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NEW SHARP EMBEDDING THEOREMS ON TUBULAR DOMAINS FOR A^p_α TYPE SPACES RELATED WITH TRACE FUNCTION

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Abstract. We provide new sharp embeddig theorem in Bergman type spaces in tubular domains over symmetric cones related with so called diagonal map. Related embeddings with a certain condition on Bergman kernel will be also provided.

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1. Introduction

Let T_λ be a tube domain over a cone $z_0 \in (0,1)$ we denote by $B(z_0, r)$ the Bergman an ball of center z_0 and radius r. We denote by dV the normalized Lebeques measure on T_λ.. Let $T_\lambda = V + i\lambda$ be the tube domains over an irreducible symmetric cone λ in the complexification V^C of an n-d dimensional Eucledean space V. We denote the rank of the cone λ by r and by Δ the determinant function on V. Letting $V \subset \mathbb{R}^n$. We have as an example of a symmetric cone on $\mathbb{R}^n$ the Lorentz cone Λ_n defined for $n \geq 1$. by $\Lambda_n = \left\{ y \in \mathbb{R}^n : y_1^2 - \cdots - y_n^2 > 0, y_1 > 0 \right\}$.

$H(T_\lambda)$ denotes the spaces of all holomorfic functions on T_λ. Also denote m product of tubes by $T^m_\lambda : T^m_\lambda = T_\lambda \times \cdots \times T_\lambda; m \in \mathbb{N}$.

Let $\{a_k\}$ be r-lattice in T_λ (see [1], [2]). For $\tau \in \mathbb{R}_+$ and associated determinant function $\Delta(x)$ we set

$$A^\infty_\tau = \left\{ f \in H(T\lambda) : ||f||_{A^\infty_\tau} = \sup_{x+iy \in T_\lambda} |f(x+iy)|(\Delta(Imy))^\tau < \infty \right\}.$$

It can be checked that this is a Banach space. Let further $\Delta_k = B(a_k, r)$ be Bergman ball and $\{a_k\}$ – r-lattice.

For $1 \leq p, q \leq \infty, \nu \in \mathbb{R}$ and $\nu > \frac{n}{r} - 1$ we denote by $(A^{p,q}_\nu)(T_\lambda)$ the mixed norm weighted Bergman space consisting of analytic functions f in T_λ so that

$$||f||_{(A^{p,q}_{\nu})} = \left(\int \lambda \left(\int\limits_{V} \|f(x+iy)|^p dx \right)^{q/p} \cdot \frac{\Delta^{\nu}(y)}{\Delta^{n/r}(y)} dy \right))^{1/q} < \infty.$$

This is also Banach space. It is known the $(A^{p,q}_{\nu})(T_{\lambda})$ space is non empty. if and only if $\nu > \frac{n}{r} - 1$ and we will assume this when $p = q$ we write $A^{p,q}_{\nu}(T_{\lambda}) = A^{p}_{\nu}(T_{\lambda})$.

When weighted Bergman projection P_{ν} is the orthogonal projection from the Hilbert space $L^2_{\nu}(T_{\lambda})$ onto its closed subspace $A^2_{\nu}(T_{\lambda})$ and is given by the following integral formula

$$(P_{\nu} f)(z) = C_{\nu} \int\limits_{T_{\lambda}} B_{\nu}(z,w) f(w) \Delta^{\nu - \frac{n}{r}}(v) du dv,$$

where

$$B_{\nu}(z,w) = \Delta^{-(\nu + \frac{n}{r})} \left(\frac{z - \bar{w}}{i} \right)$$

is a weighted Bergman kernel for $A^2_{\nu}(T_{\lambda})$. We denote by Δ^{+}_{k} enlarged Bergman ball $B(a_k, r)$ (see [1], [2]).

Let further for $m > 1, H(T^m_{\lambda})$ be the spaces of all analytic functions on $T^m_{\lambda} = T_{\lambda} \times \cdots \times T_{\lambda}, m \in \mathbb{N}$.

Let also further

$$A^p_s = \left\{ f \in H(T^m_{\lambda}) : \int\limits_{T_{\lambda}} \cdots \int\limits_{T_{\lambda}} |f(z_1, \ldots, z_m)|^p \prod_{j=1}^{m} \Delta^{s_j - \frac{n}{r}}(Imz) dv(z) < \infty \right\},$$

$s_j > \frac{n}{r} - 1, s = (s_1, \ldots, s_m), 0 < p < \infty, j = 1, \ldots, m.$

Let further $L^p(T_{\lambda}, \lambda\mu)$ be a space of all measurable functions on T_{λ}, so that

$$\int\limits_{T_{\lambda}} |f(z)|^p d\mu(z) < \infty$$

for any positive measure μ on tubular domain $T_{\lambda}, 0 < p < \infty$.

Let $\{a_k\}$ be lattice in T_{λ},. We put $|\Delta_k| = V(B(a_k, r)), \Delta_k = B(a_k, r), k = 1, 2, \ldots$

One of the goals of this paper is to consider new trace map in the trace problem related with embedding theorems for the case of the tubular domains are symmetric cones, it is a map

$$T_r(z) = f(z, ..., z), z \in T_\lambda, f \in X \subset O(T_\lambda^m)$$

for a certain quazinormed analytic space X on T_λ^m.

On traces of analytic functions in tubular domains are symmetric cones. This type of maps (diagonal map) previously considered by various authors in particular cases where $D = U$ (unit disk) and when $D = B$ (unit ball) (see, for example [1], [2]) and various references there applications of this map to various problem in the complex function theory are also known (see [1],[2], and references there). We give precise definitions.

Let $H(T_\lambda^m)$ be the space of all analytic functions in $T_\lambda^m : T_\lambda^m = T_\lambda \times \cdots \times T_\lambda; m \in \mathbb{N}$.

We say trace $X = Y$ of X is a quazinormed subspace of $H(T_\lambda^m)$ and Y is certain fixed quazinormed subspace of $H(T_\lambda^m)$, if for each $F, F \in X, F(z, \ldots, z) = f(z), f \in Y$, and the reverse is also true, for each $g, g \in Y$, there is a function $F, F \in X$, such that $F(z, \ldots, z) = g(z)$; for all $z \in T_\lambda$.

We denote various constants in this paper by c, C, C_1.

2. Main results

Theorem 1. *Let $0 < p < \infty, 0 < q_i, \sigma_i < \infty, \alpha_j > -1, j = 1, \ldots, m$, $q_i \le p, i = 1, \ldots, m$. Let μ be a positive measure on T_λ. Then the following two conditions are equivalent.*

For any analytic functions $f(z_1, \ldots, z_m)$

1) On $T_\lambda^{(m)}$ that splits into a product of analytic functions $fi(z_i) \in H(T_\lambda)$, $i = 1, \ldots, m$, i.e. $f(z_1, \ldots, z_m) = \left(\prod_{j=1}^m\right) f_j(z_j); z_j \in T_\lambda, j = 1, \ldots, m$ we have that

$$\left(\int_{T_\lambda} |T_r f(z)|^p d\mu(z)\right)^{1/p} \le \left(\prod_{j=1}^m\right)\left(\int_{T_\lambda}\left(\int_{B(w,z)} |f_i(z)|^{\sigma_i} \cdot \Delta^{\alpha_i}(Imz) dv(z)\right)^{q_i/\sigma_i} dv(w)\right)^{1/q_i} < C;$$

2) The positive Borel measure μ satisfies the following Carleson type condition

$$\mu(\tilde{B}(a_k, r)) \le \tilde{\tilde{C}} \left[\Delta(Ima_k)\right]^{p\sum_{i=1}^m\left(\frac{\frac{2n}{r}+\alpha_i}{\sigma_i}+\frac{\frac{2n}{r}}{q_i}\right)}; k \ge 1.$$

Theorem 2. *Let $0 < p < \infty, s_j > -1, j = 1, \ldots, m$. Let μ be a positive Borel measure on T_λ. Then the following are equivalent:*

1) $\mu(\Delta_k) \leq C|\Delta_k|^{\frac{2n}{r}m+\sum_1^m(s_j)}; k \geq 0;$
2) $Trace(A^p_{\vec{s}}) \subset L^p(T_\lambda, \lambda_\mu);$
3) $\int\limits_{T_\lambda} \prod\limits_{j=1}^m |f_j(z)|^p d\mu(z) \leq \tilde{C} \prod\limits_{j=1}^m ||f||^p_{A^p_{s_j}}; f \in A^p_{s_j}; 1 \leq j \leq m.$

Proof. Proofs of the Theorems 1-2.

In this section we prove the main to results find of this paper. We find complete characterization of μ Borel positive measure on D so that

$$\int\limits_D |(T_r)f(z)|^p(d\mu(z)) = \int\limits_{T_\lambda} |f(z,\dots,z)|^p d\mu(z) \leq C||f||_{\gamma(T^m_\lambda)},$$

where $0 < p < \infty$ and γ is a certain quazinormed subspace of

$$H(T^m_\lambda) = H(T_\lambda \times \dots \times T_\lambda), m \geq 1$$

.

Note for $m = 1$ we discussed this problem in detail above. For future we use the following notation.

$Q_w = B(w,r), w \in T_\lambda, \{a_k\}$ is a fixed r- lattice in T_λ and

$$\Delta^*_k = |B^*(a_k,R)|, R = \frac{1+r}{2}, \Delta_k = |B(a_k,r)|.$$

Note in our theorems below the following estimate is very important (see for example [1], [2]).

$$\int\limits_{B(z,r)} |B_\tau(\tilde{z},w)|^p(\Delta^\alpha(Imz))dV(z) \leq C|B_{\tilde{\tau}}(w,z)|; \qquad (A)$$

$\tilde{\tau} = \tau p - \alpha - \frac{2n}{r}, 0 < p < \infty, \alpha > -1. \tau > 0, w, z \in T_\lambda$, τ is large enough.
Proof of theorem 1.
Set $a = \frac{2n}{r}$

$$\Theta = p\sum_{i=1}^m \left(\frac{a+\alpha_i}{\sigma_i} + \frac{a}{q_i}\right), \Theta_i = p\left(\frac{a+\alpha_i}{\sigma_i} + \frac{a}{q_i}\right), 1 \leq i \leq m.$$

Using standard covering argument one easily shows that for any positive measurable function $u : u : D \to R$ we have

$$\sum_{k=1}^\infty [\Delta(Ima_k)]^a \times \left(\int\limits_{\Delta^+_k} u(z)dV_\alpha(z)\right)^\beta \asymp \sum_{k=1}^\infty \Delta(Ima_k)^a \times \left(\int\limits_{\Delta_k} u(z)dV_\alpha(z)\right)^\beta; \beta > 0, a > 0.$$

Let us assume (2) holds. Then we have using standard arguments based on r-lattices and their applications to embedding theorem (see, for example, [1], [2]) the following chain of estimates.

$$\int_{T_\lambda} |(T_r)f(z)|^p d\mu(z) = \sum_{k=1}^{\infty} \int_{B(a_k,r)} |(T_r f)(z)|^p d\mu(z) \leq$$

$$\leq \sum_{k=1}^{\infty} \mu(\Delta_k)(\max_{z\in\Delta_k})|(T_r)f(z)|^p \leq C_1 \sum_{k=1}^{\infty}(\Delta(Ima_k)^{\Theta}) \prod_{i=1}^{m}(\max_{z\in B(a_k,r)})|f_i(z)|^{\sigma_i}/)^{p/\sigma_i}$$

$$\leq C_1 \sum_{k=1}^{\infty}\prod_{i=1}^{m}(\Delta(Ima_k)^{\Theta_i})(\max_{z\in B(a_k,r)})|f_i(z)|^{\sigma_i}/)^{p/\sigma_i},$$

where $\Delta_k = B(a_k, r), k = 0, 1, 2, 3, \dots$

$$C_2 \sum_{k=1}^{\infty}\prod_{i=1}^{m} \Delta(Ima_i)^{\Theta_i} \left(\Delta(Ima_k)^{-a-\alpha_i} \int_{\Delta_k^*} |f_i(z)|^{\sigma_i} dv_{\alpha_i}(z) \right)^{p/\sigma_i} =$$

$$= C_2 \sum_{k=1}^{\infty}\prod_{i=1}^{m} \Delta(Ima_i)^{\frac{pa}{q_i}} \left(\int_{\Delta_k^*} |f_i(z)|^{\sigma_i} dv_{\alpha_i}(z) \right)^{p/\sigma_i} = C_2 \sum_{k=1}^{\infty} x_{1,k}^p \dots x_{m,k}^p;$$

$$x_{i,k} = (\Delta(Im(a_k)))^{\frac{a}{q_i}} \cdot \left(\int_{\Delta_k^*} |f_i(z)|^{\sigma_i} dv_{\alpha_i}(z) \right)^{1/\sigma_i}.$$

Using the fact that for $q_i \leq p < \infty, x_{i,k} \geq 0, 1 \leq i \leq m, k \geq 1$

$$\left(\sum_{k=1}^{\infty} x_{1,k}^p \dots x_{m,k}^p \right)^{1/p} \leq \prod_{k=1}^{m} \left(x_{i,1}^{q_i} + x_{i,2}^{q_i} + \dots \right)^{q/q_i},$$

which follows from the special case $q_i = p, i = 1, \dots, m$ which is obviously valid and the formulation of our theorem we have that ($|\Delta_k| = |B(a_k, r)|$).

$$\left(\int_{T_\lambda} |T_r f(z)|^p d\mu(z) \right)^{1/p} \leq C \prod_{i=1}^{m} \left(\sum_{k=1}^{\infty} |\Delta_k|^a \left(\int_{B(a_k,r)} |f_i(z)|^{\sigma_i} dv_{\alpha_i}(z) \right)^{\frac{q_i}{\sigma_i}} \right)^{1/q_i} \leq$$

$$\leq \tilde{C}\prod_{i=1}^{m}\left(\sum_{k=1}^{\infty}|\Delta_k|^{\alpha}\left(\int\limits_{B(a_k,r)}|f_i(z)|^{\sigma_i}dv_{\alpha_i}(z)\right)^{\frac{q_i}{\sigma_i}}\right)^{1/q_i}$$

and then using that

$$\sum_{k=1}^{\infty}|\Delta_k|^{a}\left(\int\limits_{B(a_k,r)}|f(z)|^{\sigma}dv_{\alpha}(z)\right)^{\frac{q}{\sigma}} \leq C\int\limits_{T_\lambda}\left(\int\limits_{B(w,r)}|f(z)|^{\sigma}dv_{\alpha}(z)\right)^{\frac{q}{\sigma}}dv, a=\frac{2n}{r},$$

$f \in H(T_\lambda), \alpha > -1, 0, q, \sigma < \infty$

We get what we need.

To show the reverse we use (A) and we have to use standard test function $|B_{\tau(z,w)}|\delta^{\tilde{\tau}}(w), \tau > 0, \tilde{\tau} > 0$ for some $z, w \in T_\lambda$ then standard arguments (see [1], [2]) and Forelly-Rudin type estimate

$$\int\limits_{T_\lambda}\frac{\Delta^{\alpha}(Imz)dv(z)}{\Delta^{\beta}\left(\frac{z-\bar{w}}{i}\right)} \leq C\Delta^{\alpha-\tilde{\beta}-a}(Imw) \qquad (B)$$

$\alpha-\tilde{\beta}-a<0, w\in T_\lambda, \alpha>-1, \tilde{\beta}>\alpha-a, \tilde{\beta}=\frac{2n}{r}+\beta,$

combined with estimates from below of Bergman ball (see [1], [2]).

Indeed let $\tau, \tilde{\tau}$ be large enough. Then we have putting for $i = 1, \ldots, m, f_i = B_\tau(a_k, z)(\delta^{\tilde{\tau}}(a_k))$

$$\left(\int\limits_{T_\lambda}|T_r f(z)|^p d\mu(z)\right)^{1/p} \geq \left(\int\limits_{B(a_k,r)}|T_r f(z)|^p d\mu(z)\right)^{1/p} \geq C\left(\mu(\Delta_k)^{1/p}\right)\cdot\Delta^{(p\tau+\tilde{\tau}p)(m/p)}(Ima_k);$$

$k = 1, 2, \ldots$ for some fixed $\{a_k\}$ - r-lattice in T_λ (see [1],[2]).

And using (B) we have that

$$\prod_{i=1}^{m}\left(\int\limits_{T_\lambda}\left(\int\limits_{B(w,r)}|f_i(z)|^{\sigma_i}du_{\alpha_i}(z)\right)^{\frac{q_i}{\sigma_i}}du(w)\right)^{1/q_i} \leq C(\delta(a_k))^{\sum\limits_{i=1}^{m}\left(\frac{a+\alpha_i}{\sigma_i}+\frac{a}{q_i}\right)+\tilde{\tilde{C}}},$$

where $\tilde{\tilde{C}} = \left(\frac{m}{p}\right)(p\tau + p\tilde{\tau}), \tilde{\tau}$ - large enough.

We used that

$$\int\limits_{B(w,r)} B_\tau(a_k, z)dv(z) \le B_{\tau-a}(a_k, w), a_k, w \in T_\lambda$$

and that

$$\int\limits_{T_\lambda} B_\lambda(z, w)dv(z) \le C\Delta^\tau(Imz), \lambda = \tau + \frac{2n}{r}, z \in T_\lambda,$$

Theorem 1 is proved.

Remark 1. *Final remarks.*

We can set all our theorem in more general form replacing left side and formulate in a bit general form. We mean expressions

$$\sum_{k=1}^{\infty}\left(\int\limits_{B(a_k,r)} |(T_r f)(z)|^p d\mu(z)\right)^{\frac{q}{p}}; \left(\int\limits_{T_\lambda}\left(\int\limits_{B(\tilde{z},r)} |(T_r)(f)(z)|^p d\mu(z)\right)^{\frac{q}{p}} dv(\tilde{z})\right)$$

We give examples below.

We can show easily that the following estimate is

$$\sum_{k=1}^{\infty}\left(\int\limits_{B(a_k,r)} |f(z)|^p d\mu(z)\right)^{\frac{q}{p}} \le C\left(\int\limits_{T_\lambda} |f(z)|^p d\mu(z)\right)$$

for all $0 < p < \infty$ *and for any positive Borel measure* μ *on* $T\lambda$ *if only* $p/q \le 1$ *based on estimate* $(\sum\limits_{k\ge 0} a_k)^s \le \sum\limits_{k\ge 0} a_k^s; s \le 1$.

So in Theorem 2 we can replace

$$\int\limits_{T_\lambda} \prod_{k=1}^{m} |f_k(z)|^p d\mu(z) \quad by \quad \sum_{k=1}^{\infty}\left(\int\limits_{B(a_k,r)} \prod_{k=1}^{m} |f_k(z)|^p d\mu(z)\right)^{q/p},$$

where $p/q \le 1$*, so get a more general form of Theorem 2.*

We can also replace the A_s^p *function space in this Theorem by* $(A_{s_1}^{p,q})$ *, using known embedding for* $(A_{s_1}^{\tilde{p};\tilde{q}})$ *spaces; with some restrictions on* $\tilde{p}, \tilde{q}, s_1, p, q$ *to get* *another general form of Theorem 2. These results are also sharp and they have similar proofs. In addition we have that.*

In Theorem 1 we can replace similarly

$$\int\limits_{T_\lambda} |(Trf)(z)|^p d\mu(z) \quad by \quad \sum_{k=1}^{\infty} \left(\int\limits_{B(a_k,r)} |Trf)(z)|^p d\mu(z) \right)^{q/p}$$

to get a more general form of Theorem 1 for $(p,q) \leq 1$. The proof of this assertion is very similar to the proof of Theorem 1 and we simply omit details here.

Proof. Proof of Theorem 2.

Let us show 1) $\Rightarrow$ 2). Let use choose $f, f \in A^p_{\vec{s}}$. We have by properties of r-lattice the following chain of estimates for some values of τ_1, τ_2.

$$||(T_\lambda f)||^p_{L^p(\mu)} = \int\limits_{T_\lambda} |f(z,\ldots,z)|^p d\mu(z) = \sum_{k=1}^{\infty} \int\limits_{B(a_k,r)} |f(z,\ldots,z)|^p d\mu(z) \leq$$

$$\leq C \sum_{k=1}^{\infty} \mu(B(a_k,r)) \sup_{z \in B(a_k,r)} ||f(z,\ldots,z)||^p \leq C_0 \sum_{k=1}^{\infty} \mu(B(a_k,r)) (\sup_{\substack{z_1,\ldots,z_m \\ z_j \in B(a_k,r)}}) ||f(z_1,\ldots,z_m)||^p \leq$$

$$\leq C_1 \sum_{k=1}^{\infty} \mu(B(a_k,r)) \left(\frac{\mu(\Delta_k)}{|\Delta_k|^{ma}} \right) |\Delta_k|^{\tau_2} \cdot \int\limits_{\Delta_k^1} \cdots \int\limits_{\Delta_k^\tau} |f(\vec{z})|^p \prod_{j=1}^{m} dv(z_j) \leq$$

$$\leq C_2 \sum_{k=1}^{\infty} \left(\frac{1}{|\Delta_k|^{\tau_1}} \right) \cdot \int\limits_{\Delta_k^1} \cdots \int\limits_{\Delta_k^\tau} |f(z_1,\ldots,z_m)|^p \prod_{j=1}^{m} dv(z_j) \leq C_3 ||f||_{A^p_{\vec{s}}}.$$

The implication (2) $\Rightarrow$ (3) is standard just put $f = \prod\limits_{j=1}^{m} f_j(z_i)$ and note $||f||_{A^p_{\vec{s}}} = \prod\limits_{j=1}^{m} ||f_j||_{A^p_{s_j}}$.

The implication (3) $\Rightarrow$ (1) follows from standard from estimates related with the test function and known vital estimates from below of Bergman kernel on so called Bergman balls (see [1]). Indeed we have by (A), (B)

$$\mu(\Delta_k)|\Delta_k^{\tilde{\delta}}| \leq C|| \prod_{j=1}^{m} f_i||_{L^p(\mu)} \leq \tilde{\tilde{C}} \prod_{j=1}^{m} ||f_i||^p_{A^p_{s_j}} \leq C_3 \delta^\nu(a_k).$$

Note that (see [1]) $|\Delta_k| = \Delta^a(Ima_k.)$. Theorem 2 is proved.

Remark 2. *Final remarks.*

Note that (see [1], [2])

$$|f(w)|^p \leq C \int\limits_{B(w,r)} |f(z)|^p dv(z)(L)$$

for all $0 < p < \infty$*, where* $\Delta(B(w,r)) = (\Delta(Imw))^a; w \in T_\lambda$ *for some constant* C*. So based on this estimatewe can replace on Theorem 1,2.*

$$\left(\int\limits_{T_\lambda} |f|^p d\mu\right) \quad by \quad \int\limits_{T_\lambda} \left(\int\limits_{B(w,r)} |f|^p\right) d\mu(w)(L), 0 < p < \infty,$$

and also $||f||_{A^p_s}$ *replace by expression*

$$\int\limits_{T_\lambda} \left(\int\limits_{B(w,r)} |f|^p dv(w)\right) dv_{\tilde{s}}(z)$$

for some $\tilde{s}, 0 < p < \infty, s > -1, \tilde{s} > -1$.

And as a result new a bit different results can be easily recovered from both Theorems 1,2, where again μ *is any positive Borel measure on* T_λ*. We set easily new (not sharp) results (at least one part necessary or sufficiency). To prove that such results are sharp (or not) probably a separate paper will be needed (and a separate investigation) .*

Note also space that with (norm or quazinorms)

$$\sum_{k=0}^{\infty} \left(\int\limits_{B(a_k,r)} |f(z)|^p dv_\alpha(z)\right)^{q/p} \quad or \quad \int\limits_{T_\lambda} \left(\int\limits_{B(z,r)} |f(z)|^p dv_\alpha(z)\right)^{q/p} dv(z)$$

are called usually Herz type analytic spaces and were considered previously in various papes of the author (see [1], [2]) $0 < p < \infty, \alpha > -1, dv_\alpha(z) = \Delta^\alpha(Imz)dv(z), z \in T_\lambda$.

References

[1] R. F. Shamoyan, O. Mihic, *Embedding theorems for weighted spaces of holomorphic functions in tubular domain*, Romai. J. V.13 (2017), 93–115.

[2] R. F. Shamoyan, O. Mihic, *On embedding theorems for weighted spaces of holomorphic functions in tubular domain*, Boletin de Matematicas. V.25 (1) (2018), 1–11.

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