset N_{p,\delta}(g)$.

Proof:
Suppose that $f(z) \in S_p^m(\vartheta,\beta,\gamma,\varphi)$ and $f(z)$ is given by (3). Then, Theorem 1 immediately yields

$$(p+1)(1+\varphi\vartheta)\left[\frac{(p+\lambda_1+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2)^m}\;\frac{\Gamma(n+p+1)}{\Gamma(p+n)}\right]\sum_{k=p+1}^{\infty} a_k \le \varphi(\vartheta p+\beta-\gamma). \tag{7}$$

And so

$$\sum_{k=p+1}^{\infty} a_k \leq \frac{\varphi(\vartheta p+\beta-\gamma)}{(p+1)(1+\varphi\vartheta)\left[\frac{(p+\lambda_1+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2)^m}\ \frac{\Gamma(n+p+1)}{\Gamma(p+n)}\right]}. \tag{8}$$

Now subsisting (8) into (7) and using (6), we get

$$\sum_{k=p+1}^{\infty} ka_k \leq \frac{\varphi\left(\vartheta p+\beta-\gamma\right)}{(1+\varphi\vartheta)\left[\frac{(p+\lambda_1+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2)^m}\ \frac{\Gamma(n+p+1)}{\Gamma(p+n)}\right]} \quad .$$

That is

$$\sum_{k=p+1}^{\infty} ka_k \leq \frac{\varphi\left(\vartheta p+\beta-\gamma\right)}{(1+\varphi\vartheta)\left[\frac{(p+\lambda_1+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2)^m}\ \frac{\Gamma(n+p+1)}{\Gamma(p+n)}\right]} = \delta.$$

Thus, from (4) $f(z) \in N_{p,\delta}(g)$.

Theorem 4. *If* $g(z) \in S_p^{m,\eta}(\vartheta,\beta,\gamma,\varphi)$, *and*

$$\vartheta = 1-\frac{\delta}{p+1}\frac{(p+1)\left[\frac{(p+\lambda_1+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2)^m}\ \frac{\Gamma(n+p+1)}{\Gamma(p+n)}\right](1+\varphi\vartheta)}{(p+1)\left(1+\varphi\vartheta\right)\left[\frac{(p+\lambda_1+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2)^m}\ \frac{\Gamma(n+p+1)}{\Gamma(p+n)}\right]-\varphi\left(?p+\beta-\gamma\right)} \quad , \tag{13}$$

then $N_{p,\delta}(g) \subset S_p^{m,\eta}(\vartheta,\beta,\gamma,\varphi)$.

Proof.
Suppose that $f(z) \in N_{p,\delta}(g)$, then by Definition (3), we have

$$\sum_{k=p+1}^{\infty} k\left|a_k-b_k\right| \leq \delta,$$

which readily implies the coefficient inequality given by

$$\sum_{k=p+1}^{\infty} \left|a_k-b_k\right| \leq \frac{\delta}{p+1} \qquad (p\in \mathrm{N}) \ . \tag{9}$$

Now, since $g(z) \in S_p^m(\vartheta,\beta,\gamma,\varphi)$ we have from inequality (6) that

$$\sum_{k=p+1}^{\infty} b_k \leq \frac{\varphi(\vartheta p+\beta-\gamma)}{(p+1)(1+\varphi\vartheta)\left[\frac{(p+\lambda_1+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2)^m}\ \frac{\Gamma(n+p+1)}{\Gamma(p+n)}\right]}. \tag{10}$$

Therefore, from the definition of the class, we have

$$\left|\frac{f(z)}{g(z)}-1\right|<\frac{\sum_{k=p+1}^{\infty}|a_k-b_k|}{1-\sum_{k=p+1}^{\infty}b_k}. \tag{11}$$

Finally, subsisting (9),(10) in (11), we get

$$\left|\frac{f\left(z\right)}{g\left(z\right)}-1\right|\leq\frac{\delta}{p+1}\frac{(p+1)\left[\frac{(p+\lambda_1+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2)^m}\ \frac{\Gamma(n+p+1)}{\Gamma(p+n)}\right](1+\varphi\vartheta)}{(p+1)\left(1+\varphi\vartheta\right)\left[\frac{(p+\lambda_1+l)^{m-1}}{(p+l)^{m-1}(1+\lambda_2)^m}\ \frac{\Gamma(n+p+1)}{\Gamma(p+n)}\right]-\varphi\left(\vartheta p+\beta-\gamma\right)}=1-\eta\ .$$

Conclusion:
In this article, a generalized extended derivative operator has been defined successfully. Then relying on such definition, we presented generalization of classes of multivalent functions in open unit disk. Furthermore, coefficient inequities and important properties of neighborhoods for such classes have concluded. There are a lot of research papers related to study integral operator and differential operator . Those interested in studying it can view [14], [42] , [43] , [44] ,[45] and [46].

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